Session 6: Solid State Physics

Diode

Outline

- A
  - B
  - C
  - D
  - E
- F
  - G
- H
- I
- J

Definitions / Assumptions

Homojunction: the junction is between two regions of the same material
Heterojunction: the junction is between two different semiconductors

Approximations used in the step-junction model

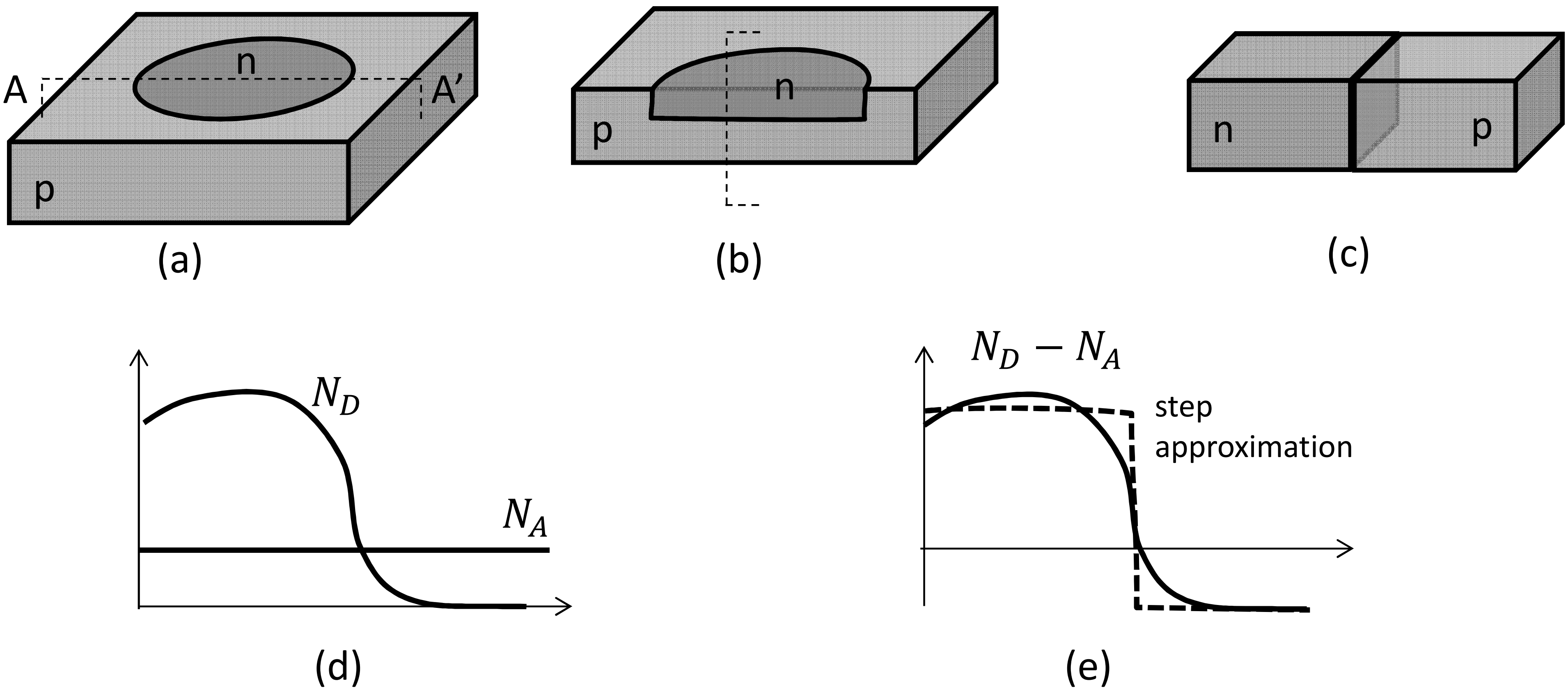
1. The doping profile is a step function. On the n-type side, $N'_D = N_D - N_A$ and is constant.

On the p side, $N'_A = N_A - N_D$ and is constant.

2. All impurities are ionized. Thus the equilibrium electron concentration on the n side is $n_{n0} = N'_D$.
The equilibrium hole concentration on the p side is $p_{p0} = N'_A$.

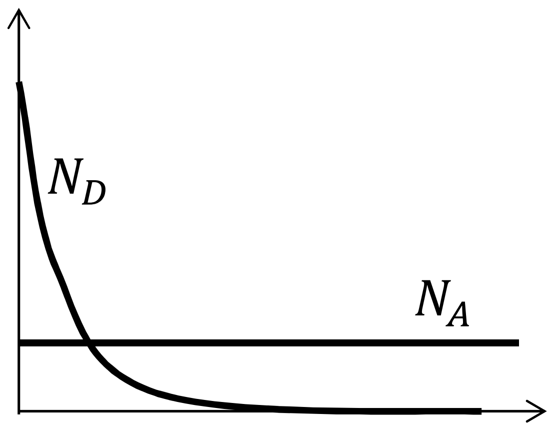
3. Impurity-induced band-gap narrowing effects are neglected.

Planar (1-D) pn Junction

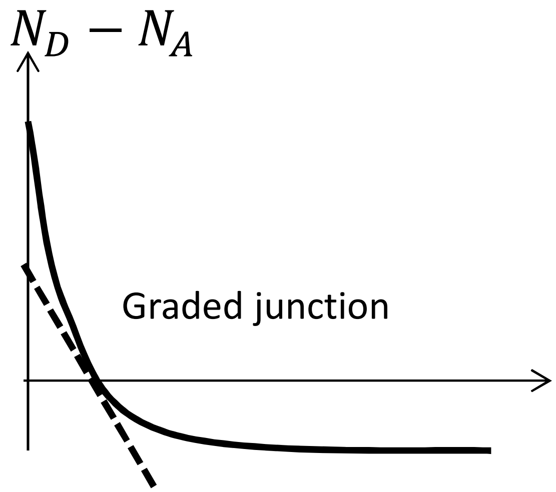


(a) The physical picture of a planar pn junction; (b) cross section through A−A'; (c) schematic representation of the pn junction; (d) typical doping profile showing a p-type substrate with implanted donors (the junction occurs where $N_D - N_A$); (e) the net doping concentration $N_D - N_A$ for this junction, and the step approximation (dashed line). (x_0= metallurgical junction)

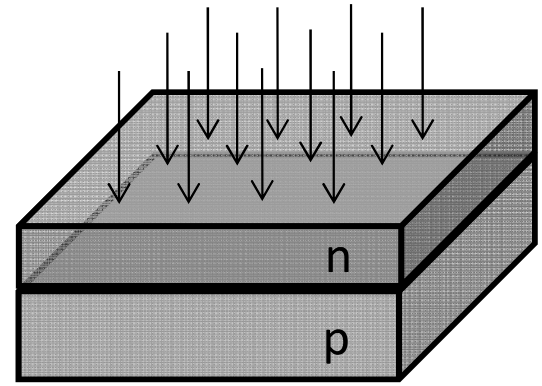
pn Junction



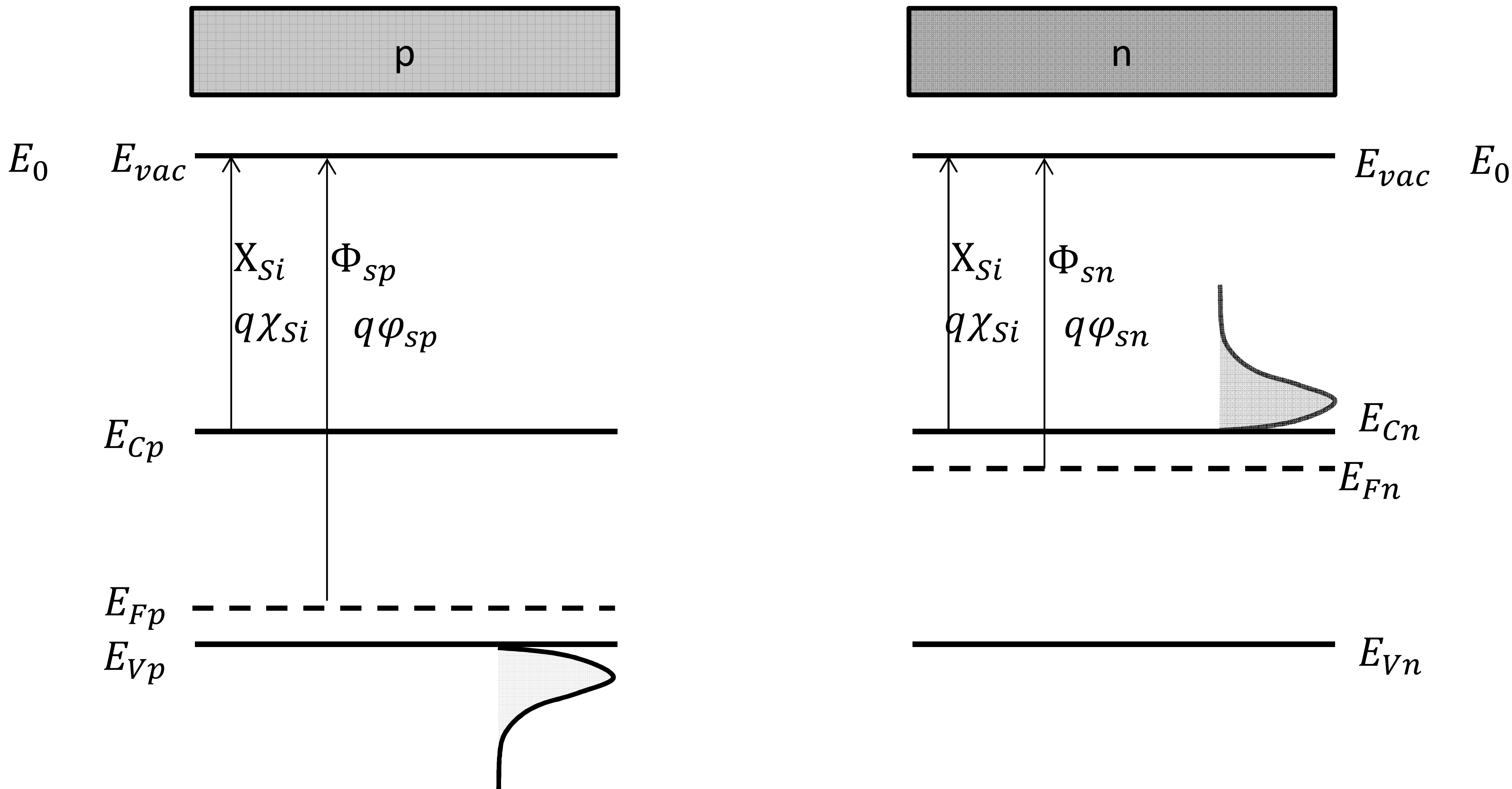
(d)



(e)



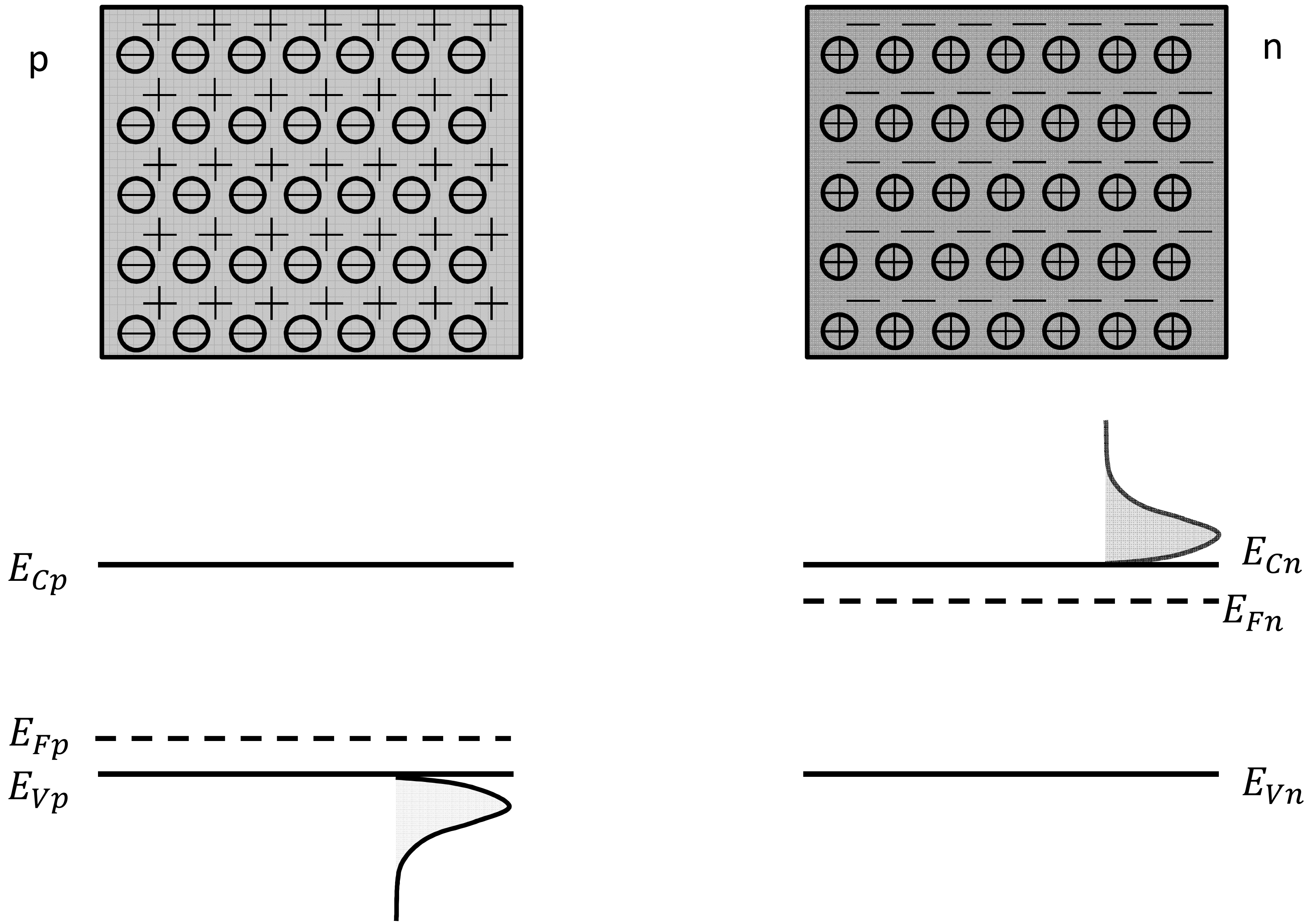
PN junctions – Before Being Joined



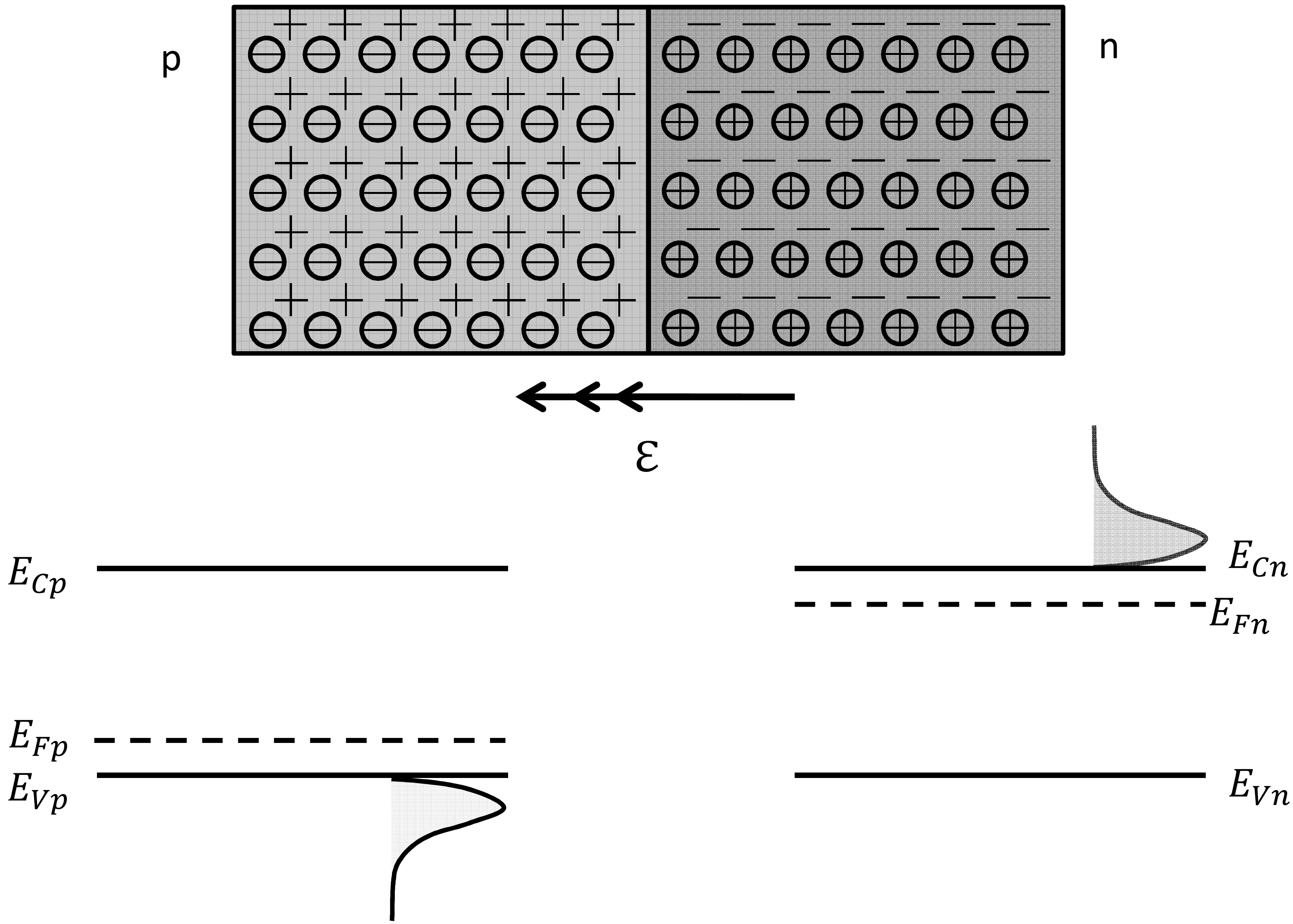
electrically neutral in every region
electron affinity : X_{Si}
work function Φ: $\Phi = E_{vac} - E_F$

