Session 1: Signals / elements R, L, C

Introduction :
Introduction, Review
Signals
R, L, C

Outline

- Introduction
  - Textbooks, course information
  - Review
- Signals
  - Sinusoidal, unit step, pulse, unit impulse, ramp
- Resistance
  - Linear, non-linear, time dependent, time independent, bilateral, unilateral,
- Capacitance
  - Initial voltage
- Inductance
  - Initial current

Introduction – Text Books

Foundations Of Analog And Digital Electronic Circuits
by Anant Agarwal, Jeffrey H. Lang, Morgan Kaufmann, 2005, 984 pages

Basic Circuit Theory
by Charles A. Desoer, Ernest S. Kuh , McGraw-Hill,
1984, 876 pages

نظریه اساسی مدارها و شبکه ها
ناشر : دانشگاه تهران - ترجمه : دکتر پرویز جبه دار مارالانی

Linear and Nonlinear Circuits
by Leon O.; Desoer, Charles A.; Kuh, Ernest S Chua,
Mcgraw-hill College, 1987

Introduction – Course Information

Course homepage:
http://ee.sharif.edu/~sarvari/Teaching.html
Refresh the page!

Grading (tentative):
Lab (20%) + 2-MidTerms (40%) + HW & Quizzes (5%) + Final (35%)
Note: to pass the course you need to score minimum 50% of (Mid & Final)

LTSPICE

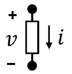
SPICE (Simulation Program with Integrated Circuit Emphasis)
a general-purpose circuit simulator

http://www.linear.com/designtools/software/#LTspice

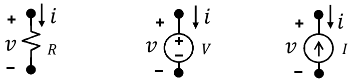
LINEAR TECHNOLOGY

Reminder - definitions

Convention:



Elements with abstract idealization:

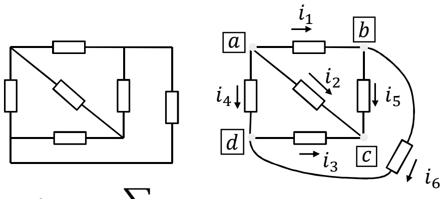


$v = Ri$
$i = Gv$

$v = V \,, \forall i$

$i = -I \,, \forall v$

Reminder – Nodal Analysis



arbitrary current directions!
no voltage!

4 nodes

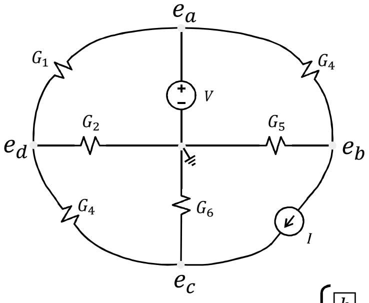
Based on KCL $\sum_{OUT} i = 0$

$\boxed{a}$: $i_1 + i_2 + i_4 = 0$
$\boxed{b}$: $-i_1 + i_5 + i_6 = 0$
$\boxed{c}$: $-i_2 - i_5 - i_3 = 0$
$\boxed{d}$: $-i_4 + i_3 - i_6 = 0$

3- independent eq.

$\sum \quad \rightarrow 0 = 0$

Reminder – Nodal Analysis



$e_a = V$

3- independent eq.
3 unknowns e_b, e_c, e_d

5 nodes

⏚ Reference node, ground

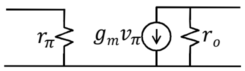
voltages relative to ground
→ KVL considered!

$$\begin{cases} \boxed{b}: & G_4(e_b - V) + G_5(e_b - 0) + I = 0 \\ \boxed{c}: & -I + G_6(e_c) + G_3(e_c - e_b) = 0 \\ \boxed{d}: & G_3(e_d - e_c) + G_2(e_d) + G_1(e_d - V) = 0 \end{cases}$$

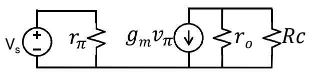
Solving circuit means finding v and i for all 8-elements

Definitions

network **?** circuit

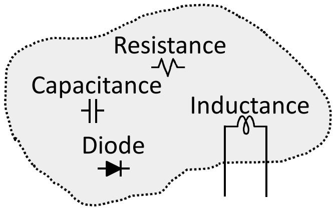


2-port network



amplifier circuit

Circuit elements:



