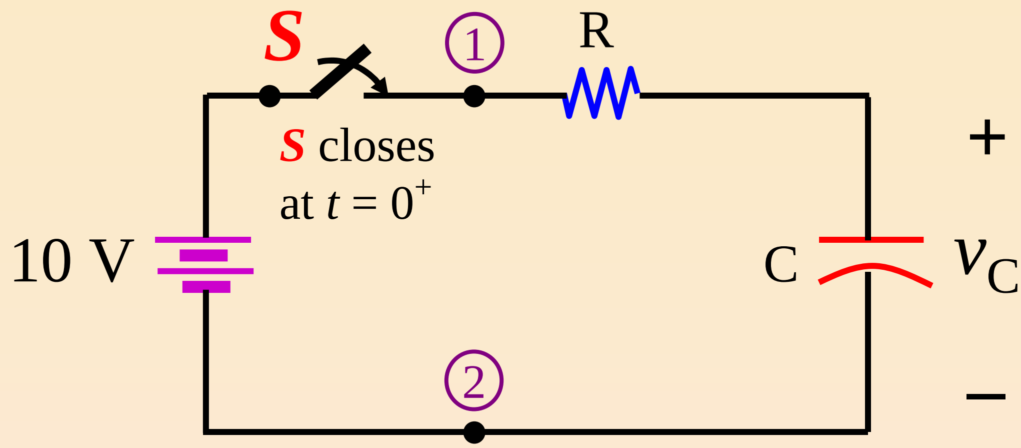
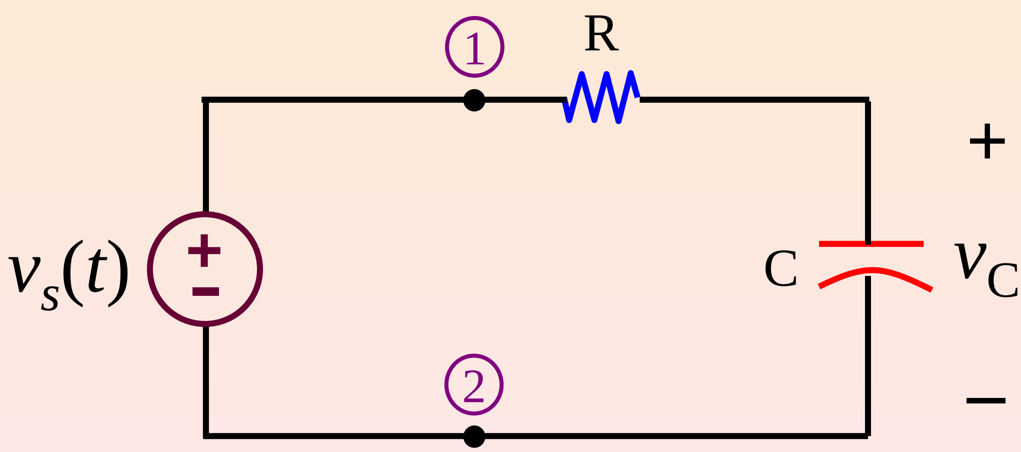
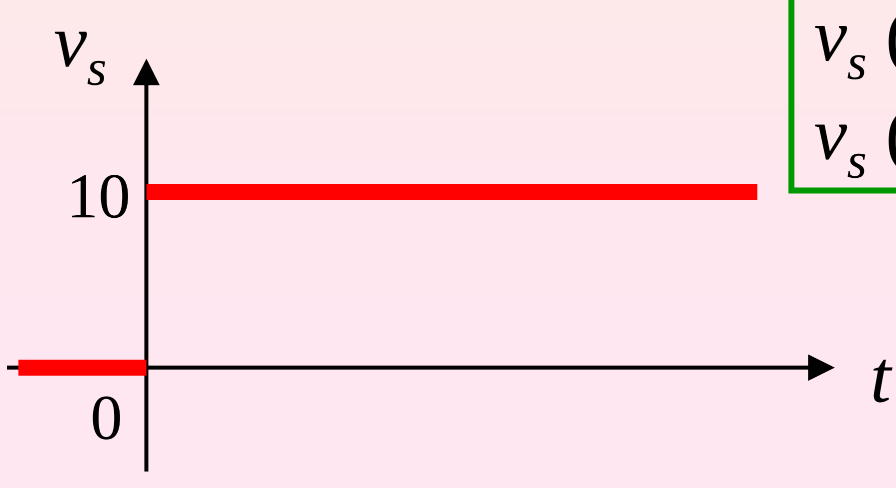
Two Equivalent Representations