$$\Phi_n \neq \Phi_p$$

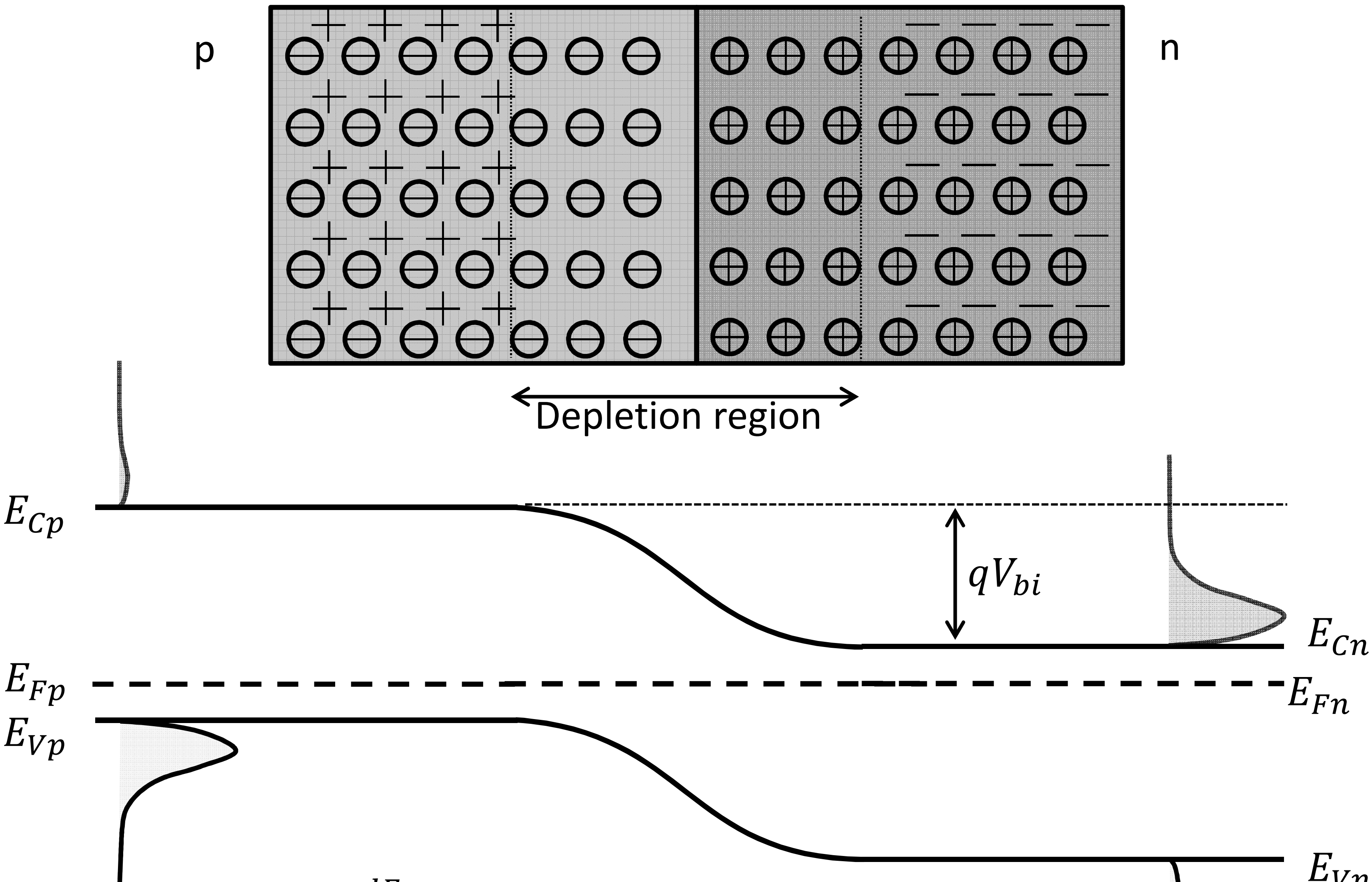
PN junctions (Qualitative)



PN junctions (Qualitative)

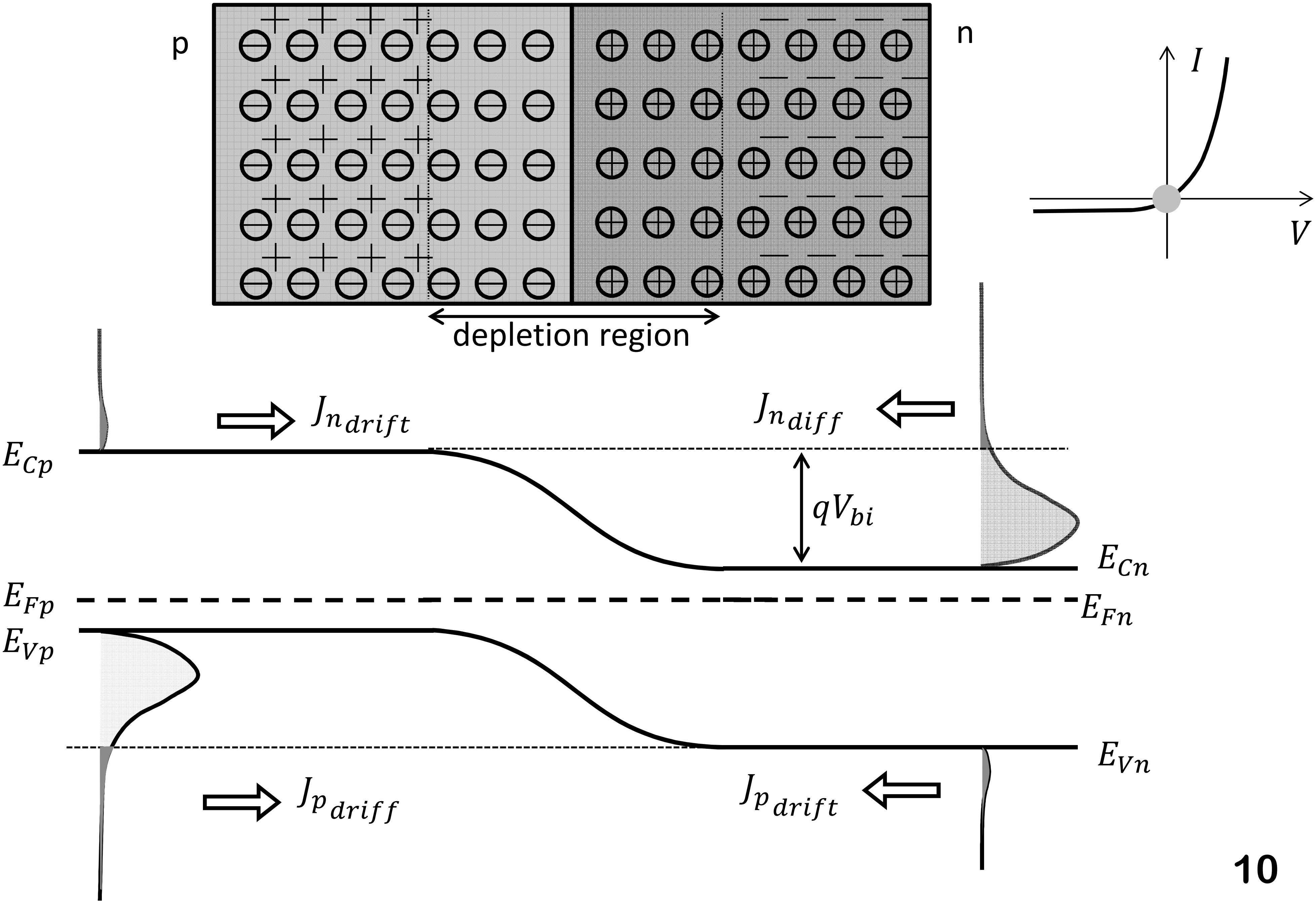


PN junctions (Qualitative)

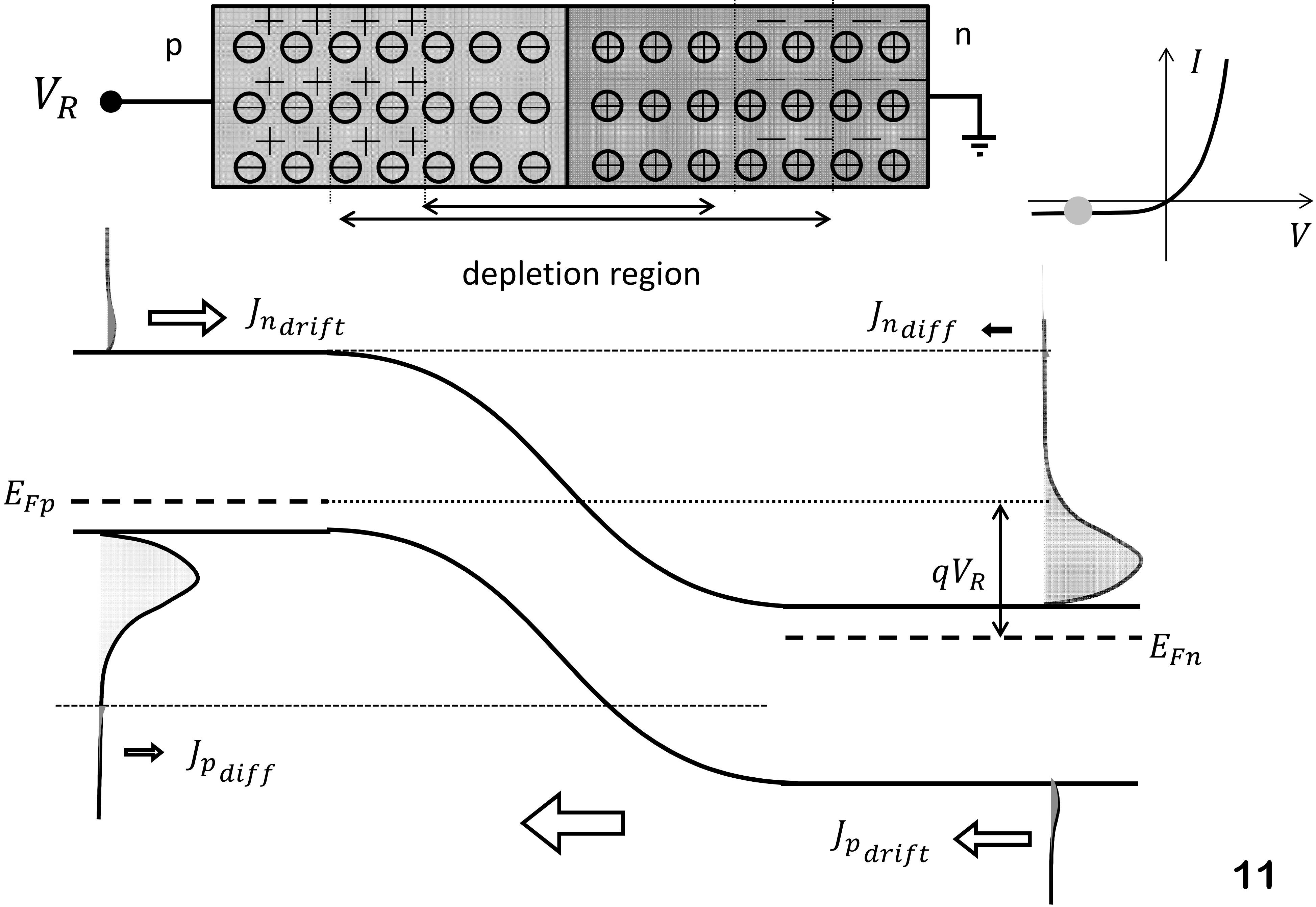


Remember $\frac{dE_F}{dx} = 0$ under equilibrium.
Band bending occurs around the metallurgical junction!

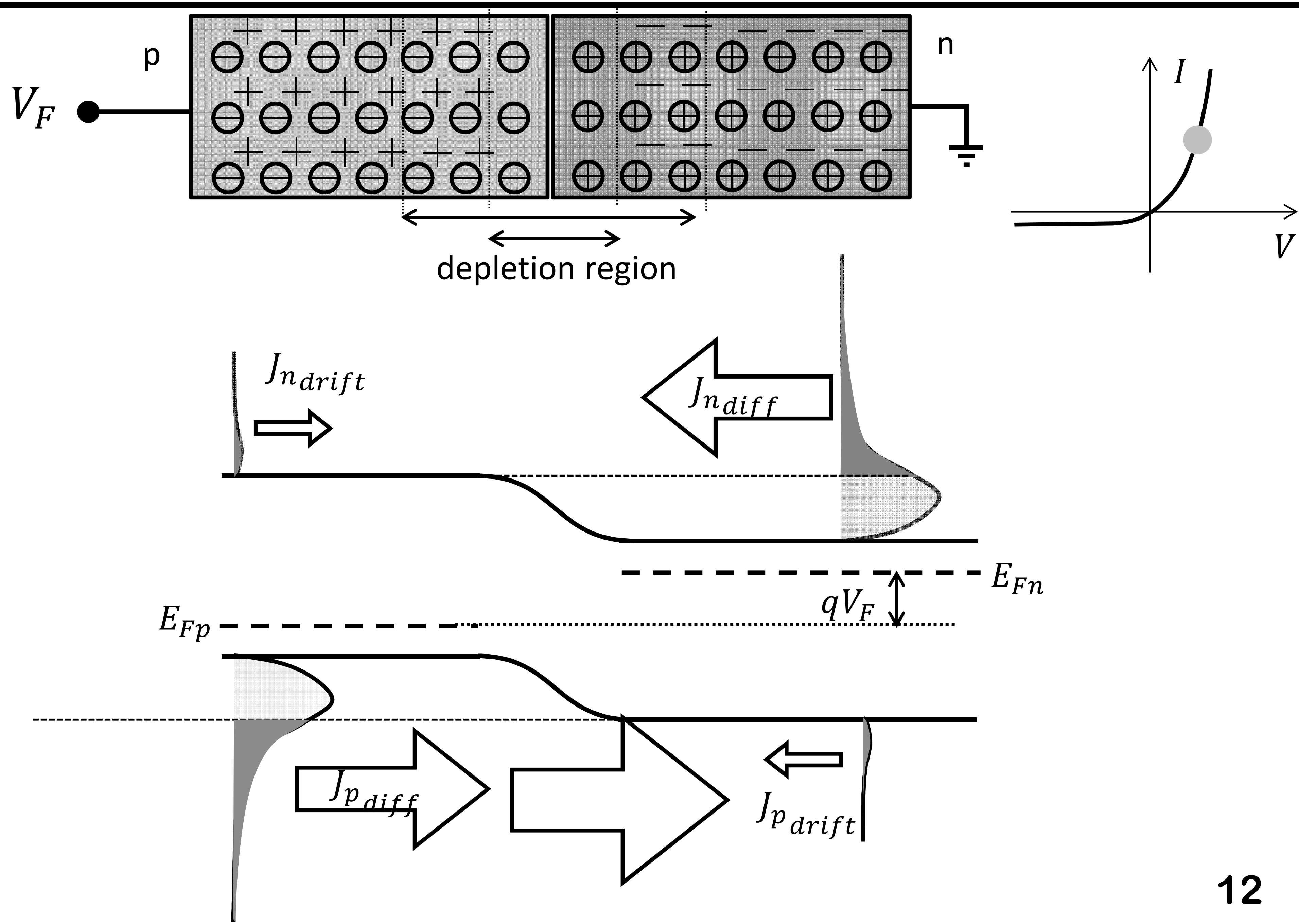
PN junctions (Qualitative)



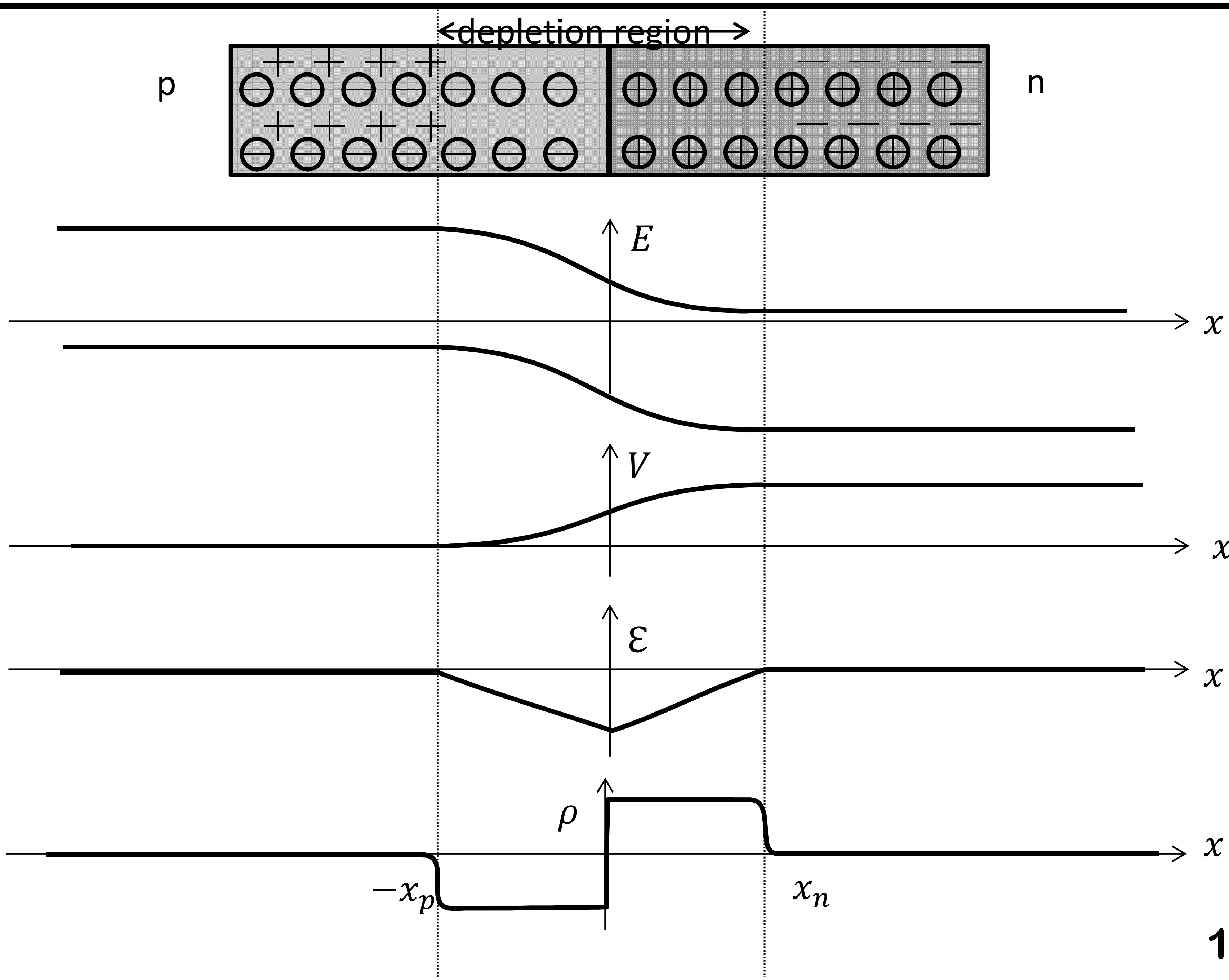
PN junctions (Qualitative) Reverse Biased