Resistance

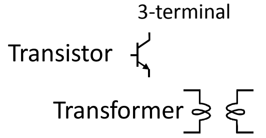
Capacitance

Inductance

Diode

two-terminal, or one-port elements

3-terminal



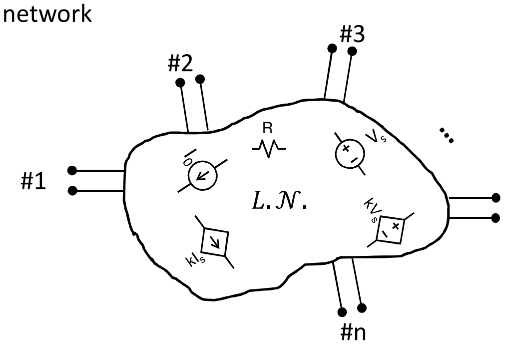
Transistor

Transformer

two-port elements

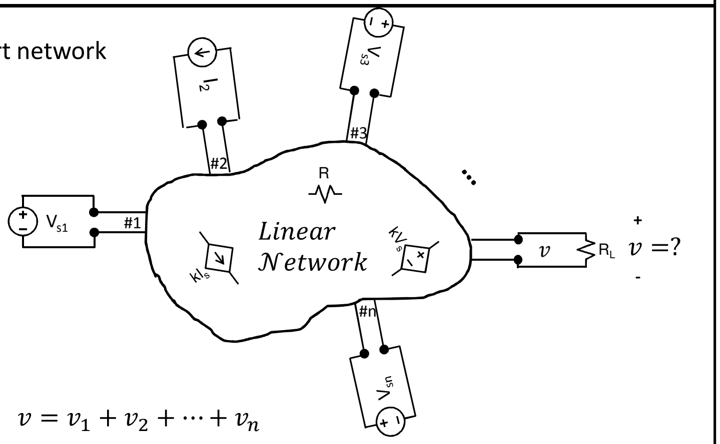
Network

n-port network



Superposition Principle

n-port network

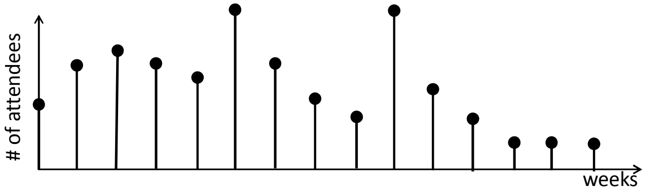


$v = v_1 + v_2 + \cdots + v_n$

signals

Signals

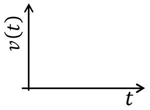
Signal : it is more general than *current* or *voltage* $f(\cdot)$

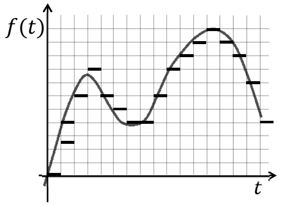


Signals

Signal : it is more general than *current* or *voltage* $f(\cdot)$

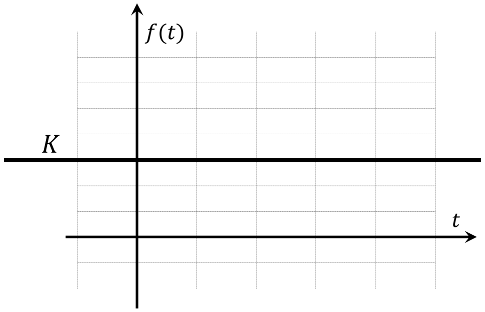
oscilloscope

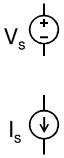




analog signal / continuous signal
digital signal / discrete signal

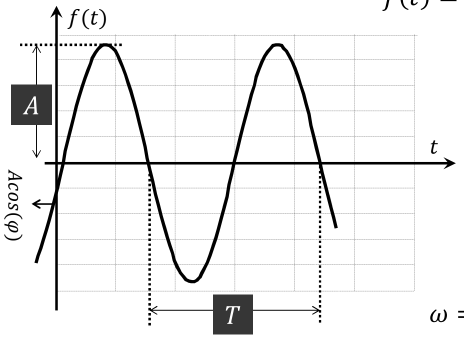
Constant





$f(t) = K$

Sinusoidal



$f(t) = Acos(\omega t + \varphi)$

amplitude

angular frequency (radians per second)