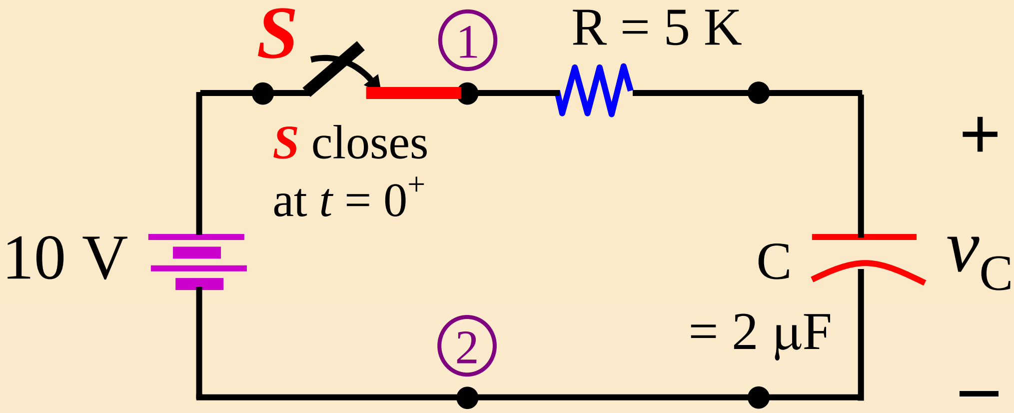




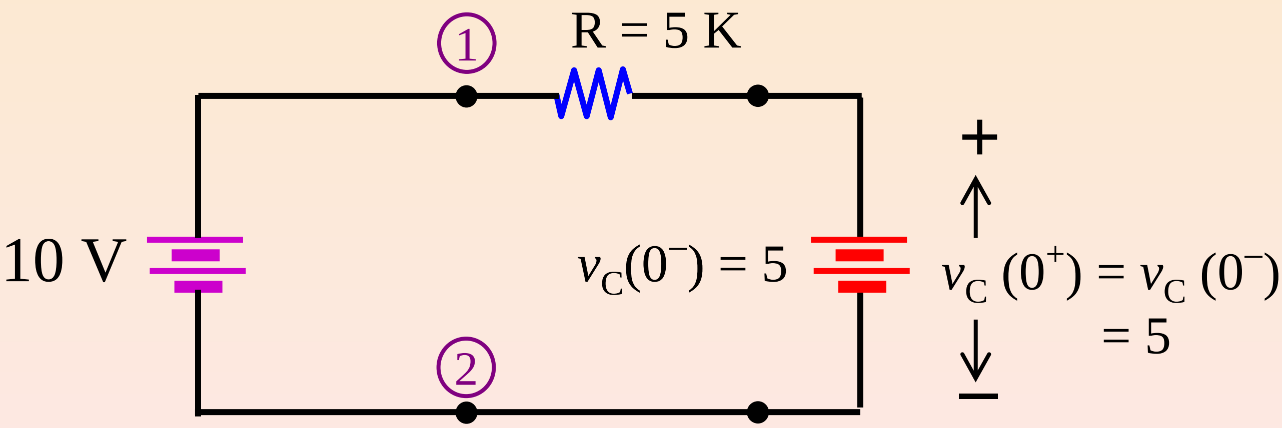


$v_s\,(0^-) = 0$

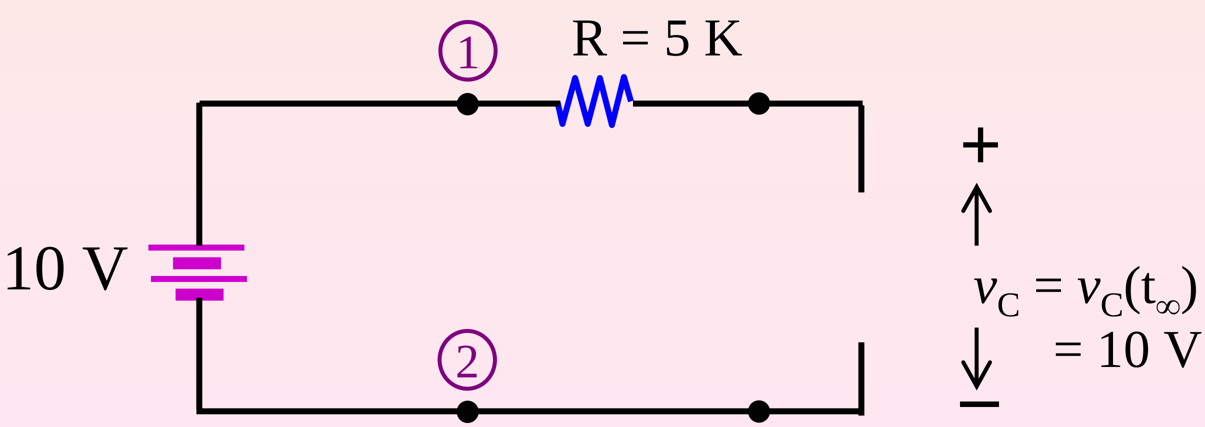
$v_s\,(0^+) = 10$



Step 1 Find $v_C(0^+)$: Replace C by battery with voltage = $v_C(0^-)$, then calculate $v_C(0^+)$.