PN junctions (Qualitative) Forward Biased



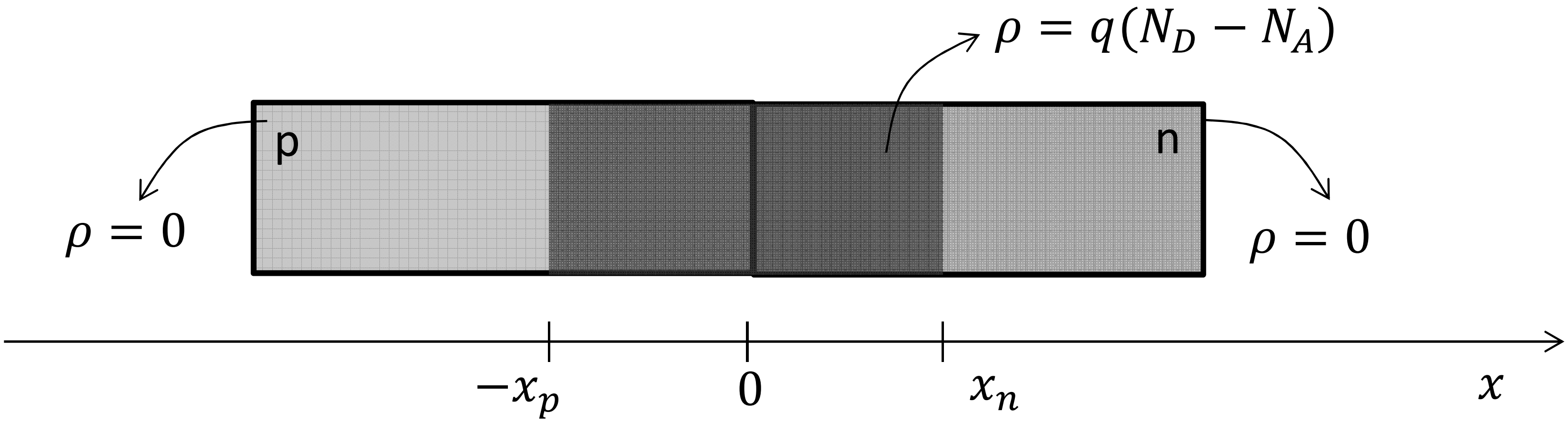
PN junctions (Qualitative)



PN junctions - Assumptions

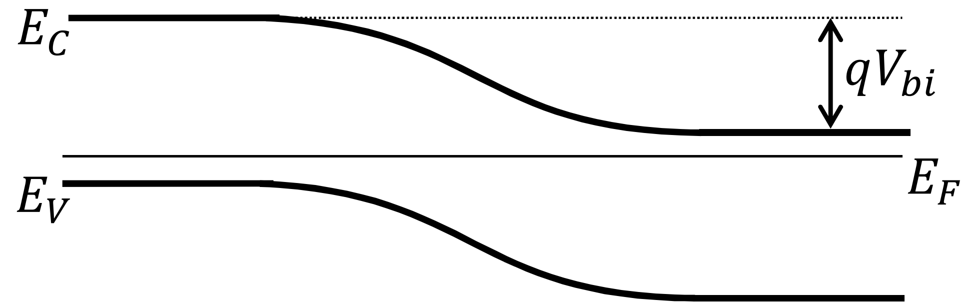
The Depletion Approximation : Obtaining closed-form solutions for the electrostatic variables

Charge Distribution : $\rho = q(p - n + N_D - N_A)$



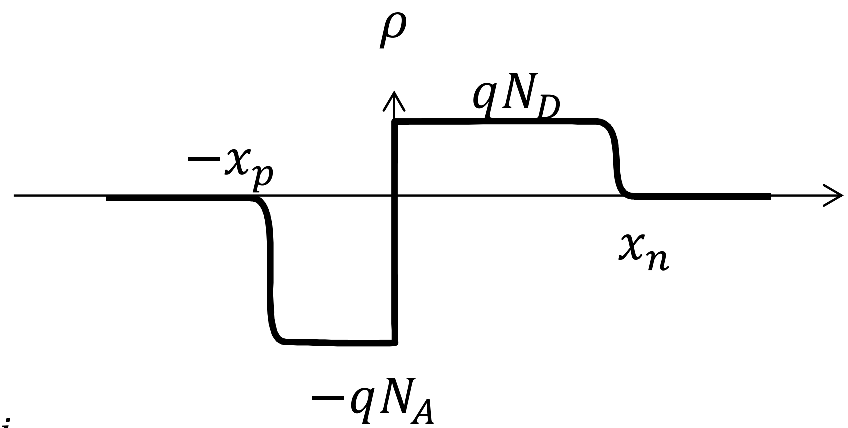
Note that
(1) $-x_p \leq x \leq x_n$: p & n are negligible ($\because \mathcal{E}$ exist).
(2) $x \leq -x_p$ or $x \geq x_n$: $\rho = 0$

How to Find $\rho(x), \mathcal{E}(x), V(x)$



1. Find the built-in potential V_{bi}

2. Use the depletion approximation → $\rho(x)$
(depletion-layer widths x_p, x_n unknown)



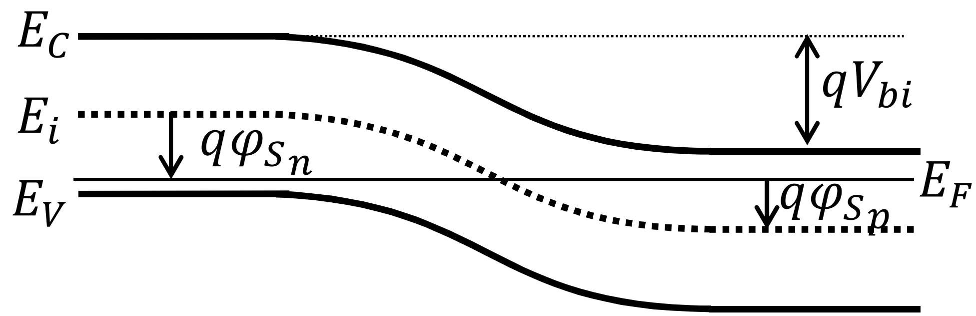
3. Integrate $\rho(x)$ to find $\mathcal{E}(x)$
boundary conditions $\mathcal{E}(-x_p) = 0, \mathcal{E}(x_n) = 0$

4. Integrate $\mathcal{E}(x)$ to obtain $V(x)$
boundary conditions $V(-x_p) = 0, V(x_n) = V_{bi}$

5. For $\mathcal{E}(x)$ to be continuous at $x = 0$,
$N_A x_p = N_D x_n$ → solve for x_p, x_n

Built-In Potential V_{bi}

$$qV_{bi} = q\varphi_{S_p} + q\varphi_{S_n}$$
$$= (E_i - E_F)_p + (E_F - E_i)_n$$



For non-degenerately doped material:

$$\left.\begin{aligned}(E_i - E_F)_p &= kT\ln\left(\frac{p}{n_i}\right) = kT\ln\left(\frac{N_A}{n_i}\right)\\(E_F - E_i)_n &= kT\ln\left(\frac{n}{n_i}\right) = kT\ln\left(\frac{N_D}{n_i}\right)\end{aligned}\right\} \rightarrow V_{bi} = \frac{kT}{q}\ln\left(\frac{N_A N_D}{n_i^2}\right)$$

What shall we do for $p^+ - n$ (or $n^+ - p$) junction?!?!?

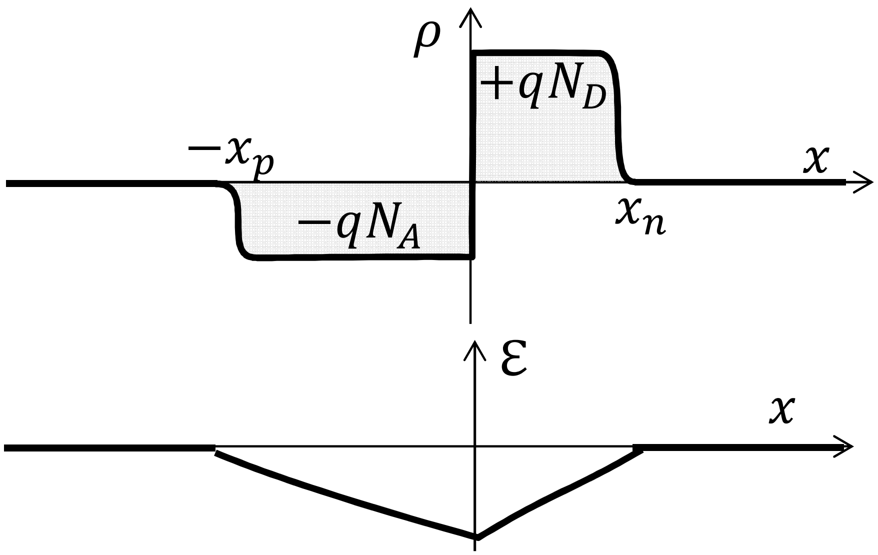
p^+:

$$(E_i - E_F)_p = \frac{E_G}{2}$$

n^+:

$$(E_F - E_i)_n = \frac{E_G}{2}$$

The Depletion Approximation



$$\frac{d\mathcal{E}}{dx} = \frac{\rho}{\epsilon}$$

$\rho = -qN_A \rightarrow$

$$\mathcal{E}(x) = \frac{-qN_A}{\epsilon} + C = \frac{-qN_A}{\epsilon}(x + x_p)$$

$\rho = qN_D \rightarrow$

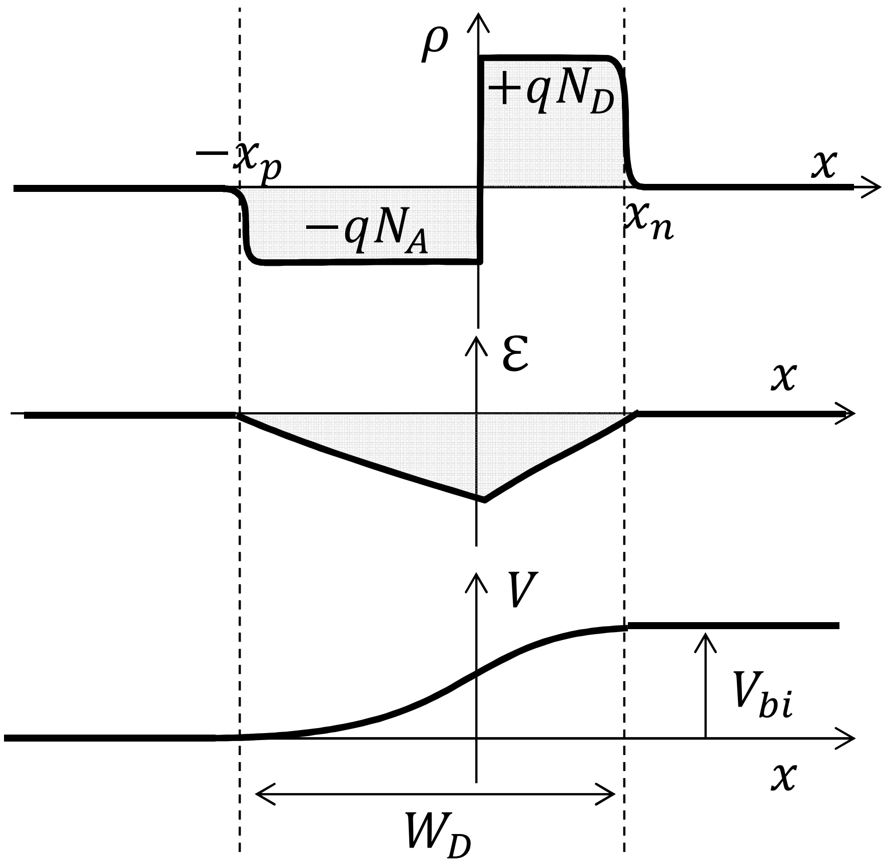
$$\mathcal{E}(x) = \frac{qN_D}{\epsilon} + C' = \frac{qN_D}{\epsilon}(x - x_n)$$

The electric field is continuous at $x = 0$

$$x_p N_A = x_n N_D$$

Charge neutrality condition as well!

Electrostatic Potential in the Depletion Layer



$$\frac{dV}{dx} = -\mathcal{E}$$

$-x_p < x < 0$:

$$\mathcal{E}(x) = \frac{-qN_A}{\epsilon}(x + x_p)$$

$$V(x) = \frac{qN_A}{2\epsilon}\left(x + x_p\right)^2 + C = \frac{qN_A}{2\epsilon}\left(x + x_p\right)^2$$

$0 < x < x_n$:

$$\mathcal{E}(x) = -\frac{qN_D}{\epsilon}(x_n - x)$$

$$V(x) = -\frac{qN_D}{2\epsilon}(x_n - x)^2 + C' = V_{bi} - \frac{qN_D}{2\epsilon}(x_n - x)^2$$

Depletion Layer Width

$$-x_p < x < 0: \quad V(x) = \frac{qN_A}{2\epsilon}\left(x + x_p\right)^2$$

$$0 < x < x_n: \quad V(x) = V_{bi} - \frac{qN_D}{2\epsilon}(x_n - x)^2$$

$$\left.\begin{matrix} V(0) = \dfrac{qN_A}{2\epsilon}x_p^2 = V_{bi} - \dfrac{qN_D}{2\epsilon}x_n^2 \\ x_pN_A = x_nN_D \end{matrix}\right\} \rightarrow \begin{cases} x_n = \sqrt{\dfrac{2\epsilon_s V_{bi}}{q}\left(\dfrac{N_A}{N_D(N_A + N_D)}\right)} \\ x_p = \sqrt{\dfrac{2\epsilon_s V_{bi}}{q}\left(\dfrac{N_D}{N_A(N_A + N_D)}\right)} \end{cases}$$

Summing, we have:

$$W = x_p + x_n = \sqrt{\frac{2\epsilon_s V_{bi}}{q}\left(\frac{1}{N_D} + \frac{1}{N_A}\right)}$$

Depletion Layer Width

If $N_A \gg N_D$ as in a $p^+ - n$ junction:

$$W = \sqrt{\frac{2\epsilon_s V_{bi}}{q}\left(\frac{1}{N_D} + \frac{1}{N_A}\right)} \quad \rightarrow W = \sqrt{\frac{2\epsilon_s V_{bi}}{qN_D}} \approx x_n$$

$$x_p N_A = x_n N_D \quad \rightarrow \quad x_p \ll x_n \quad \rightarrow x_p \approx 0$$

Note:

$$V_{bi} = \frac{E_G}{2q} + \frac{kT}{q} \ln \frac{N_D}{n_i}$$

Example

A $p^+ - n$ junction has $N_A = 10^{20}\ cm^{-3}$ and $N_D = 10^{17} cm^{-3}$. What is

a) its built in potential,

$$V_{bi} = \frac{E_G}{2q} + \frac{kT}{q} \ln \frac{N_D}{n_i} \approx 1V$$

b) W ,

$$W \approx \sqrt{\frac{2\epsilon_s V_{bi}}{qN_D}} = \sqrt{\frac{2 \times 12 \times 8.85 \times 10^{-14} \times 1}{1.6 \times 10^{19} \times 10^{17}}} = 0.12\mu m$$

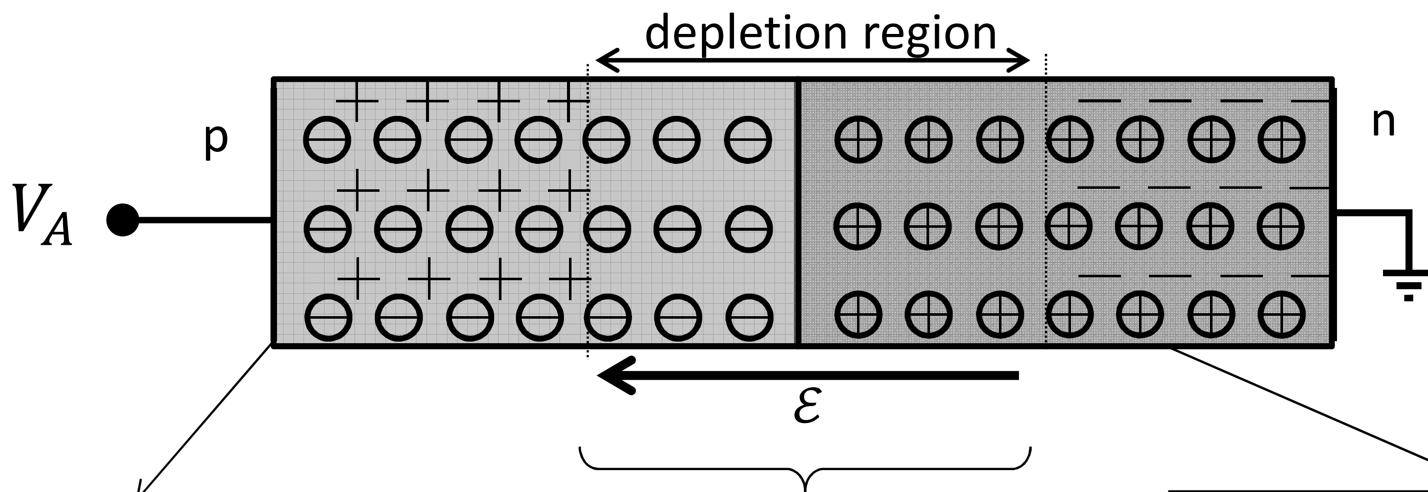
c) x_n , and

$$x_n \approx W = 0.12\mu m$$

d) x_p

$$x_p = x_n \frac{N_D}{N_A} = 1.2 \times 10^{-4}\ \mu m = 1.2\ \text{Å} \sim 0$$

Biases pn Junction (assumptions)