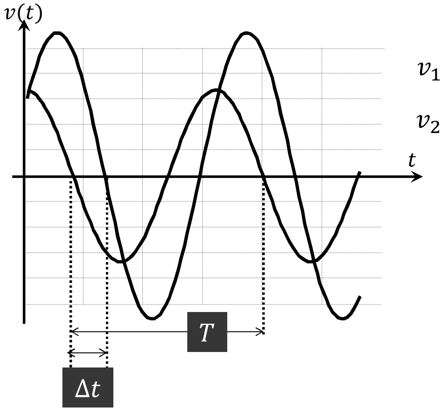
phase (radians)

$\omega = \frac{2\pi}{T} = 2\pi f$

frequency (hertz)=(1/sec)

period (sec)

Sinusoidal



$$v_1(t) = A_1 cos(\omega t + \varphi_1)$$

$$v_2(t) = A_2 cos(\omega t + \varphi_2)$$

$$\Delta\varphi = \varphi_2 - \varphi_1 = \\ = 2\pi\frac{\Delta t}{T} \\ \sim\frac{\pi}{3}$$

Sinusoidal - Fourier series

Why sinusoidal is important?!

Euler's formula: $e^{i\theta} = cos\theta + i\ sin\theta$

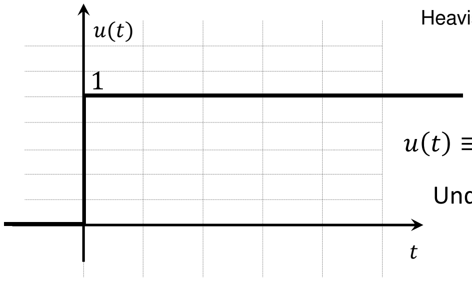
$f =$ periodic function with $T = 2\pi$

Function Generator

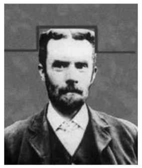
$$f(x) = a_0 + \sum_{n=1}^{\infty}(a_n \cos nx + b_n \sin nx)$$

Animated plot of the first five successive partial Fourier series

Unit Step



Heaviside function

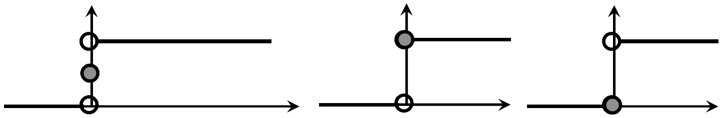


Oliver Heaviside
(850–1925)

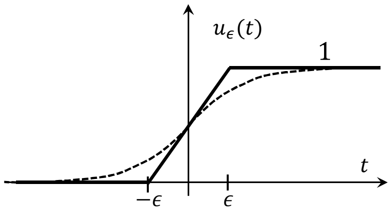
$$u(t) \equiv \begin{cases} 0 & t > 0 \\ 1 & t < 0 \end{cases}$$

Undefined at $t = 0$

continuity and limits



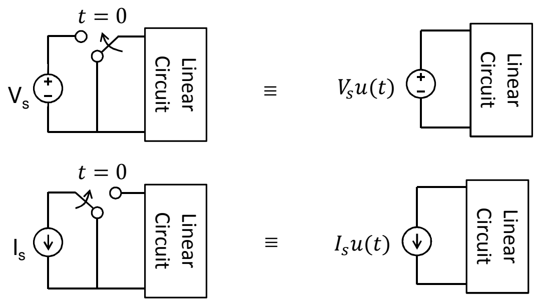
Unit Step



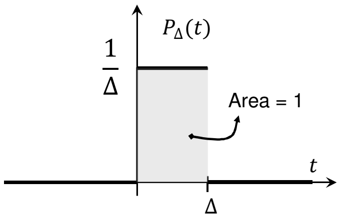
$$u(t) = \lim_{\epsilon \to 0} u_\epsilon(t)$$

$$u(t) = \lim_{k \to 0}\tfrac{1}{2}(1+\tanh kt) = \lim_{k \to 0}\frac{1}{1+e^{-2kt}}$$