Step 2 Find $v_C(t_\infty)$: Replace C by *open* circuit, then calculate $v_C = v_C(t_\infty)$.



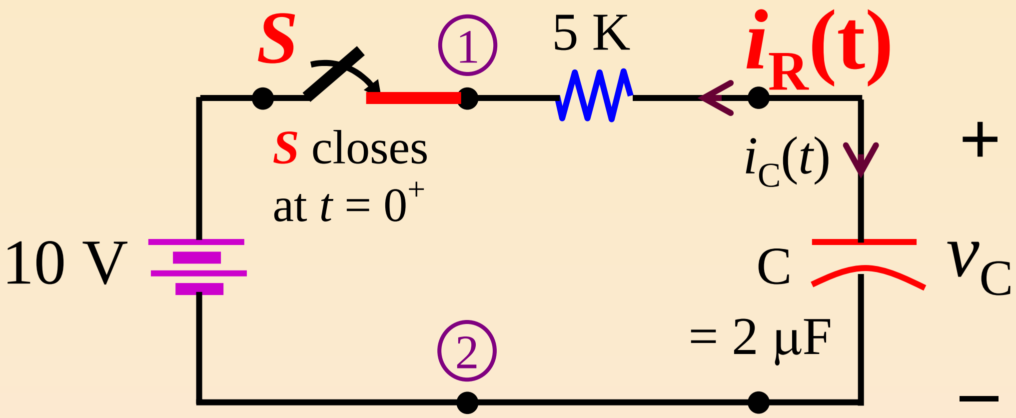
Step 3 Calculate the *time constant*

$$\tau = R\,C$$

Fundamental Behavior of 1st-order Circuits

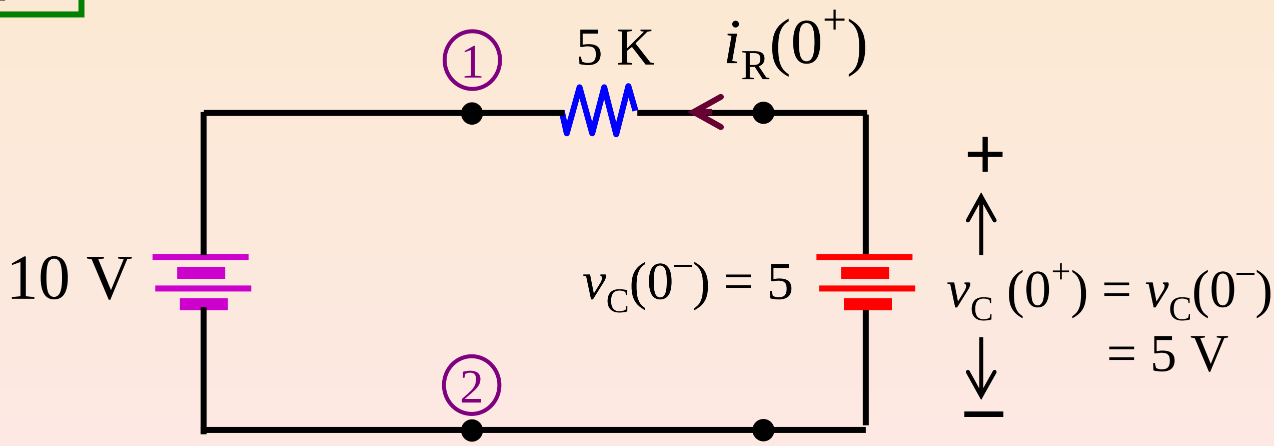
The *voltage* $v_{jk}(t)$ between any pair of nodes (j) and (k), and the *current* $i_o(t)$ at any terminal of any *linear resistive circuit* is always an *exponential waveform*, except for the trivial circuit consisting of a *current source* (resp., *voltage source*) connected across a capacitor (resp., inductor), whose solution is a linear "*ramp*" function.

Problem : Calculate $i_R(t)$ under the same setting.