Negligible voltage drop (Ohmic contact)

V_A dropped here

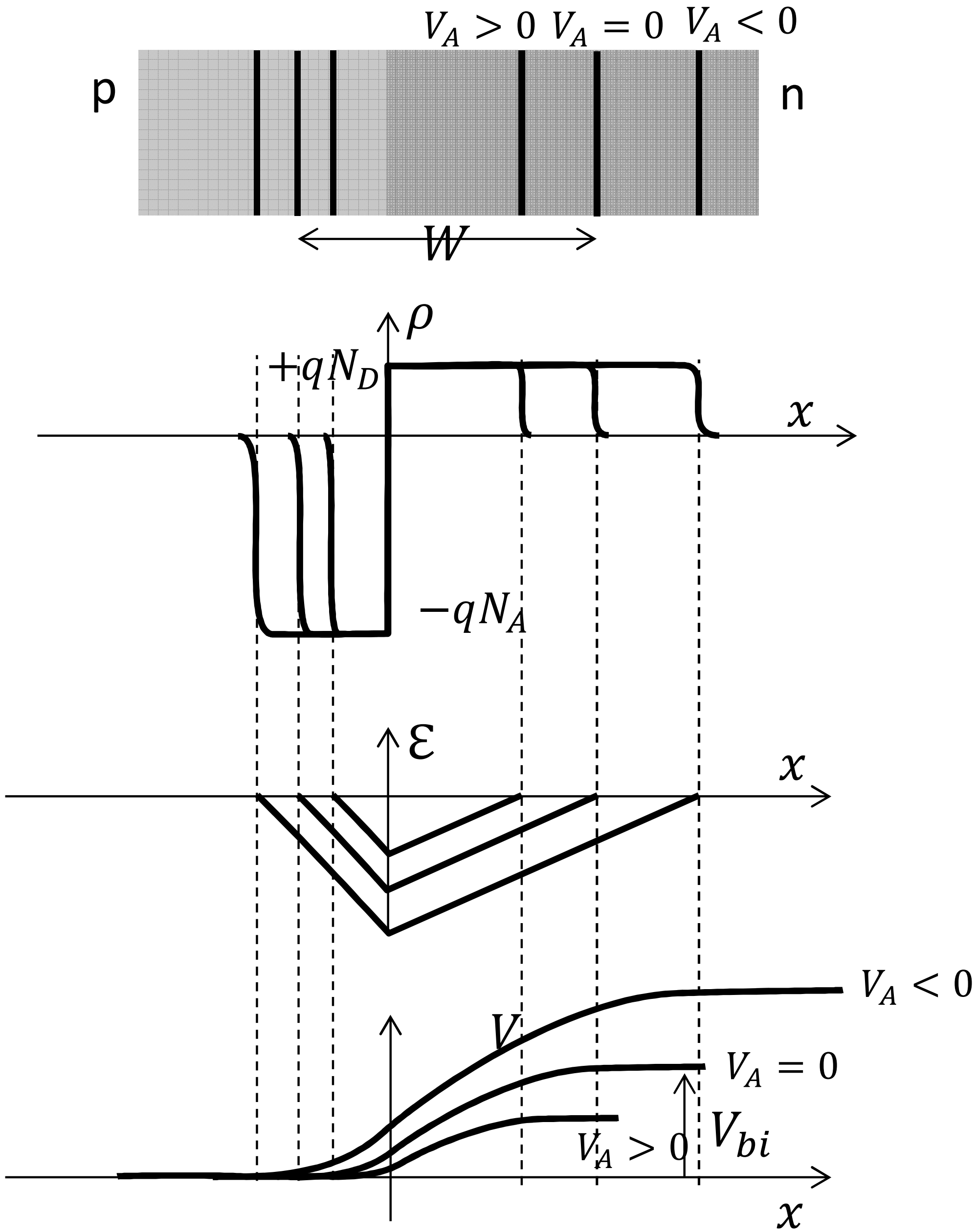
→ will apply continuity equation in this region

1) Low level injection
2) Zero voltage drop ($\mathcal{E} = 0$)

→ Since ($\mathcal{E} = 0$) may apply minority carrier diffusion equations

Note: V_A should be significantly smaller than V_{bi}(Otherwise, we cannot assume low-level injection)

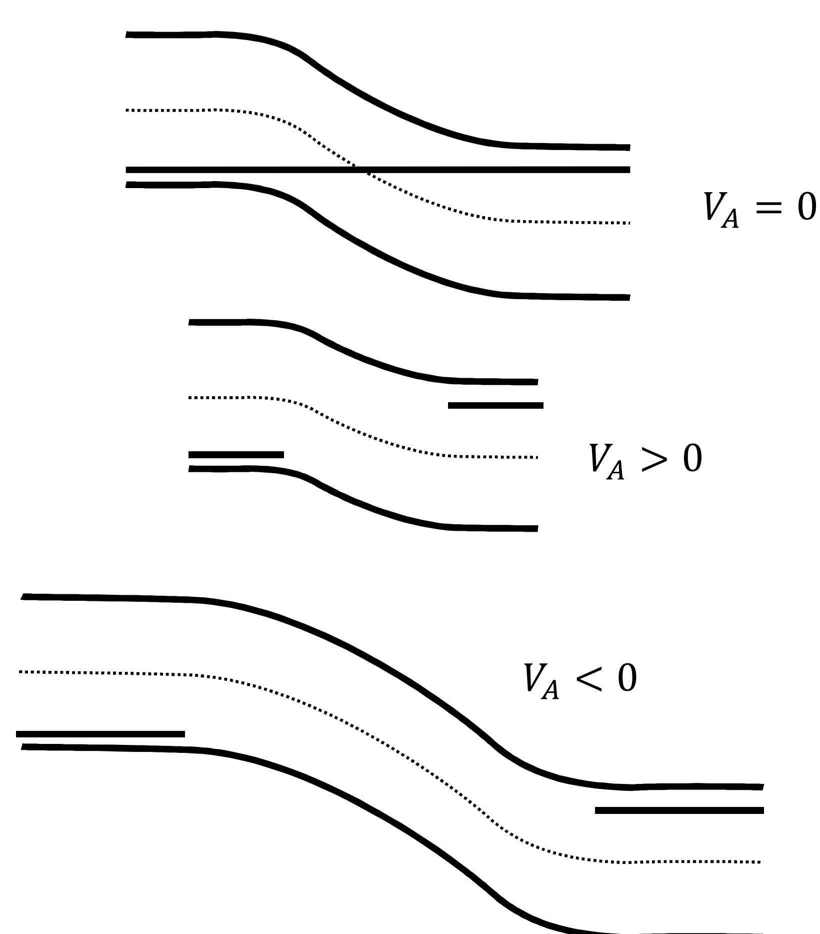
Effect of Bias on Electrostatics



Energy Band Diagram

1) The Fermi level is omitted from the depletion region because the device is no longer in equilibrium: We need the quasi Fermi energy level.

2) $E_{fp} - E_{fn} = -qV_A$



Va Applied Voltage

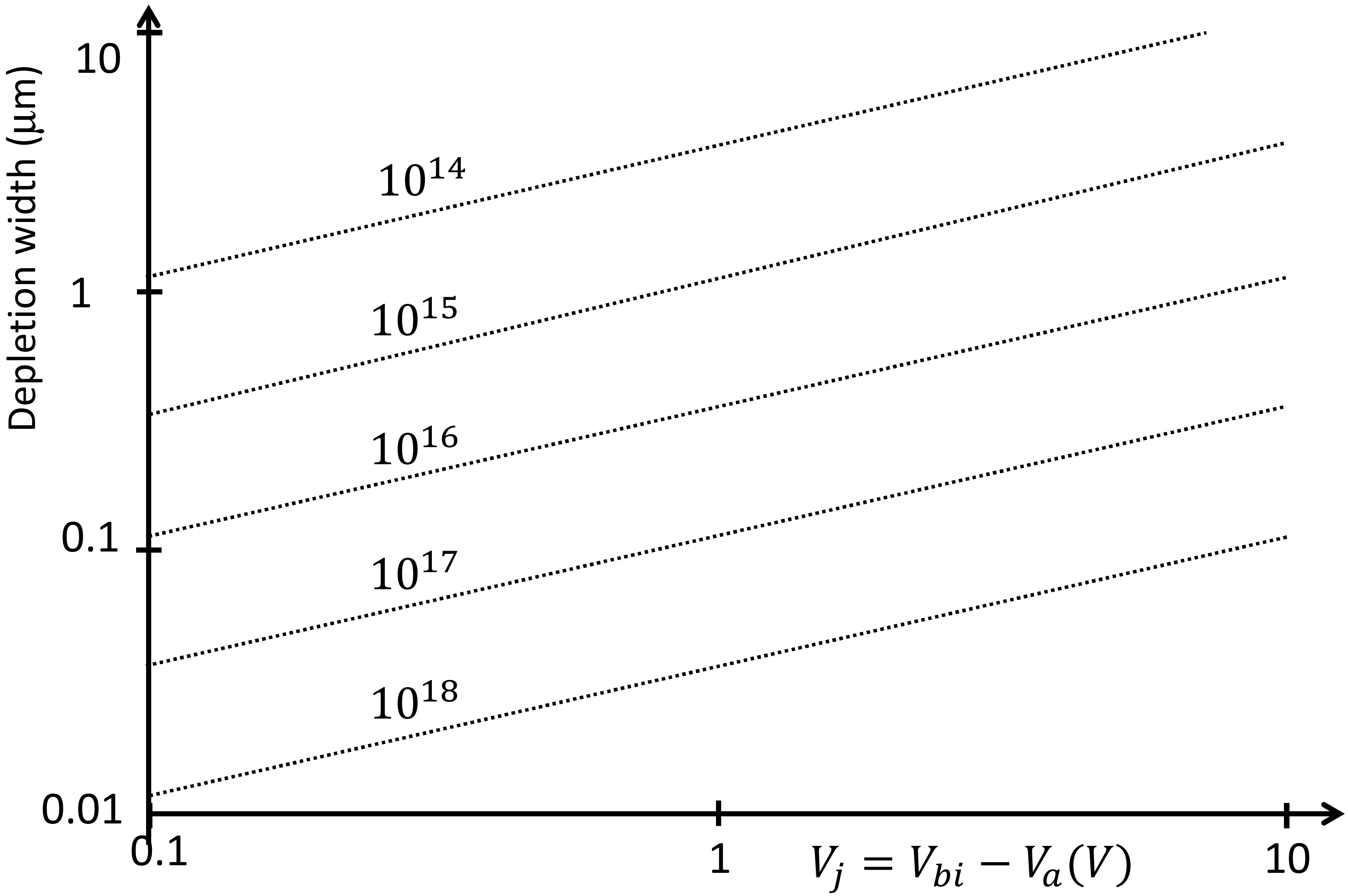
Now as we assumed all voltage drop is in the depletion region
(Note that VA ≤ Vbi)

$$x_n + x_p = W = \sqrt{\frac{2\epsilon_s(V_{bi} - V_A)}{q}\left(\frac{1}{N_D} + \frac{1}{N_A}\right)}$$

$$x_p N_A = x_n N_D$$

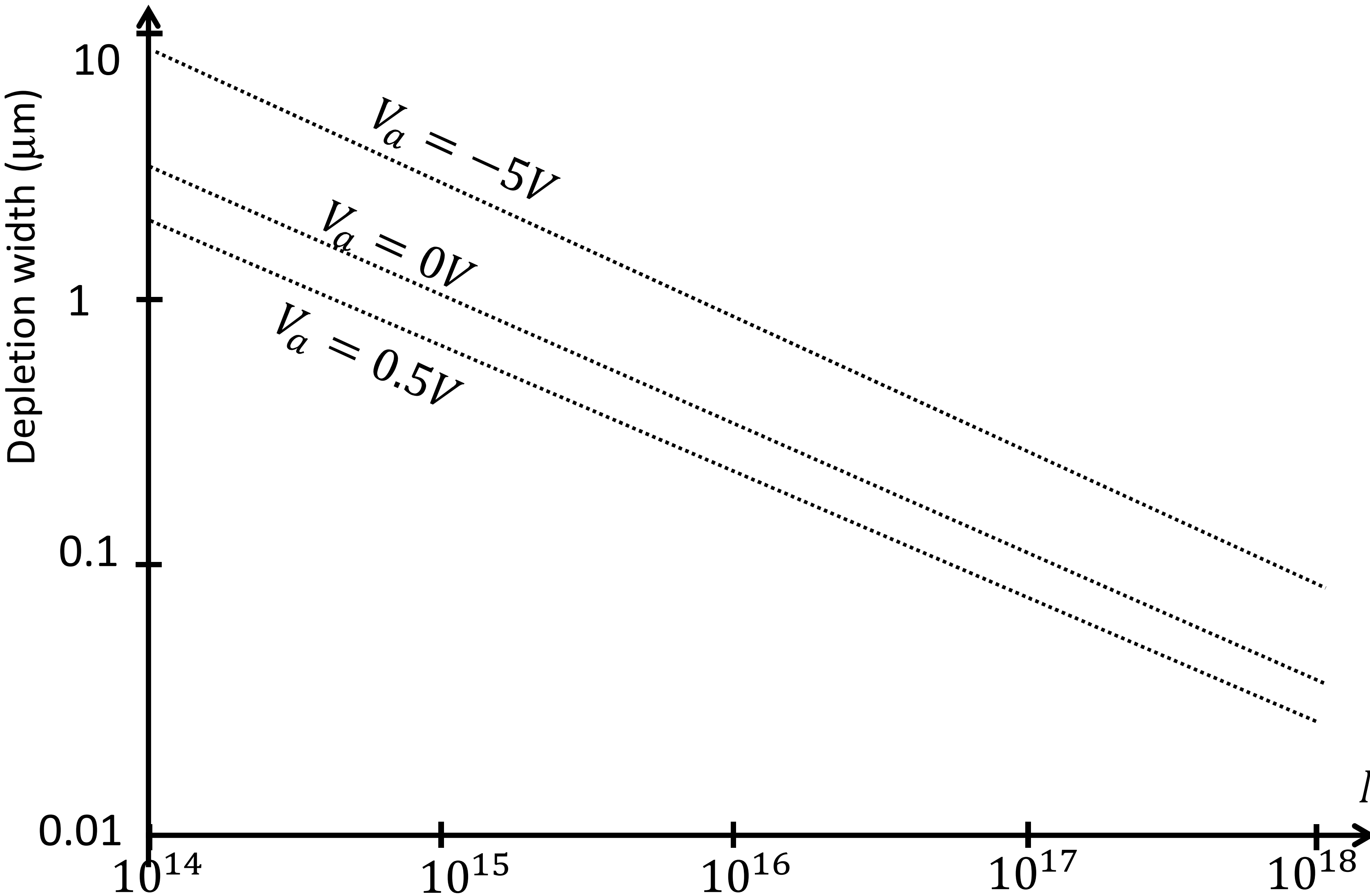
W vs. Va

The junction width for one-sided step junctions in silicon as a function of junction voltage with the doping on the lightly doped side as a parameter.



W vs. Na

Junction width for a one-sided junction is plotted as a function of doping on the lightly doped side for three different operating voltages.



pn Junction:
I-V Characteristic (assumptions)

Assumption :

1) low-level injection: $n_p \ll p_p \sim N_A$(or $\Delta n \ll p_0$, $p \sim p_0$ in p-type)

$p_n \ll n_n \sim N_D$(or $\Delta p \ll n_0$, $n \sim n_0$ in n-type)

2) In the bulk, $n_n \sim n_{n0} = N_D$, $p_p \sim p_{p0} = N_A$

3) For minority carriers $J_{drift} \ll J_{diff}$ in quasi-neutral region

4) Nondegenerately doped step junction

5) Long-base diode in 1-D (both sides of quasi-neutral regions are much longer than their minority carrier diffusion lengths, L_n or L_p)

6) No Generation/Recombination in depletion region

7) Steady state $d/dt = 0$

8) $G_{opt} = 0$

pn Junction: I-V Characteristic

Game plan:

i) continuity equations for minority carriers

$$\frac{\partial \Delta n_p}{\partial t} = \frac{1}{q}\frac{\partial J_n}{\partial x} - \frac{\Delta n_p}{\tau_n} + G_{ext}$$

$$\frac{\partial \Delta p_n}{\partial t} = -\frac{1}{q}\frac{\partial J_p}{\partial x} - \frac{\Delta p_n}{\tau_p} + G_{ext}$$

ii) minority carrier current densities in the quasi-neutral region

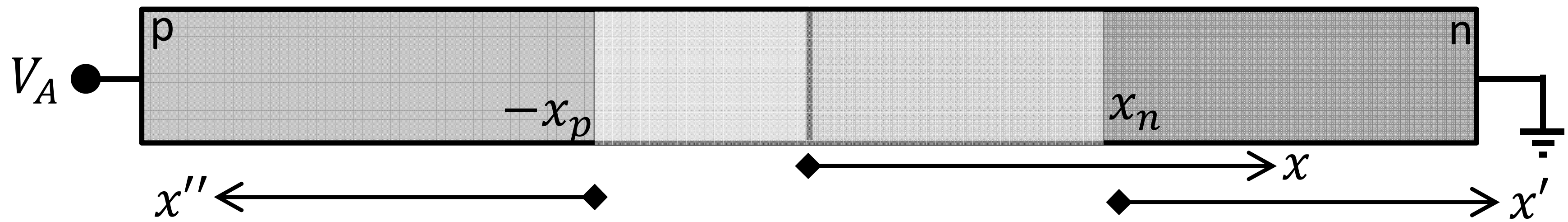
$$J_p = J_{p_{drift}} + J_{p_{diff}} = qp\mu_p\mathcal{E} - qD_p\frac{dp}{dx} \sim -\ qD_p\frac{dp}{dx}$$

$$J_n = J_{n_{drift}} + J_{n_{diff}} = qn\mu_n\mathcal{E} + qD_n\frac{dn}{dx} \sim qD_n\frac{dn}{dx}$$

pn Junction: I-V Characteristic

Steady-State solution is: $\dfrac{d^2\Delta n_p}{dx^2} = \dfrac{\Delta n_p}{D_n\tau_n} \rightarrow \Delta n_p = Ae^{x/L_n} + Be^{-x/L_n} \quad \left(L_n = \sqrt{D_n\tau_n}\right)$

diode is long enough!