Switches Representing by Unit Step



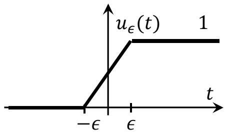
Pulse



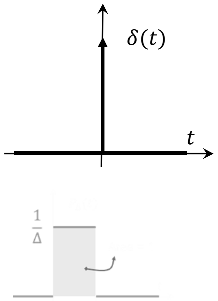
$$P_\Delta(t) \equiv \begin{cases} 0 & t < 0 \\ 1/\Delta & t = 0 \\ 0 & t > 0 \end{cases}$$

$$P_\Delta(t) = \frac{1}{\Delta}\{u(t) - u(t-\Delta)\}$$

Write $P_\Delta(t)$ in terms of $u_\epsilon(t)$



Unit Impulse



Dirac delta function

$$\delta(t) \equiv \begin{cases} 0 & t \neq 0 \\ ?\,(\infty) & t = 0 \end{cases}$$

such that $\forall \xi$: $\int_{-\xi}^{+\xi} \delta(t)dt = 1$

$$\delta(t) = \lim_{\Delta \to 0} P_\Delta(t)$$

Paul Dirac
(1902 –1984)

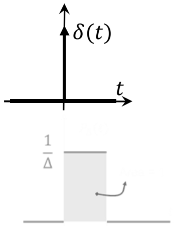
$$\boxed{\begin{aligned} u(t) &= \int_{-\infty}^{t} \delta(\epsilon)d\epsilon \\ \delta(t) &= \frac{du(t)}{dt} \end{aligned}}$$

Unit Impulse

Some properties:

$$\delta(t)x(t) = \delta(t)x(0)$$

$$\delta(at) = \frac{1}{|a|}\delta(t) \qquad \delta(-t) = \delta(t)$$



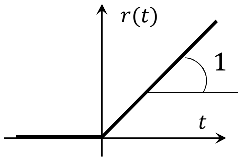
Sampling Property:

$$x(0) = \int_{-\infty}^{+\infty} x(t)\delta(t)dt \;\rightarrow x(t_0) = \int_{-\infty}^{+\infty} x(t)\delta(t-t_0)dt$$

$$\int_{-\infty}^{+\infty} x(t)\delta(t)dt = \int_{-\infty}^{+\infty} x(0)\delta(t)dt = x(0)\int_{-\infty}^{+\infty} \delta(t)dt = x(0)$$

$$\boxed{x(t) = \int_{-\infty}^{+\infty} x(\tau)\delta(\tau - t)d\tau}$$

Unit Ramp



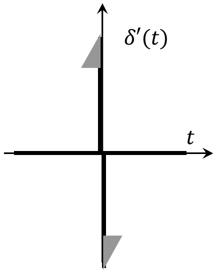
$$r(t) = tu(t)$$

$$u(t) = \int_{-\infty}^{t} \delta(\tau)d\tau \qquad r(t) = \int_{-\infty}^{t} u(\tau)d\tau$$

$$\delta(t) = \frac{du(t)}{dt} \qquad u(t) = \frac{dr(t)}{dt}$$

$$\int_{-\infty}^{t} u(\tau)d\tau = r(t) \qquad \int_{-\infty}^{t} r(\tau)d\tau = \tfrac{1}{2}r(t)^2$$

Unit Doublet



$$\delta(t) \equiv \begin{cases} 0 & t \neq 0 \\ ? & t = 0 \end{cases}$$

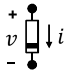
such that $\int_{-\infty}^{t} \delta'(\tau)d\tau = \delta(t)$

$$\delta'(t) = \frac{d\delta}{dt}$$

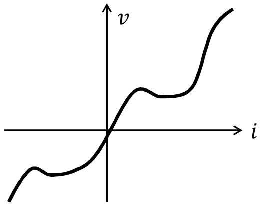
Resistance

Resistance

Definition:



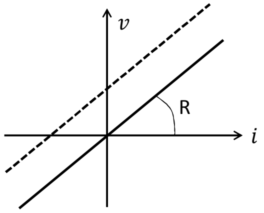
$v(t) \equiv \hat{G}\ (i(v))$ conductance



- TI: Time Independent
- TD: Time Dependent

- L: Linear
- NL: Non-linear

Resistance - LTI



LTI

$\begin{cases}\text{TI: Time Independent}\\ \text{TD: Time Dependent}\end{cases}$

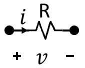
$\begin{cases}\text{L: Linear}\\ \text{NL: Non-linear}\end{cases}$

Resistance

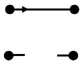
$$\text{LTI:}\ \begin{cases} v(t) = R\ i(t) \\ i(t) = G\ v(t) \end{cases}$$

Conductance

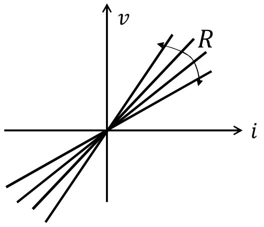
Only mathematical abstraction



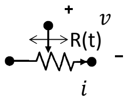
Extreme cases: $\begin{cases} R = 0 & \text{Short circuit} \\ G = 0 & \text{Open circuit} \end{cases}$



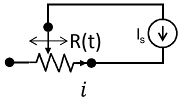
Resistance - LTD



LTD: Linear Time Dependent

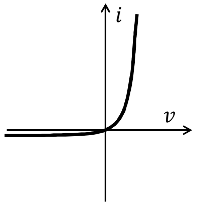


Can be used for modulation

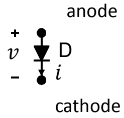


$$\left.\begin{aligned} R(t) &\propto \sin\omega_1 t \\ I_s = i(t) &\propto \sin\omega_2 t \end{aligned}\right\} \to v(t) \propto \sin(\omega_1 \pm \omega_2)t$$

Resistance - NLTI



NLTI: Diode



$$i(t) = I_s\left(e^{qv(t)/nkT} - 1\right)$$

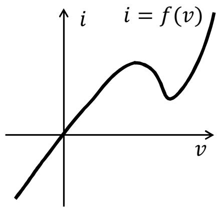
$$V_T = \frac{kT}{q}\bigg|_{300°K} = 26mV$$

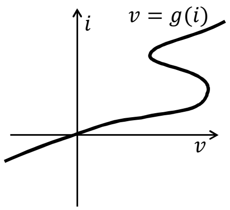
$$i(t) = I_s\left(e^{v(t)/nV_T} - 1\right) \qquad n = 1 \cdots 2$$