Step 1

Calculate $i_R(0^+)$: Replace C by a 5 V battery :

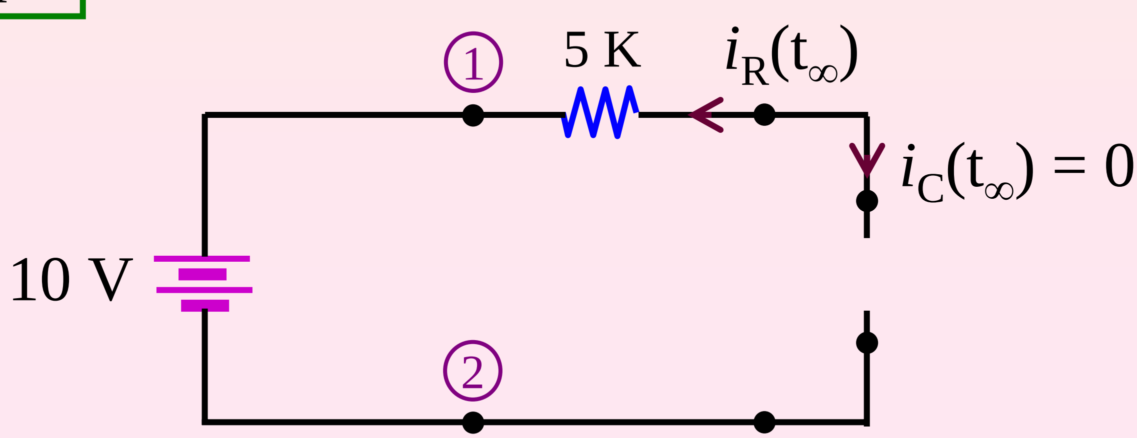


Calculate :

$$i_R(0^+) = \frac{5-10}{5K} = -1mA$$

Step 2

Calculate $i_R(t_\infty)$: Replace C by *open* circuit :

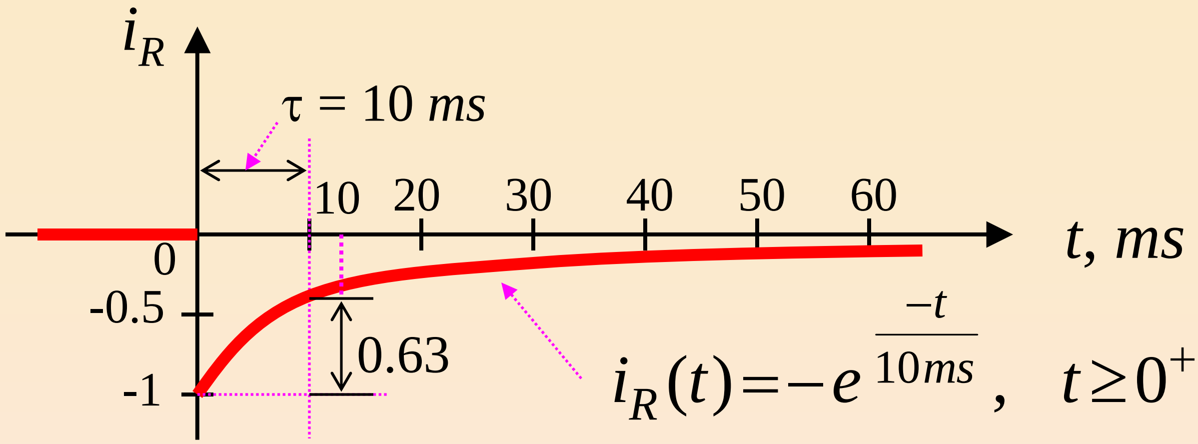


Calculate :

$$i_R(t_\infty) = 0$$

Step 3 Calculate time constant

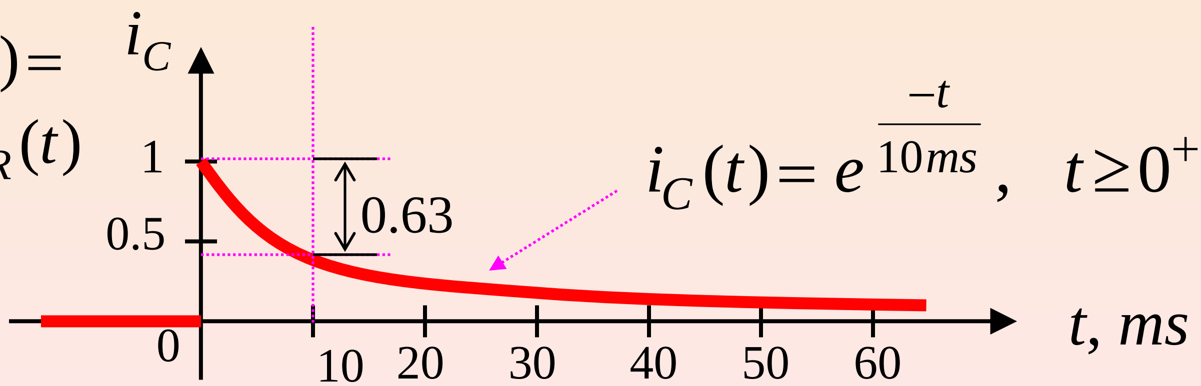
$$\tau = R\,C = (5 \times 10^{3})\,(2 \times 10^{-6}) = 10\ ms$$