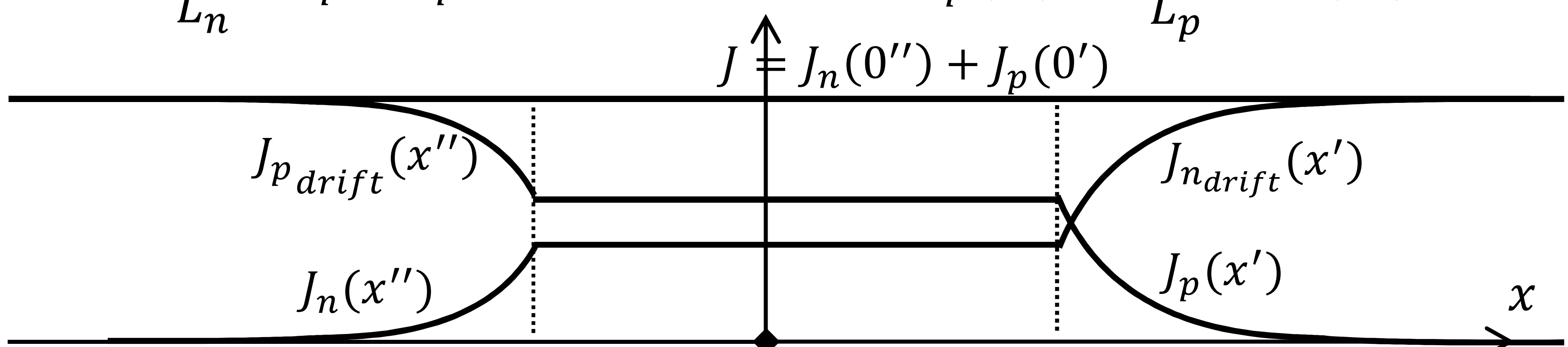


$$\Delta n_p(x'') = A''e^{-x''/L_n} \qquad \Delta p_n(x') = A'e^{-x'/L_p}$$

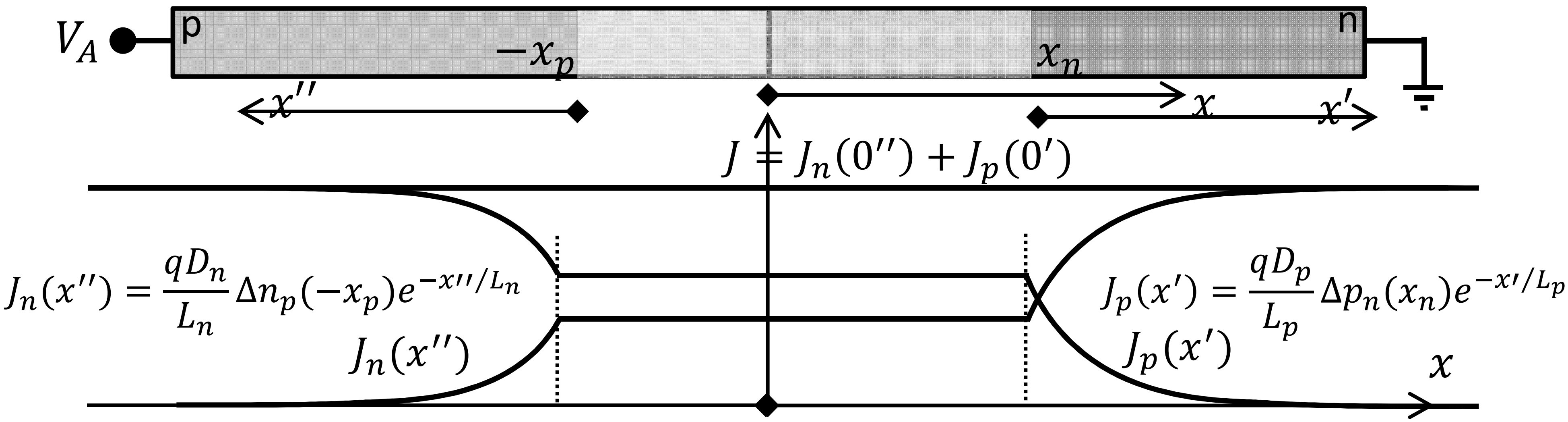
$$\Delta n_p(x'') = \Delta n_p(-x_p)e^{-x''/L_n} \qquad \Delta p_n(x') = \Delta p_n(x_n)e^{-x'/L_p}$$

$$J_n = qD_n\frac{dn}{dx} \qquad J_p = -qD_p\frac{dp}{dx}$$

$$J_n(x'') = \frac{qD_n}{L_n}\Delta n_p(-x_p)e^{-x''/L_n} \qquad J_p(x') = \frac{qD_p}{L_p}\Delta p_n(x_n)e^{-x'/L_p}$$



pn Junction: I-V Characteristic



$$J = J_n(0'') + J_p(0') = \frac{qD_n}{L_n}\Delta n_p(-x_p) + \frac{qD_p}{L_p}\Delta p_n(x_n)$$

Now! we need to find $\Delta n_p(-x_p)$ and $\Delta p_n(x_n)$ vs V

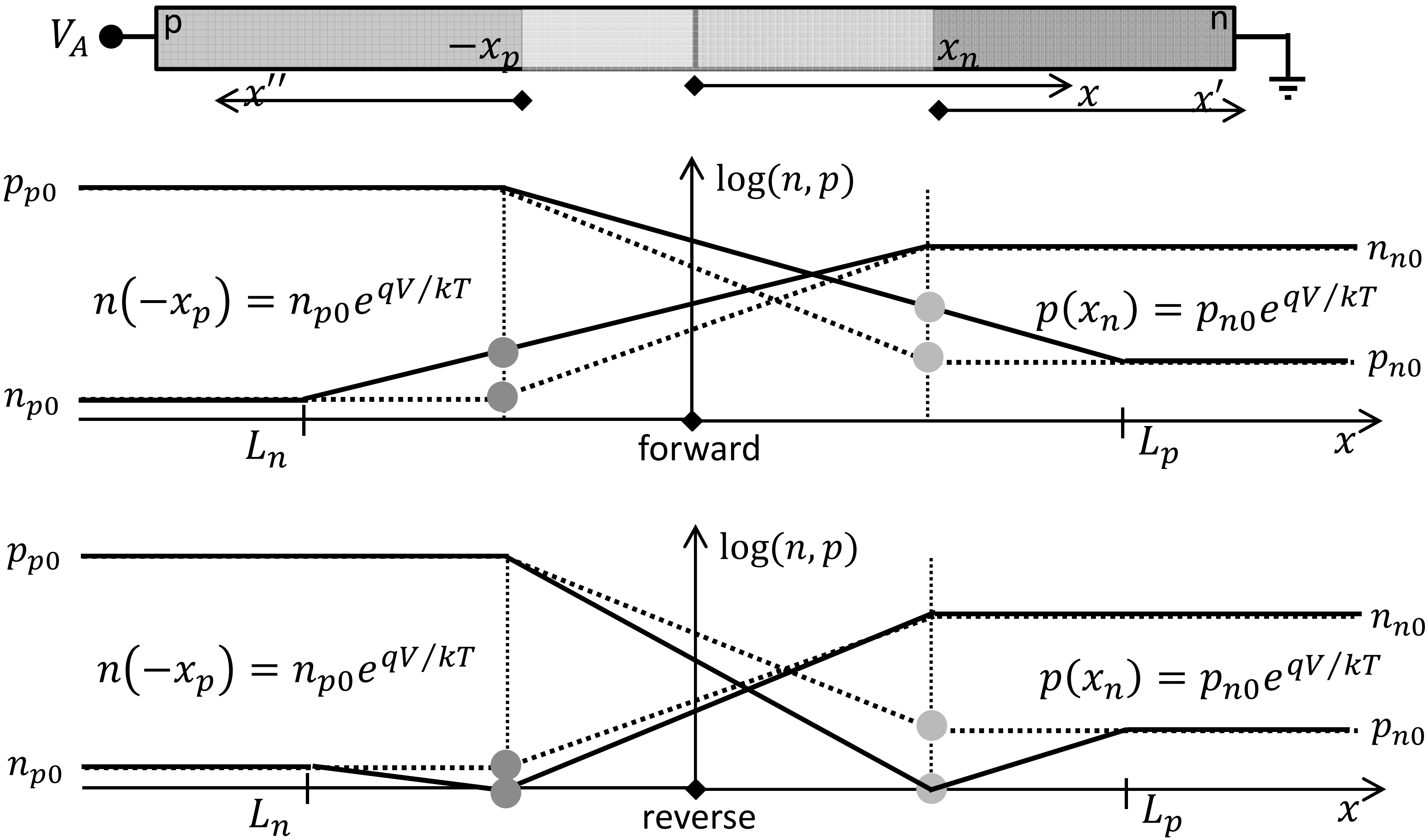
$$V_2 - V_1 = \frac{kT}{q}\ln\frac{n_2}{n_1} = \frac{kT}{q}\ln\frac{p_1}{p_2} \qquad \rightarrow V_0 - V = \frac{kT}{q}\ln\frac{n(x_n)}{n(-x_p)} = \frac{kT}{q}\ln\frac{p(-x_p)}{p(x_n)}$$

N_D ($n(x_n)$) N_A ($p(-x_p)$)

$$V_0 = \frac{kT}{q}\ln\frac{N_A N_D}{n_i^2} \quad \rightarrow$$

$$n(-x_p) = n_{p0}\, e^{qV/kT}$$
$$p(x_n) = p_{n0}\, e^{qV/kT}$$

pn Junction: I-V Characteristic



pn Junction: I-V Characteristic

$$J = J_n(0'') + J_p(0') = \frac{qD_n}{L_n}\Delta n_p(-x_p) + \frac{qD_p}{L_p}\Delta p_n(x_n)$$

$$n(-x_p) = n_{p0}\, e^{qV/kT} \quad ; \quad \Delta n_p(-x_p) = n - n_{p0} = n_{p0}(e^{qV/kT} - 1) \quad ; \quad n_{p0} = n_i^2/N_A$$

$$p(x_n) = p_{n0}\, e^{qV/kT} \quad ; \quad \Delta p_n(x_n) = p - p_{n0} = p_{n0}(e^{qV/kT} - 1) \quad ; \quad p_{n0} = n_i^2/N_D$$

$$J = q\left(\frac{D_n}{L_n} n_{p0} + \frac{D_p}{L_p} p_{n0}\right)\left(e^{qV/kT} - 1\right) \qquad I = AJ$$

$$I = qA\left(\frac{D_n}{L_n}\frac{n_i^2}{N_A} + \frac{D_p}{L_p}\frac{n_i^2}{N_D}\right)\left(e^{qV/kT} - 1\right) = I_0\left(e^{qV/kT} - 1\right)$$

$$I_0 = qAn_i^2\left(\sqrt{\frac{D_n}{\tau_n}}\frac{1}{N_A} + \sqrt{\frac{D_p}{\tau_p}}\frac{1}{N_D}\right)$$

pn Junction: I-V Characteristic

$$I = qA\left(\frac{D_n}{L_n}\frac{n_i^2}{N_A} + \frac{D_p}{L_p}\frac{n_i^2}{N_D}\right)\left(e^{qV/kT} - 1\right) = I_0\left(e^{qV/kT} - 1\right)$$

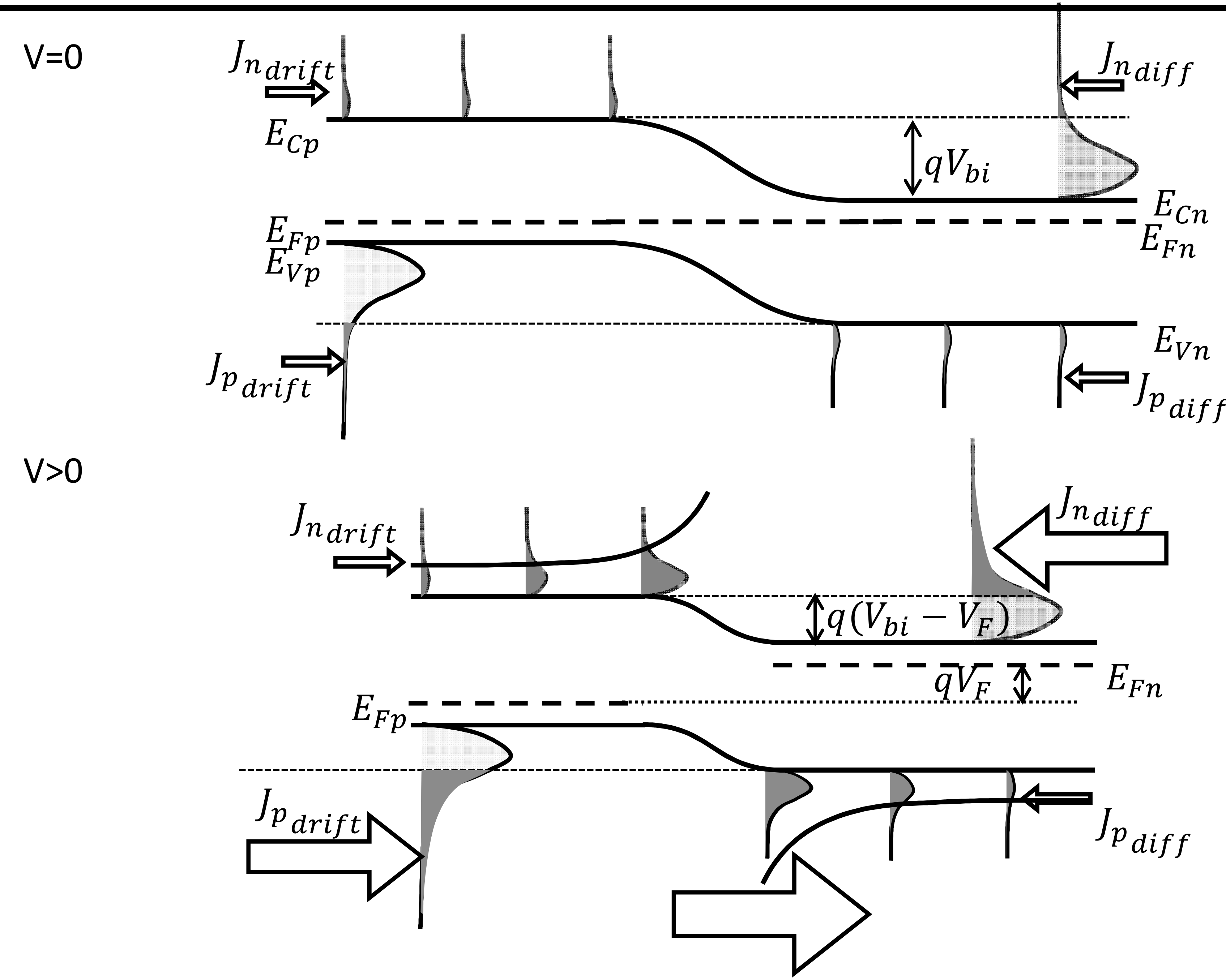
asymmetrically doped junction

If $p^+ - n$ diode ($N_A \gg N_D$), then $$I_0 \approx qA\frac{D_p}{L_p}\frac{n_i^2}{N_D}$$

If $n^+ - p$ diode ($N_D \gg N_A$), then $$I_0 \approx qA\frac{D_n}{L_n}\frac{n_i^2}{N_A}$$

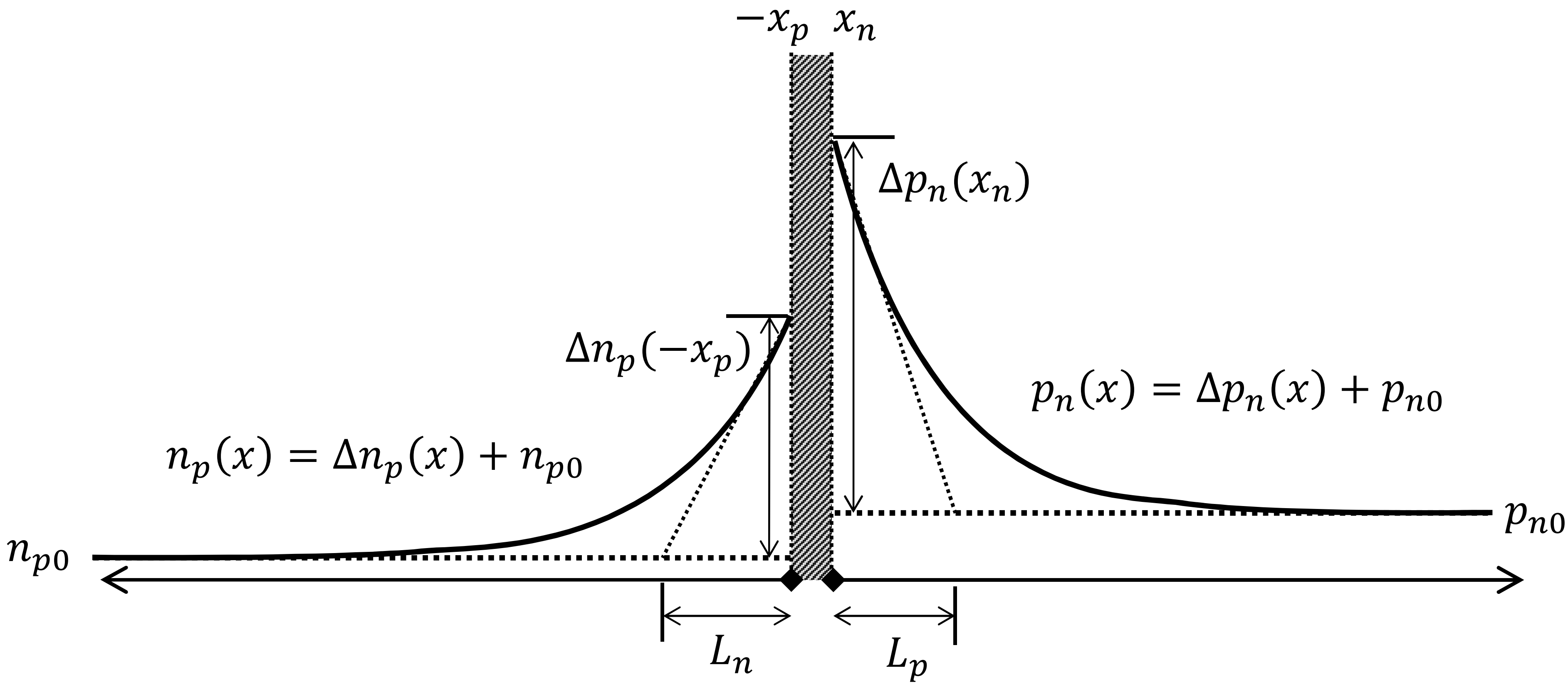
That is, one has to consider only the lightly doped side of such junction in working out the diode I-V characteristics.