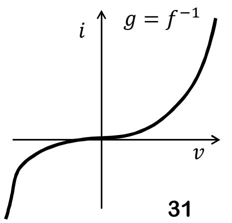
voltage controlled

current controlled

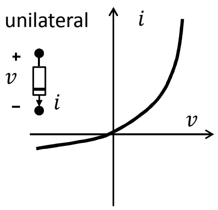
both

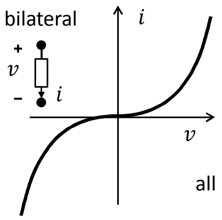






Resistance - NL

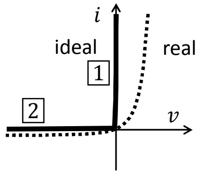




$i = f(v)$

$-i = f(-v)$

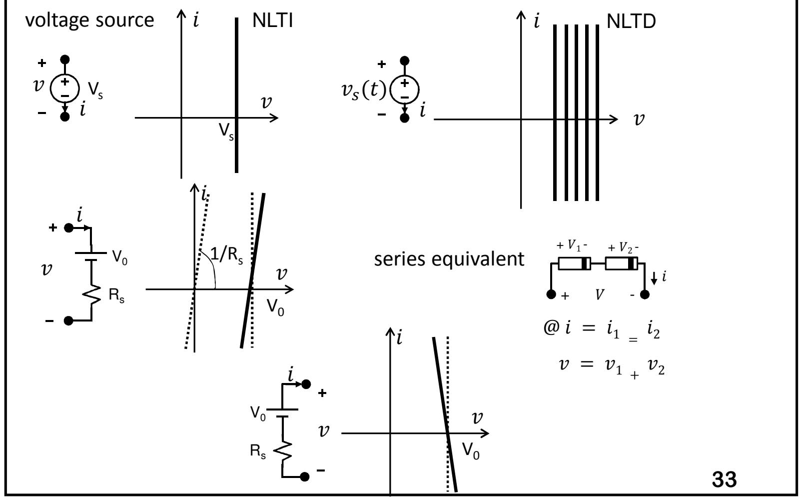
all linear Resistors



Ideal approximation: neither voltage controlled nor current controlled

conditional function $\begin{cases} \boxed{1}\ \text{short circuit ; } v = 0 & \text{if } i \geq 0 \\ \boxed{2}\ \text{open circuit ; } i = 0 & \text{if } v \leq 0 \end{cases}$