$$i_R(t) = -e^{\frac{-t}{10ms}}, \quad t \geq 0^+$$

Verification of solution

$$i_C(t) = -i_R(t)$$



$$i_C(t) = e^{\frac{-t}{10ms}}, \quad t \geq 0^+$$

$$v_C(t) = v_C(0^-) + \frac{1}{C}\int_{0^+}^{t} e^{\frac{-t}{10\times10^{-3}}}dt$$

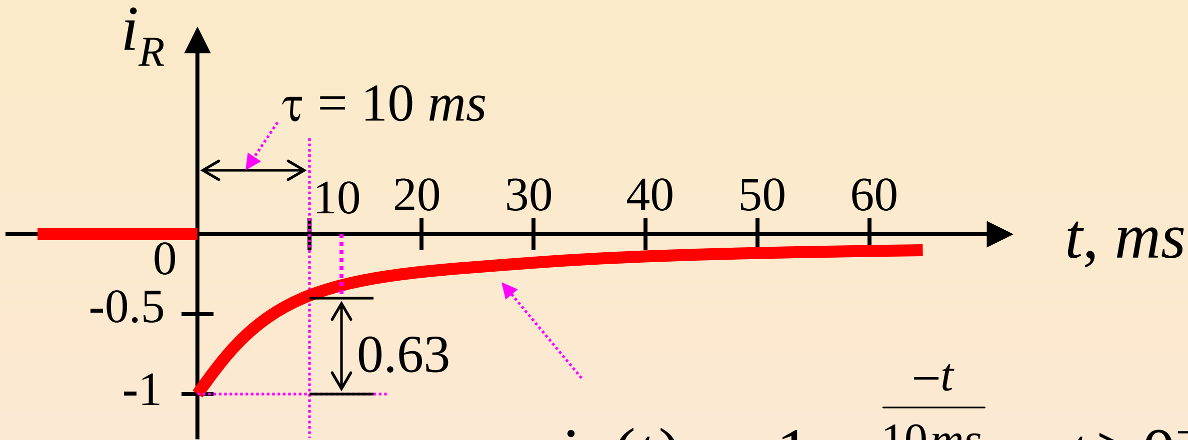
$$= 5 + \frac{1}{2\times10^{-3}}\int_{0^+}^{t} e^{\frac{-t}{10\times10^{-3}}}dt$$

$$= 5 + \left[-5e^{\frac{-t}{10\times10^{-3}}} + 5\right]$$

$$= 10 - 5e^{\frac{-t}{10\times10^{-3}}}, \; t \geq 0^+$$

Step 3 Calculate time constant

$$\tau = R\,C = (5 \times 10^{3})\,(2 \times 10^{-6}) = 10\ ms$$



$$i_R(t) = -1 \bullet e^{\frac{-t}{10ms}}, \quad t \geq 0^{+}$$

Verification

Let us calculate :

$$i_C(t) = -\,i_R(t) = e^{\frac{-t}{10ms}}$$

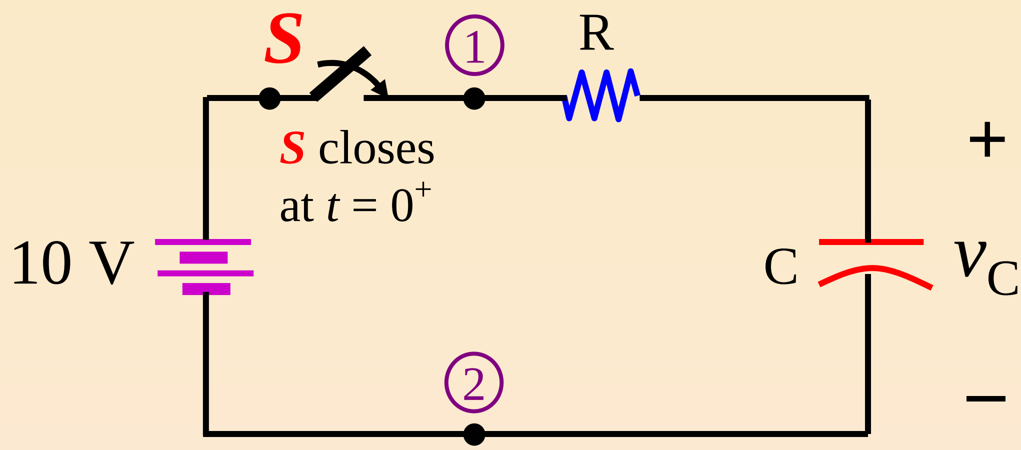
$$v_C(t) = v_C(0^{-}) + \frac{1}{C}\int_{0^{+}}^{t} i_C(\tau)\,d\tau$$