pn Junction: I-V Characteristic

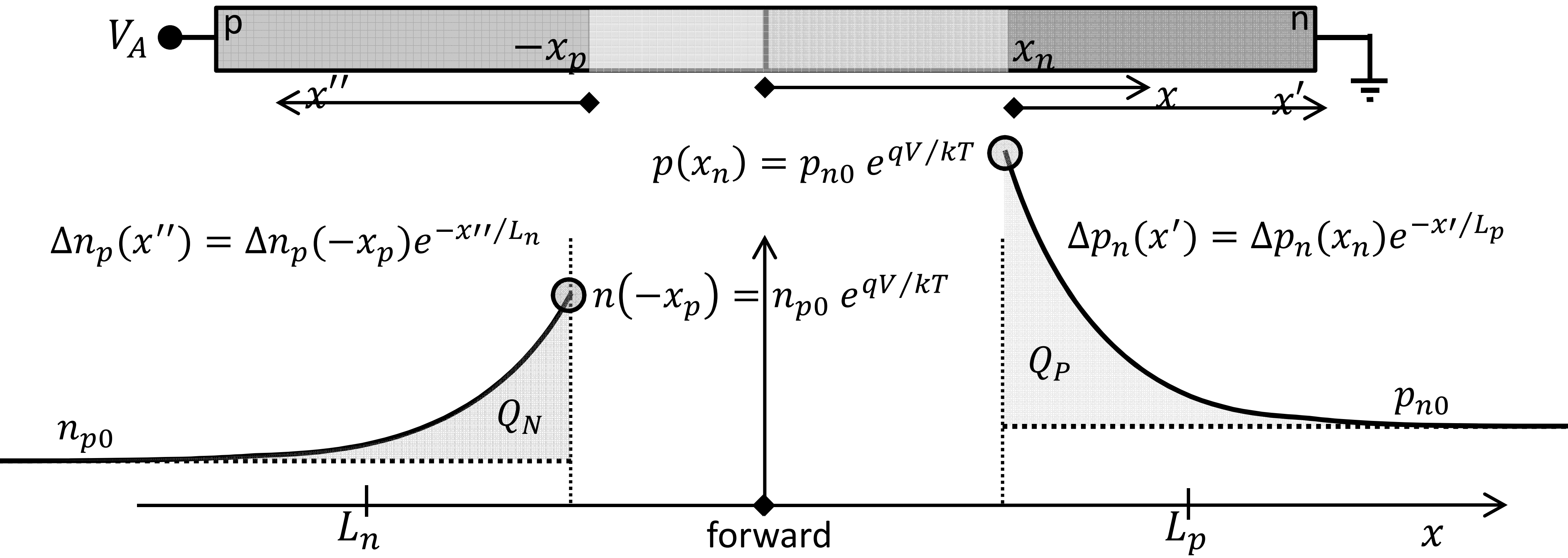


pn Junction: I-V Characteristic

The minority carrier concentrations on either side of the junction under forward bias



Minority-Carrier Charge Storage

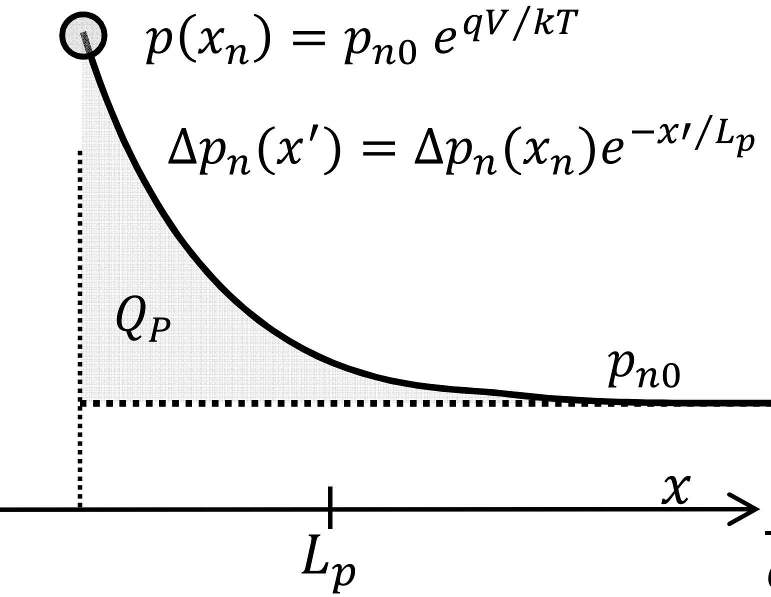


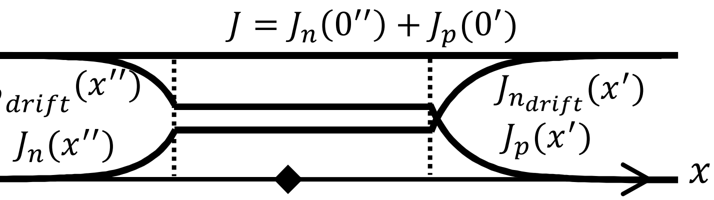
$Q_N = -qA\Delta n_p(-x_p)L_n$ $\qquad$ $Q_P = -qA\Delta p_n(x_n)L_P$

$$\frac{\partial \Delta n_p}{\partial t} = \frac{1}{q}\frac{\partial J_n}{\partial x} - \frac{\Delta n_p}{\tau_n} + \cancel{G_{ext}}$$

$$\frac{\partial \Delta p_n}{\partial t} = -\frac{1}{q}\frac{\partial J_p}{\partial x} - \frac{\Delta p_n}{\tau_p} + \cancel{G_{ext}}$$

Charge Control Model





In general: $\Delta p_n(x,t)$

$$\frac{\partial \Delta p_n}{\partial t} = -\frac{1}{q}\frac{\partial J_p}{\partial x} - \frac{\Delta p_n}{\tau_p}$$

$$\frac{\partial (qA\Delta p_n)}{\partial t} = -A\frac{\partial J_p}{\partial x} - \frac{qA\Delta p_n}{\tau_p}$$

$$\frac{\partial}{\partial t}\underbrace{\left[qA\int_{x_n}^{\infty}\Delta p_n dx\right]}_{Q_P} = -A\int_{J(x_n)}^{J(\infty)} dJ_p - \frac{1}{\tau_p}\left[qA\int_{x_n}^{\infty}\Delta p_n dx\right]$$

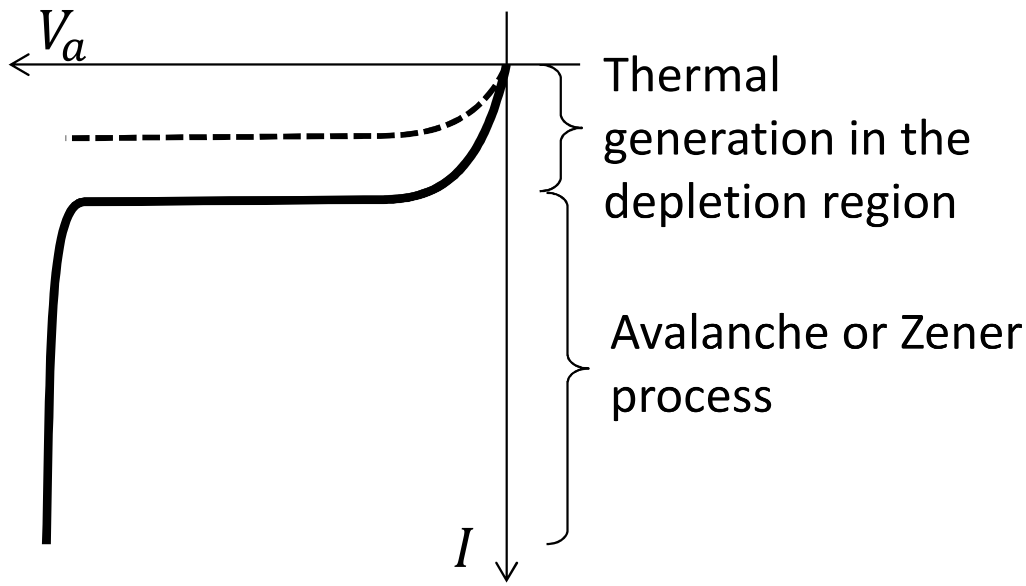
$$\frac{d}{dt}Q_P = AJ_p(x_n) - \frac{Q_P}{\tau_p}$$

$$\frac{d}{dt}Q_P = I_p(x_n) - \frac{Q_P}{\tau_p}$$

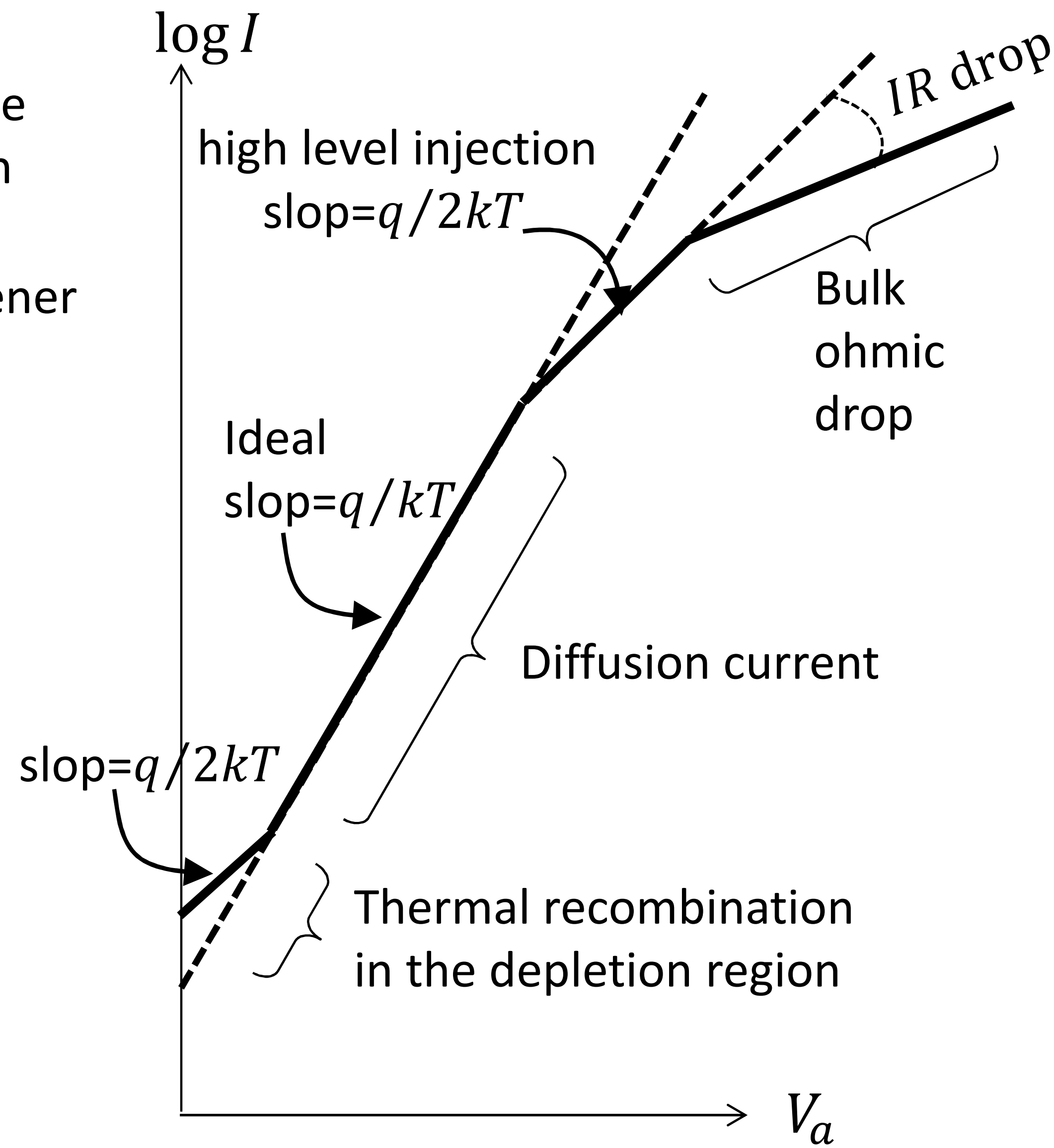
Steady state: $\frac{d}{dt} = 0$

$I_p(x_n) = \frac{Q_P}{\tau_p}$ similarly $I_n(-x_p) = \frac{Q_P}{\tau_n}$

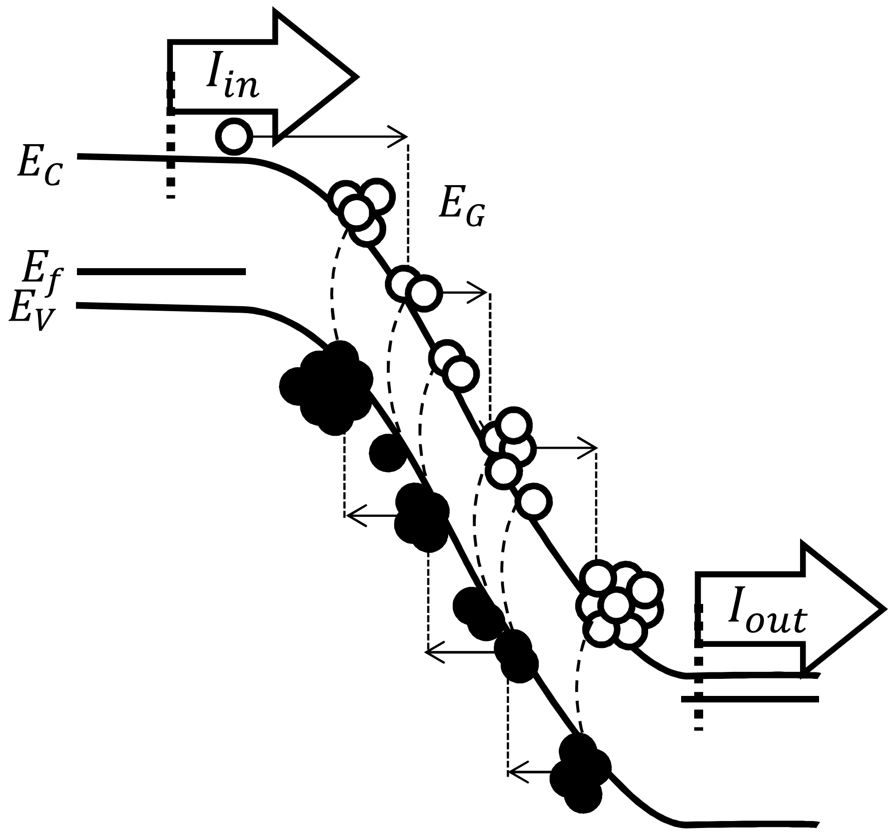
Deviations from Ideal I-V



Diode in break down has application!



Avalanche Breakdown



occurs when the minority carriers that cross the depletion region under the influence of the electric field gain sufficient kinetic energy to be able to break covalent bands in atoms with which they collide.

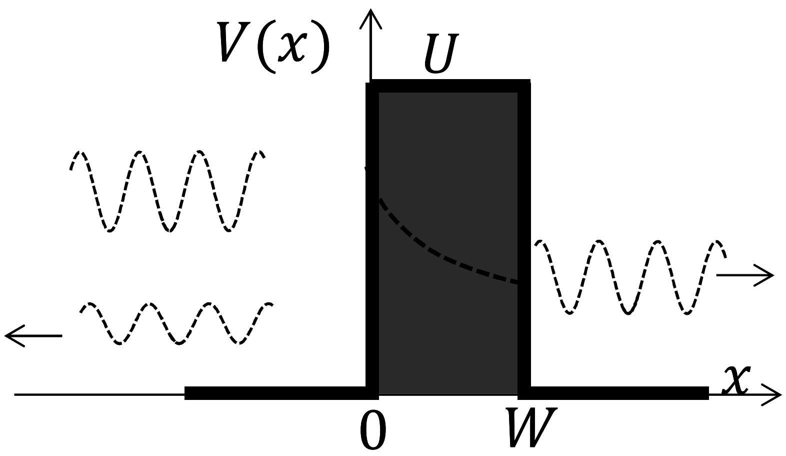
multiplication factor :

$$M = \frac{I_{out}}{I_{in}} = \frac{1}{1 - \left(\frac{V_A}{V_{BR}}\right)^m} \quad (3 < m < 6)$$

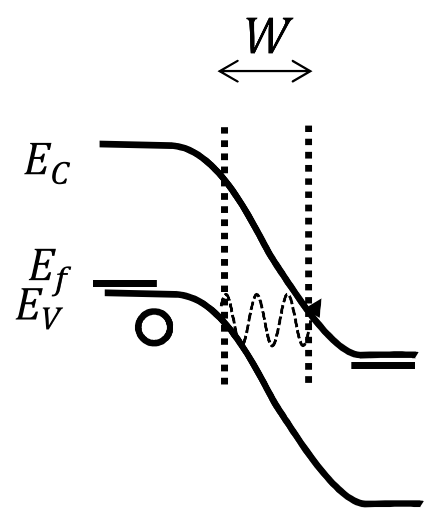
$$|\mathcal{E}_{max}| = \sqrt{\frac{2\,q(V_{bi} - V_A)}{\epsilon_s}\left(\frac{1}{N_D} + \frac{1}{N_A}\right)}$$

$$|\mathcal{E}_{max}| = cte \quad \rightarrow \quad V_{BR} \propto \frac{N_A + N_D}{N_A N_D}$$