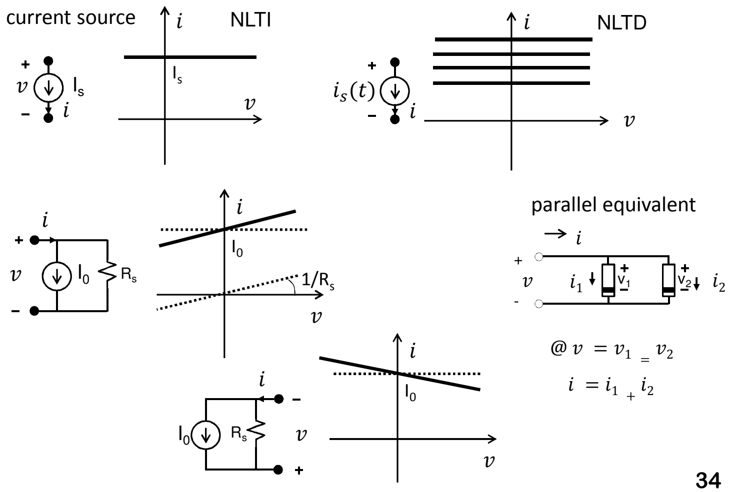
Resistance – NL – Voltage Source



$@\, i = i_1 = i_2$

$v = v_1 + v_2$

Resistance – NL – Current Source



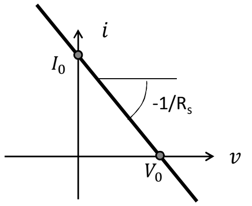
$@\, v = v_1 = v_2$

$i = i_1 + i_2$

Reminder – Thevenin , Norton

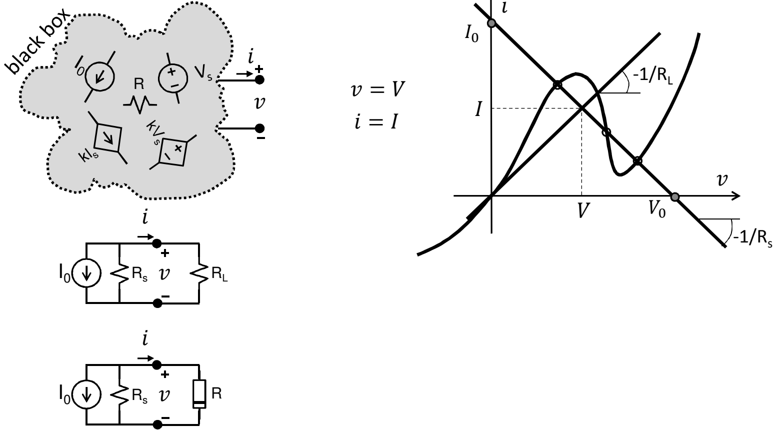
$$R_s = R_{Thevenin} = R_{Norton}$$

$$I_0 = V_0/R_s$$



Linear Circuit

Linear 2-terminal network



$v = V$

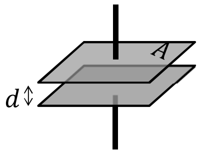
$i = I$

Capacitor

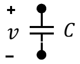
Capacitance

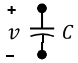
capacitor $q(t) \equiv \hat{C}\,(v(t))$

Stores energy in form of electric filed $\mathcal{E}$

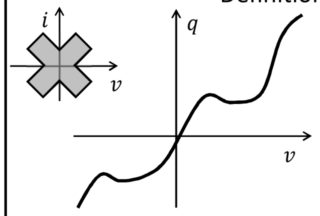


$$C = \epsilon \frac{A}{d}$$



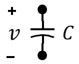


Capacitance



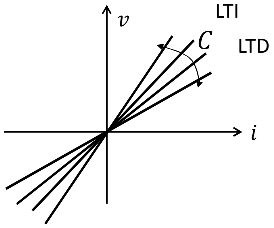
Definition: $q(t) \equiv \hat{C}\,(v(t))$

capacitance



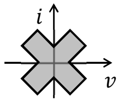
Whatever we said about $i - v$ (resistance) can be repeated here about $q - v$ (capacitance)

$\begin{cases} \text{TI: Time Independent} \\ \text{TD: Time Dependent} \end{cases}$ $\begin{cases} \text{L: Linear} \\ \text{NL: Non-linear} \end{cases}$



application:

Capacitance



LTI $q(t) = Cv(t)$ C: Capacitance [F] [μF]

$$i(t) = \frac{dq}{dt} = C\frac{dv(t)}{dt}$$ memory!

$$\Lsh\ v(t) = v(0) + \frac{1}{C}\int_0^t i(t')dt'$$

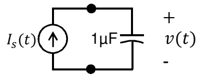
Big **?**: linear or nonlinear

Definition of linear function: $\begin{cases} f(ax) = af(x) \\ f(x_1 + x_2) = f(x_1) + f(x_2) \end{cases}$

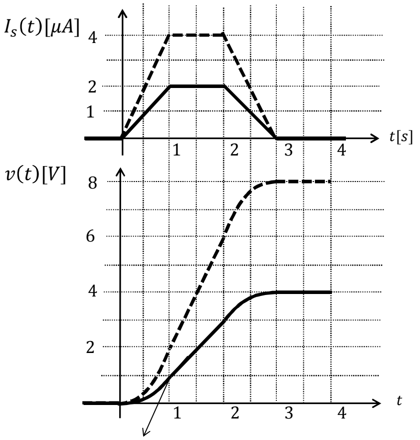
linear only if $v(0) = 0$

$$v(t) = \frac{1}{C}\int_0^t i(t')dt'$$

Capacitance - Example



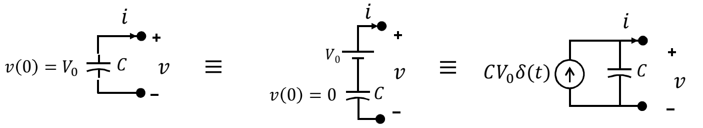
$v(0) = 0$



Slope = 2

Repeat this for $V(0) = 2V$
What is the difference?

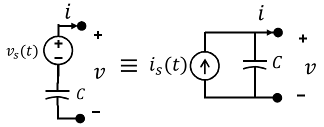
Capacitance - Thevenin , Norton



$$v(t) = \underbrace{v(0)}_{V_0} + \underbrace{\frac{1}{C}\int_0^t i(t')dt'}$$

series

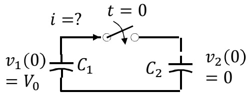
Thevenin / Norton



$$v_s(t) = \frac{1}{C}\int_0^t i_s(t')dt'$$

$$i_s(t) = C\frac{dv_s(t)}{dt}$$

Capacitance - Switches



$v_1(0^-) = V_0$ $\quad v_2(0^-) = 0$

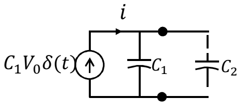
$v_1(0^+) = v_2(0^+) = V = ?$

KCL

$$v(t) = v(0) + \frac{1}{C}\int_0^t i(t')dt' \qquad i_1 = \alpha\delta(t) = -i_2$$

$$v_1(0^+) = v_1(0^-) + \frac{1}{C_1}\int_{0^-}^{0^+} i_1(t')dt' \;= V_0 + \frac{\alpha}{C_1} = V \;= 0 - \frac{\alpha}{C_2}$$