$$= 5 + \frac{1}{2 \times 10^{-3}}\int_{0^{+}}^{t} e^{\frac{-t}{10 \times 10^{-3}}}dt$$

$$= 5 + \left[-5e^{\frac{-t}{10 \times 10^{-3}}} + 5 \right]$$

$$= 10 - 5e^{-\frac{t}{\tau}}, \ t \geq 0^{+}$$

Finding Solutions by Inspection



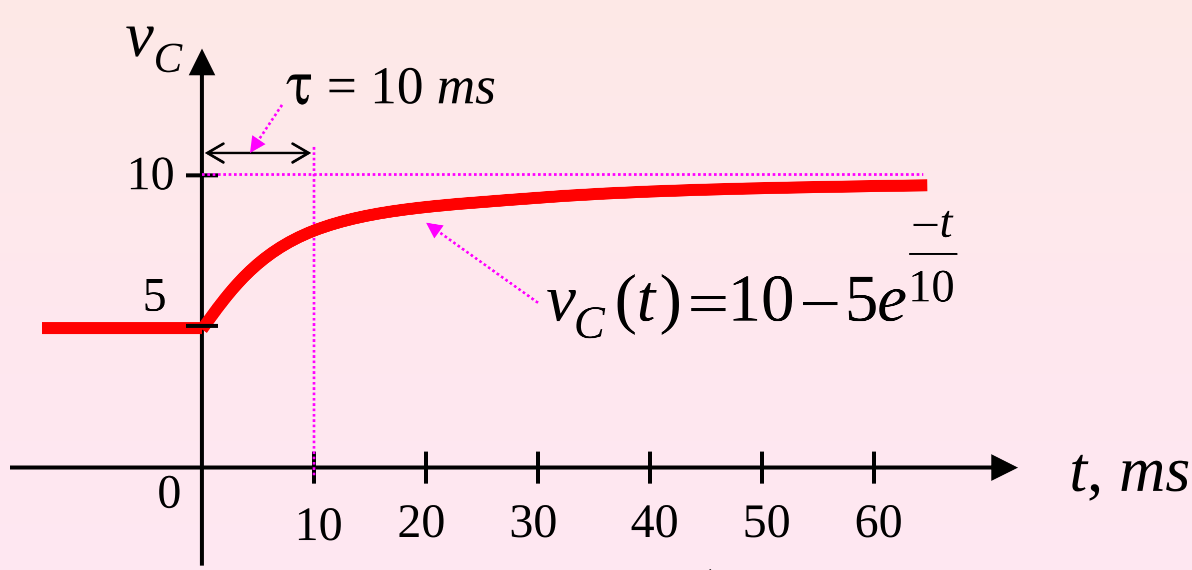
Problem

Assume $v_C(0^-) = 5$ V, where $t = 0^-$ denotes the time just before switch ***S*** made contact with resistor R.

Sketch $v_C(t)$ for $t \geq 0^+$, where $t = 0^+$ denotes the instant switch ***S*** made contact with resistor R.

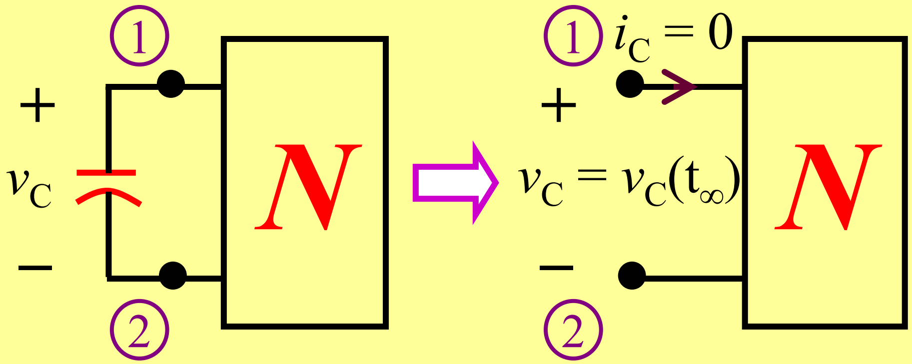
Solution $v_C(0^+) = v_C(0^-) = 5$ V, $v_C(t_\infty) = 10$ V

$$\tau = R\,C = 5(10^3)\,[2(10^{-6})] = 10\ ms$$



Calculate : $v_C(t) = 10 - 5e^{\frac{-t}{10}},\ t \geq 0^+$

How to Find $v_C(t_\infty)$?