Zener Breakdown



$$T \sim \exp[-\frac{2W}{\hbar}\sqrt{2m(U-E)}]$$ For $U \gg E$



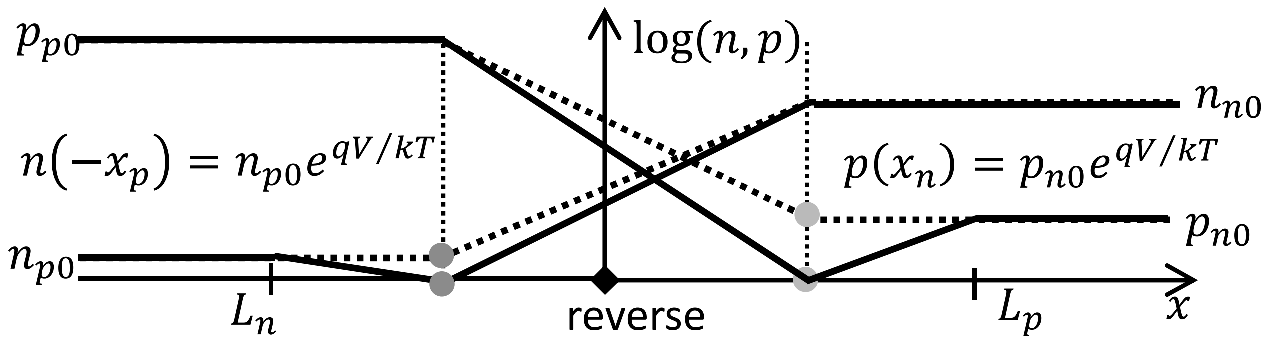
For non-degenerately doped material:

$$N_A\,, N_D \nearrow \;\Longrightarrow\; W \searrow \;\Longrightarrow T \nearrow$$

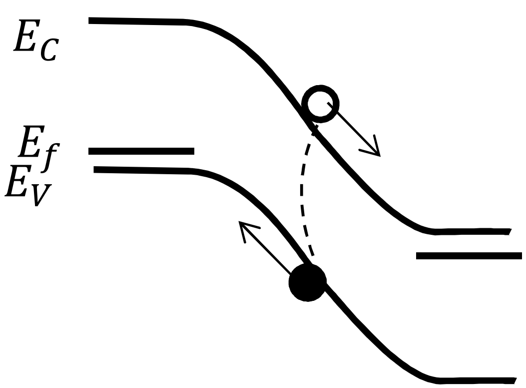
Generation in Depletion Region

Reminder1: $r_i = \alpha_i np$ $g_i = \alpha_i n_i^2$ Thermal equilibrium $r_i = g_i$

Reminder 2:



In depletion region: $np < n_i^2$ $\rightarrow r < g$ Generation > Recombination



$$I_G = -qA \int_{-x_p}^{x_n} G\, dx = -qA \frac{n_i}{2\tau_0} W$$

Effective carrier life time $\tau_0 = \frac{1}{2}(\tau_n + \tau_p)$

$$I = I_0\left(e^{qV/kT} - 1\right) + I_G$$

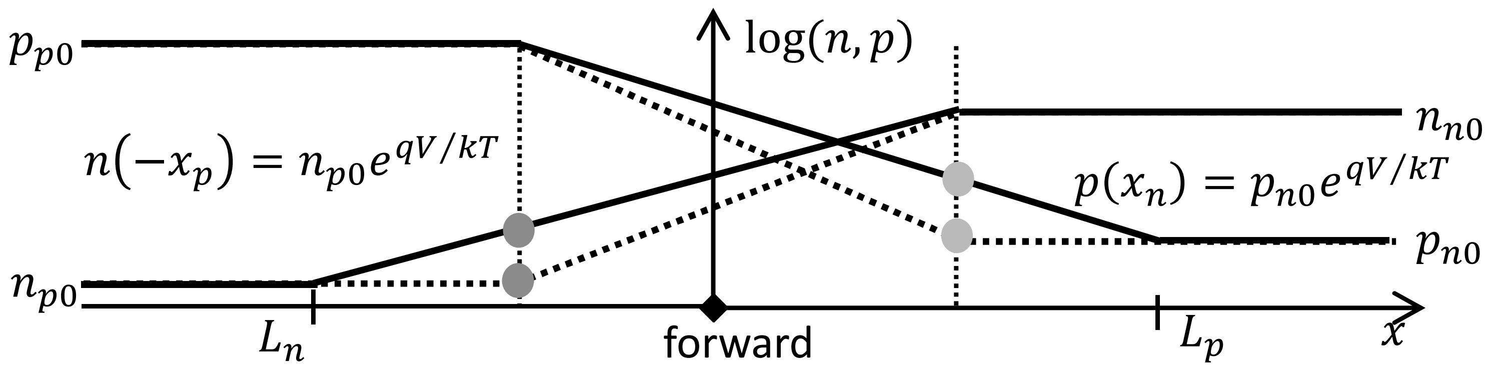
Recombination in Depletion Region

Reminder1: $r_i = \alpha_i np$

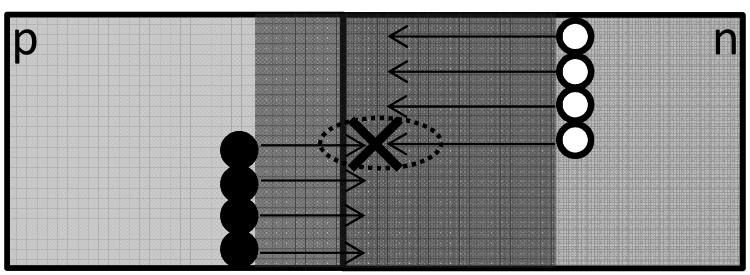
$g_i = \alpha_i n_i^2$

Thermal equilibrium $r_i = g_i$

Reminder 2:



In depletion region: $np > n_i^2$ $\rightarrow r > g$ Recombination > Generation

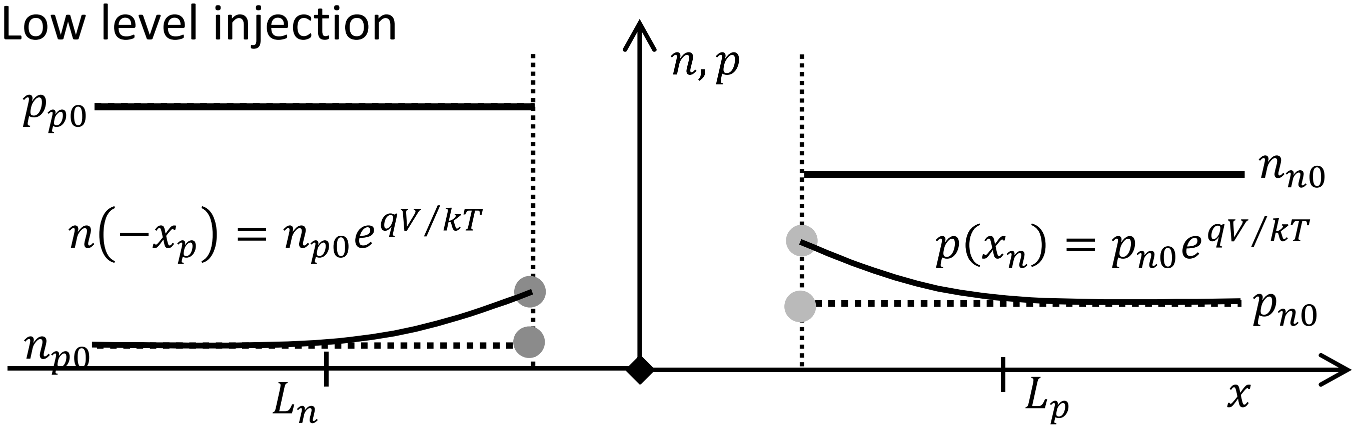


$$I_R = qA\frac{n_i}{2\tau_0}W\left(e^{qV/2kT} - 1\right)$$

$$I = I_0\left(e^{qV/kT} - 1\right) + I_R$$

High Level Injection

Low level injection



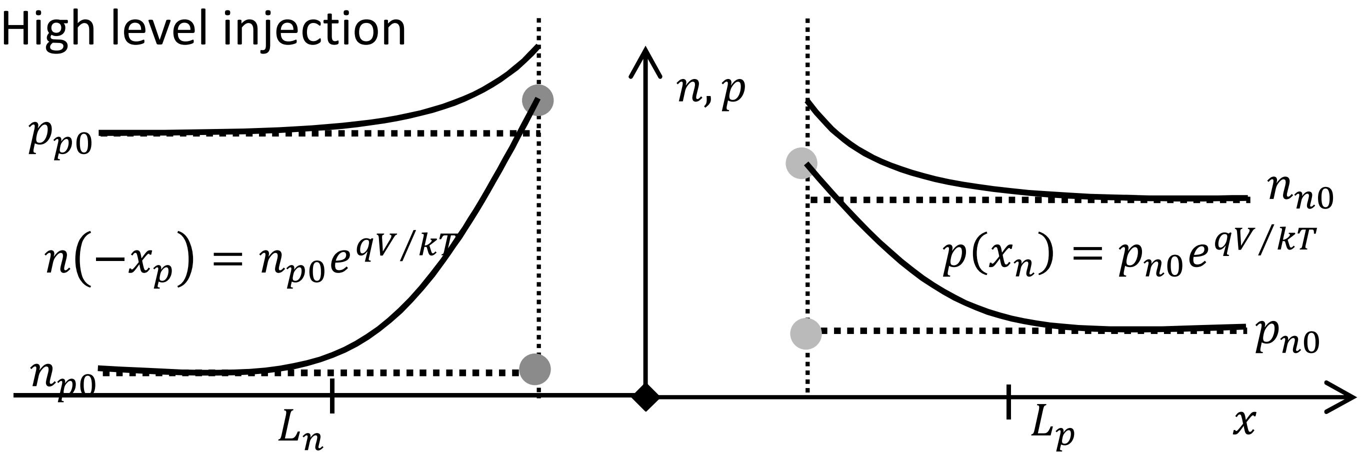
All of the relations was based on the low level injection condition as:

$$n_p + \delta n_p \ll p_p$$
$$p_n + \delta p_n \ll n_n$$

Minority << Majority

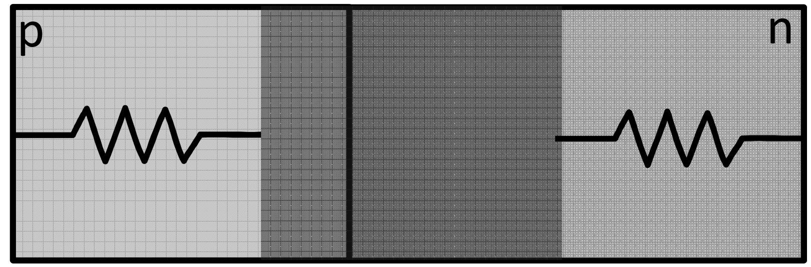
In High level injection condition we should add recombination current to the continuity equations for the minority carriers, result will be as: $I \propto e^{qV/2kT}$

High level injection



Series Resistance

We assumed that the electric field outside the depletion region is zero; which means as semiconductor is treated as a perfect(ideal) conductor.
But actually the conductivity is limited to

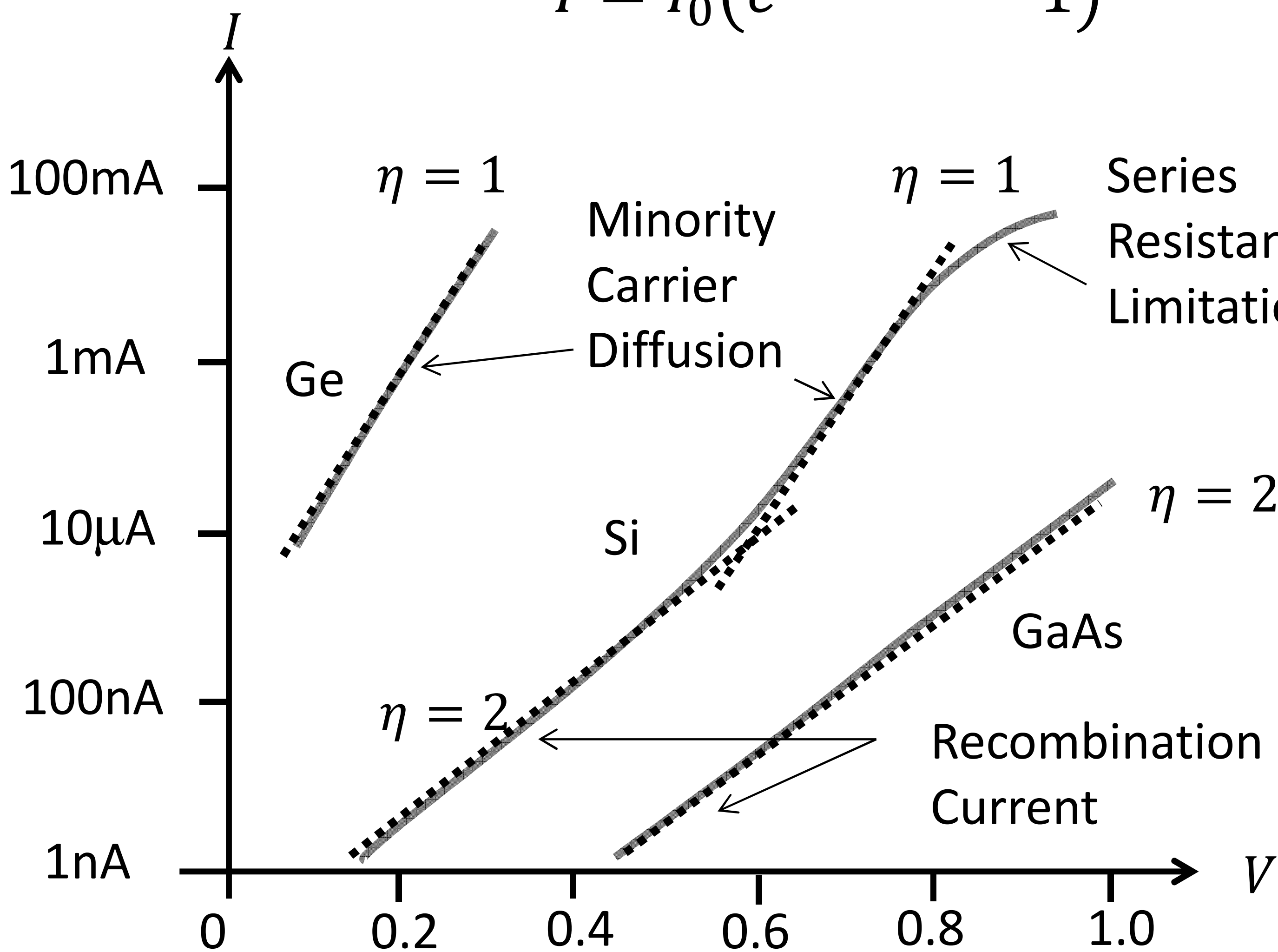


$$\sigma = q(\mu_n n + \mu_p p)$$

Hence the “ohmic voltage drop” outside depletion region becomes considerable

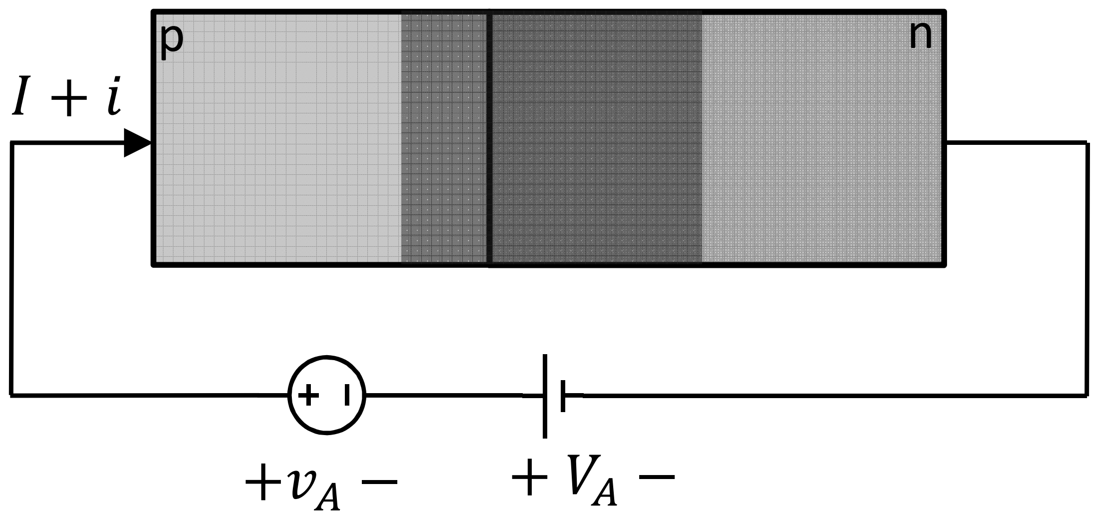
Forward Bias

$$I = I_0\left(e^{qV/\eta kT} - 1\right)$$



10

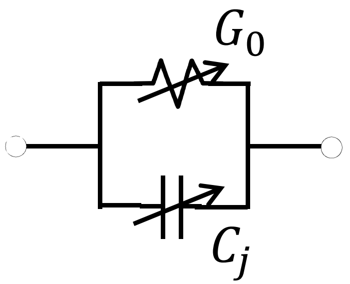
Small Signal



A small ac signal (v_A) is superimposed on the DC bias.
This results in ac current (i). Then, admittance Y is given by

$$Y = G + j\omega C = \frac{i}{v_A}$$

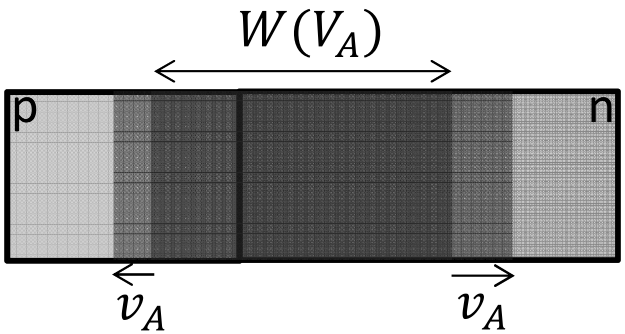
Reverse Bias Admittance



$$Y = G_0 + j\omega C_j$$

C_j : Junction (depletion layer) capacitance
G_0: Reverse bias conductance

A pn junction under reverse bias behaves like a capacitor.
Such capacitors are used in ICs as voltage-controlled capacitors.