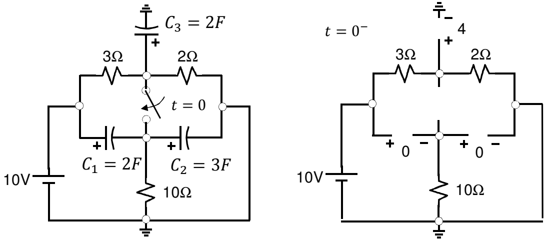
$$\alpha = \frac{-C_1C_2}{C_1 + C_2}V_0 \qquad V = \frac{C_1}{C_1 + C_2}V_0$$

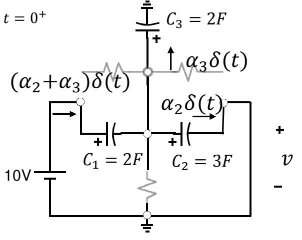


$$v(0^+) = \frac{1}{C_1 + C_2}\int_{0^-}^{0^+} C_1V_0\delta(t)dt'$$

$$= \frac{C_1}{C_1 + C_2}V_0$$

Example





$$v = 0 + \frac{\alpha_2}{3} = 4 + \frac{\alpha_3}{2} = v_2(0^+) = v_3(0^+)$$

$$v_1(0^+) = 10 - v = \frac{\alpha_2 + \alpha_3}{2}$$

$$v_2(0^+) = v_3(0^+) = \frac{28}{7} \qquad v_1(0^+) = \frac{42}{7}$$

Capacitance - LTD

LTD: $q(t) = C(t)v(t)$

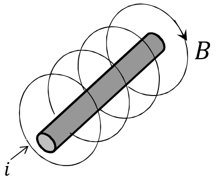
$$i(t) = \frac{dq}{dt}$$

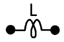
$$= C(t)\frac{dv(t)}{dt} + \frac{dC(t)}{dt}v(t)$$

Inductor

Inductance

Inductor $\varphi(t) \equiv \hat{L}\,(i(t))$

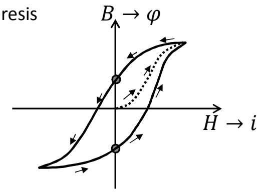




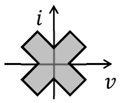
Stores energy in form of magnetic filed B

$$B = \mu n I = \mu\frac{N}{l}I$$
$$L = \mu\frac{N^2}{l}A$$

ferrite core: hysteresis



Inductance



LTI $\varphi(t) = Li(t)$ L: Inductance [H]

$$v(t) = \frac{d\varphi}{dt} = L\frac{di(t)}{dt}$$ memory!

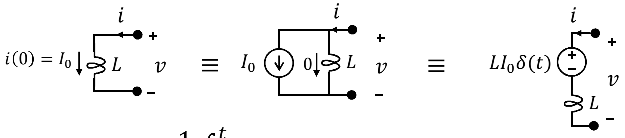
$$\Lsh\; i(t) = i(0) + \frac{1}{L}\int_0^t v(t')dt'$$

linear if $i(0) = 0$

$$i(t) = \frac{1}{L}\int_0^t v(t')dt'$$

built in 1877 by Alfred Apps. Wound with 450 km of wire, could produce a 1,200,000 volts spark

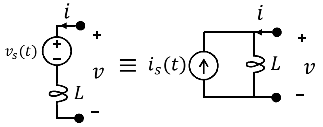
Inductance - Thevenin , Norton



$$i(t) = \underbrace{i(0)}_{I_0} + \underbrace{\frac{1}{L}\int_0^t v(t')dt'}_{}$$

parallel

Thevenin / Norton



$$i_s(t) = \frac{1}{L}\int_0^t v_s(t')dt'$$

$$v_s(t) = L\frac{di_s(t)}{dt}$$

Inductance - LTD

LTD: $\varphi(t) = L(t)i(t)$

$$v(t) = \frac{d\varphi}{dt}$$
$$= L(t)\frac{di(t)}{dt} + \frac{dL(t)}{dt}i(t)$$