Step 1 :

Open capacitor

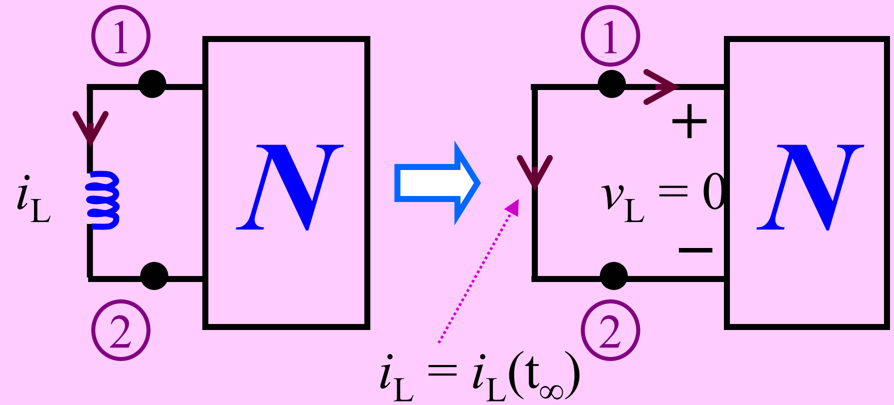
Step 2 :

Calculate : $v_C|_{\text{Open capacitor}}$

Then

$$v_C(t_\infty) = v_C\,|_{\textbf{Open capacitor}}$$

How to Find $i_L(t_\infty)$?



Step 1 :

Short capacitor

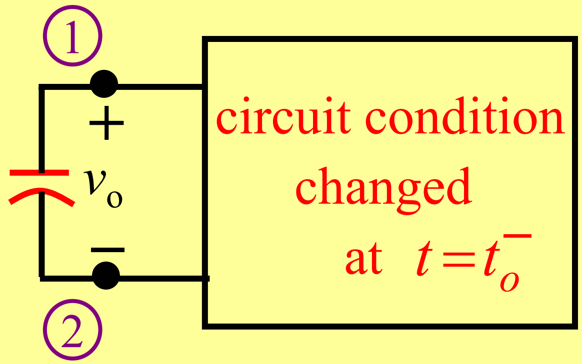
Step 2 :

Calculate : $i_L|_{\text{Short inductor}}$

Then

$$i_L(t_\infty) = i_L\,|_{\textbf{Short inductor}}$$

How to Find $v_c(t_o^+)$?



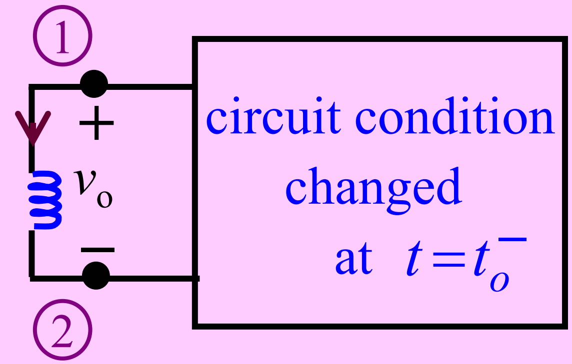
Step 1 :

Calculate : $v_C(t_o^-)$

Step 2 :

$$v_C(t_o^+) = v_C(t_o^-)$$

How to Find $i_L(t_o^+)$?



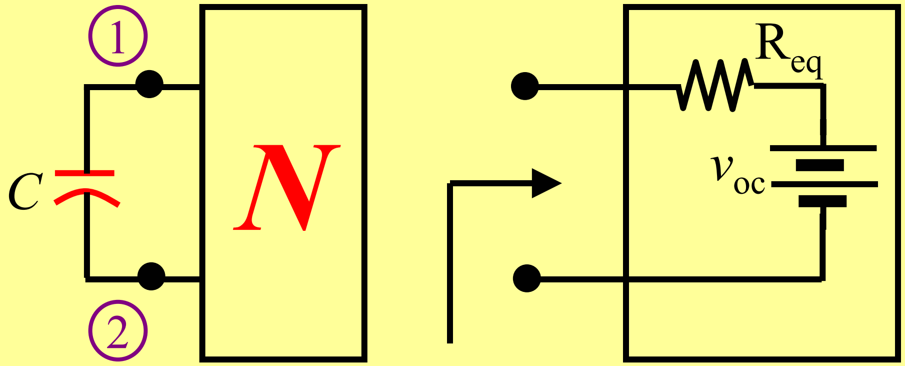
Step 1 :

Calculate : $i_L(t_o^-)$

Step 2 :

$$i_L(t_o^+) = i_L(t_o^-)$$

How to Find $\tau = R\,C$?



Thevenin equivalent circuit

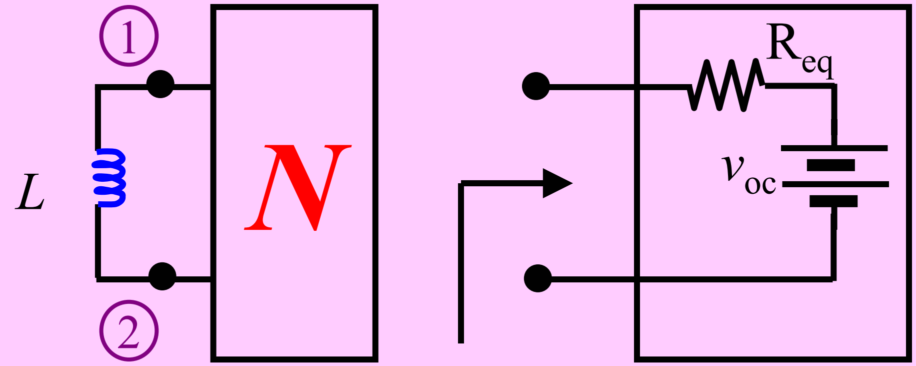
Step 1 :

Find *Thevenin* equivalent circuit of N.

Step 2 :

$$\tau = R_{eq}\,C$$

How to Find $\tau = G\,L = \frac{L}{R}$?



Thevenin equivalent circuit

Step 1 :

Find *Thevenin (or Norton)* equivalent circuit of N.

Step 2 :

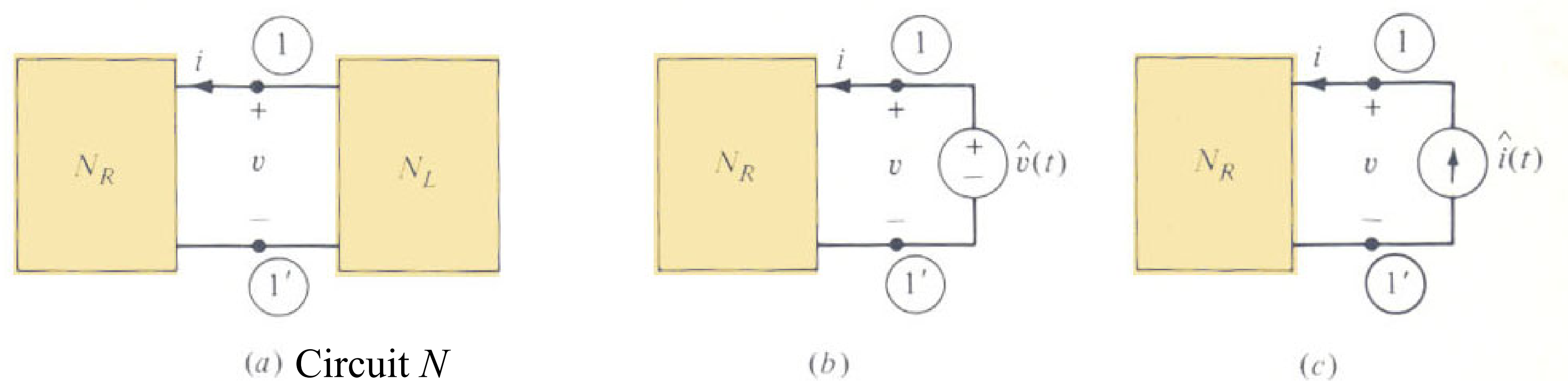
$$\tau = \frac{L}{R_{eq}}$$

Substitution Theorem

Let N be a circuit made of a nonlinear resistive one-port N_R terminated in an arbitrary one-port N_L , as shown in Fig. (a).

1. If N has a *unique* solution $v = \hat{v}(t)$ for all t, then N_L may be substituted by a voltage source $\hat{v}(t)$ without affecting the branch voltage and branch current solution inside N_R, provided the substituted circuit N_v in Fig. (b) has a *unique* solution for all t.

2. If N has a *unique* solution $i = \hat{i}(t)$ for all t, then N_L may be substituted by a current source $\hat{i}(t)$ without affecting the branch voltage and branch current solution inside N_R, provided the substituted circuit N_i in Fig. (c) has a *unique* solution for all t.



(a) Circuit $\mathcal{N}$. (b) Substituted circuit $\mathcal{N}_v$. (c) Substituted circuit $\mathcal{N}_i$.