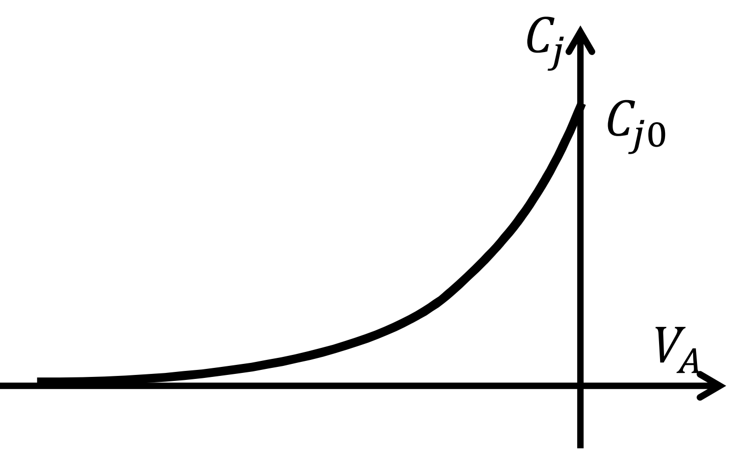


$$W = \sqrt{\frac{2\epsilon_s(V_{bi} - V_A)}{q}\left(\frac{1}{N_D} + \frac{1}{N_A}\right)}$$

$$C_j = \frac{\epsilon_s A}{W} = \frac{C_{j0}}{\left(1 - \frac{V_A}{V_0}\right)^{1/2}}$$

where

$$C_{j0} = \epsilon_s A \Big/ \sqrt{\frac{2\epsilon_s V_{bi}}{q}\left(\frac{1}{N_D} + \frac{1}{N_A}\right)}$$



$$C_j = \frac{C_{j0}}{\left(1 - \frac{V_A}{V_0}\right)^m} \quad \begin{cases} m = 1/2 & \text{step junction} \\ m = 1/3 & \text{linear junction} \end{cases}$$

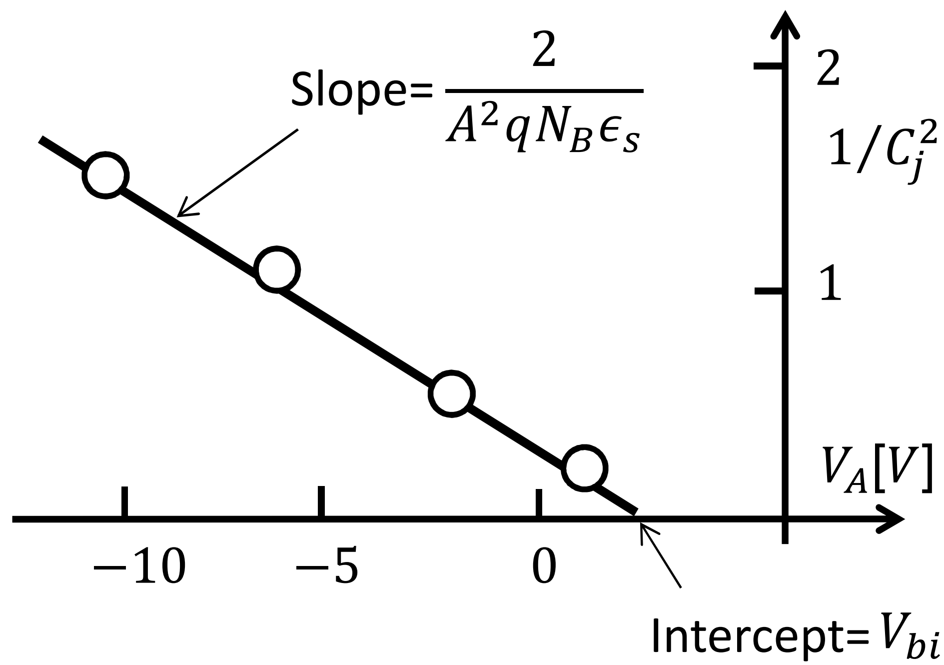
C-V curve is very useful for characterization of the devices

Reverse Bias Admittance - Characterization

C-V data from a pn junction is routinely used to determine the doping profile on the lightly doped side of the junction.

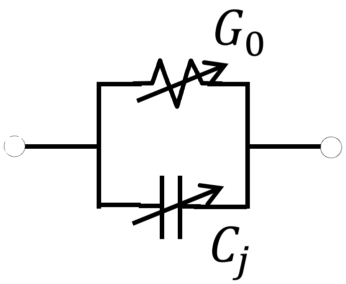
$$C_j = \frac{\epsilon_s A}{W} = A\sqrt{\frac{\epsilon_s q N_B}{2(V_{bi} - V_A)}}$$

$$\frac{1}{C_j^2} = \frac{2}{A^2 q N_B \epsilon_s}(V_{bi} - V_A)$$



If the doping on the lightly doped side is uniform, a plot of $1/C_j^2$ versus V_A should be a straight line with a slope inversely proportional to N_B and an extrapolated $1/C_j^2 = 0$ intercept equal to V_{bi}.

Reverse Bias Admittance



$$Y = G_0 + j\omega C_j$$

C_j : Junction (depletion layer) capacitance

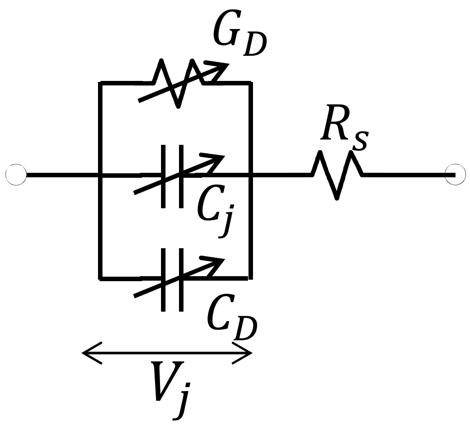
G_0: Reverse bias conductance

$$G_0 = \frac{i}{v_A} = \frac{dI}{dV} = I_0 \frac{q}{kT} e^{qV/kT} \quad \rightarrow \quad r = \frac{1}{G_0} = \frac{kT/q}{I - I_0}$$

Hence , in reverse bias, ideally

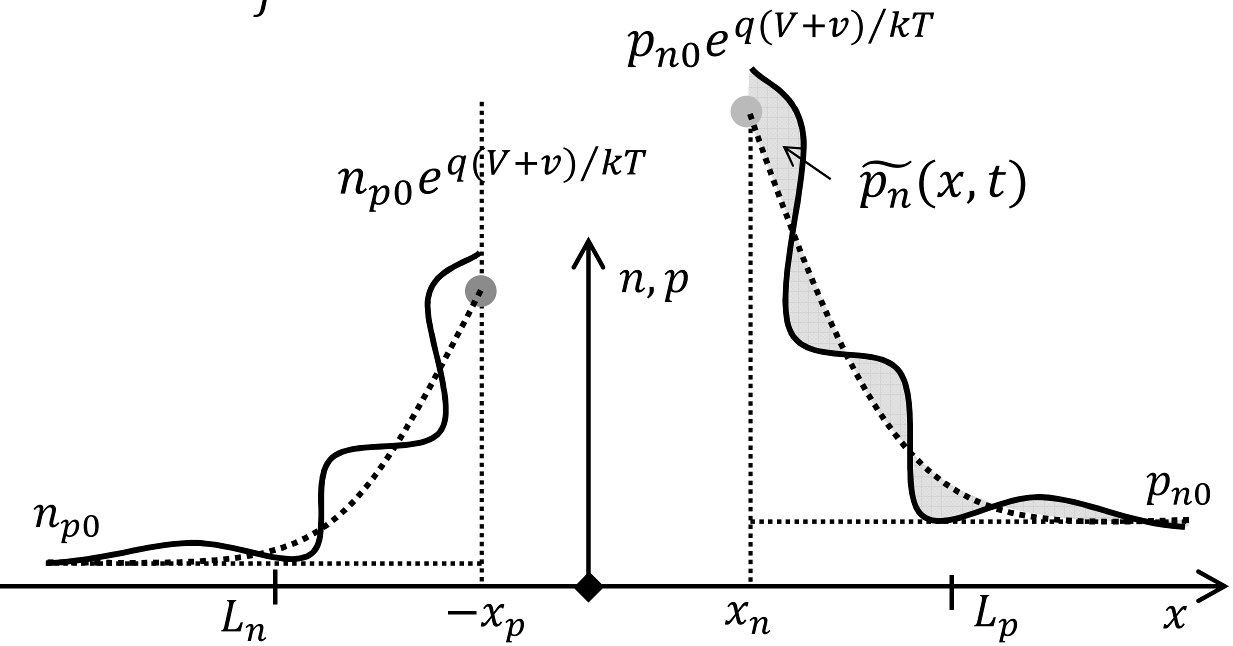
$$I \sim I_0 \quad \rightarrow \quad G_0 \sim 0$$

Forward Bias Admittance



R_s: ohmic (physical) resistance
C_j : Junction capacitance
G_D: diffusion conductance
C_D: diffusion capacitance

} Function of bias point and frequency



$$p_n(x) \mapsto p_n(x,t)$$

$$\Delta p_n(x,t) = \Delta\overline{p_n}(x) + \widetilde{p_n}(x,t)$$

$$\frac{\partial \Delta p_n}{\partial t} = D_p \frac{\partial^2 \Delta p_n}{\partial x^2} - \frac{\Delta p_n}{\tau_p}$$

$$\begin{cases} 0 = D_p \dfrac{\partial^2 \Delta\overline{p_n}}{\partial x^2} - \dfrac{\Delta\overline{p_n}}{\tau_p} \\ \dfrac{\partial \widetilde{p_n}}{\partial t} = D_p \dfrac{\partial^2 \widetilde{p_n}}{\partial x^2} - \dfrac{\widetilde{p_n}}{\tau_p} \end{cases}$$

Forward Bias Admittance

$$\frac{\partial \widetilde{p_n}}{\partial t} = D_p \frac{\partial^2 \widetilde{p_n}}{\partial x^2} - \frac{\widetilde{p_n}}{\tau_p}$$

Phasor representation $\widetilde{p_n}(x,t) = \widehat{p_n} e^{j\omega t}$

$$\frac{d^2 \widehat{p_n}}{dx^2} = \frac{\widehat{p_n}\,(1 + j\omega\tau_p)}{D_p \tau_p} = \frac{\widehat{p_n}}{{L_p^*}^2}$$

where ${L_p^*}^2 = \frac{L_p^2}{1 + j\omega\tau_p}$

$$\widehat{p_n}(x) = \cancel{B_1} e^{x/L_p^*} + B_2 e^{-x/L_p^*}$$

$$p_n(0,t) = p_{n0} e^{q(V+v)/kT} \approx p_{n0} e^{qV/kT} (1 + \frac{qv(t)}{kT})$$

$$\rightarrow B_2 = p_{n0} e^{qV/kT} \frac{qv}{kT}$$

one-sided diode

$$i = -qAD_p \left.\frac{d\widehat{p_n}}{dx}\right|_{x=0} = qA \frac{D_p}{L_p^*} p_{n0} e^{qV/kT} \frac{qv}{kT}$$

$$Y = \frac{i}{v} = \frac{q}{kT} A(q \frac{D_p}{L_p^*} p_{n0})\, e^{qV/kT} = \underbrace{\frac{q}{kT} A(q \frac{D_p}{L_p} \sqrt{1 + j\omega\tau_p} p_{n0})\, e^{qV/kT}}$$

$$Re\{\quad\} = G \qquad Im\{\quad\} = \omega C$$

Forward Bias Admittance

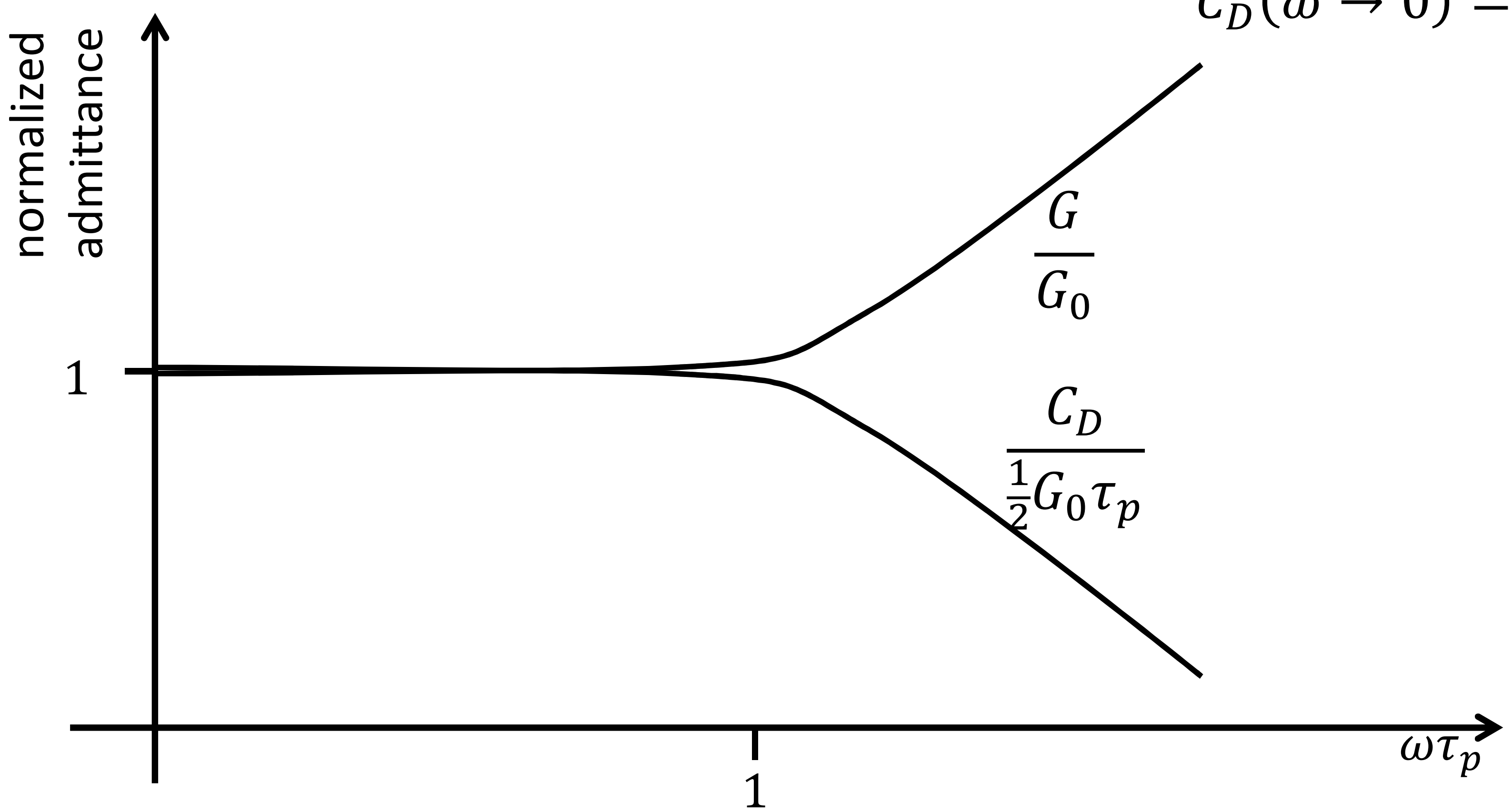
$$C_D = \frac{Im\{Y\}}{\omega}$$

$$G = Re\{Y\}$$

$\omega \to 0:$ $$G_0 = \frac{qA}{kT}\left(q\,\frac{D_p}{L_p}\,p_{n0}\right) e^{qV/kT}$$

$$Y = G_0\sqrt{1 + j\omega\tau_p}$$

$$C_D(\omega \to 0) = \tfrac{1}{2}G_0\tau_p$$

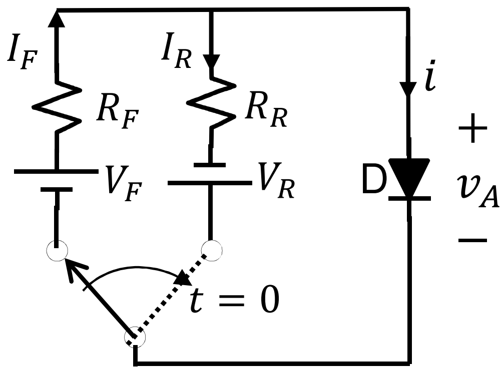


pn Junction Transient Response

Turn-off transient

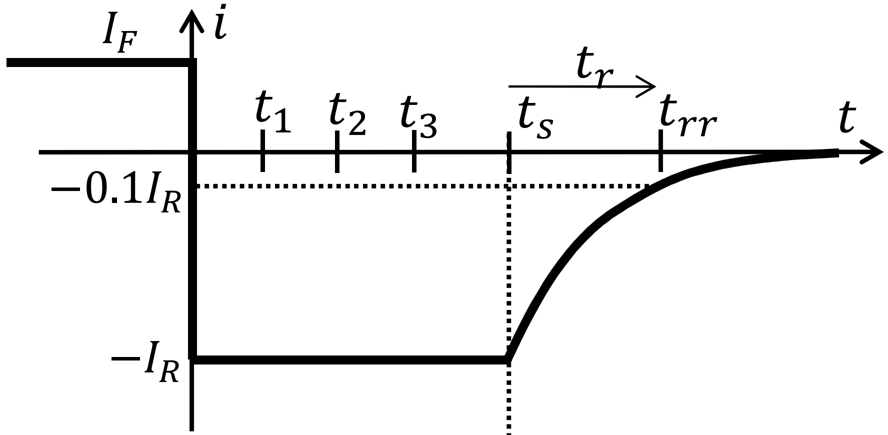
t_s : storage time

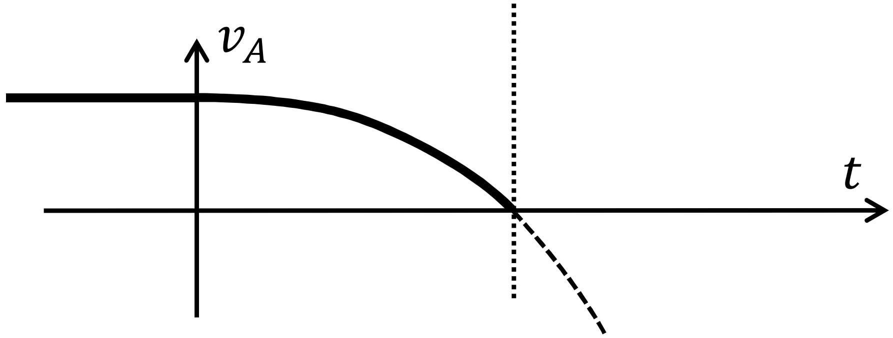
t_r : recovery time

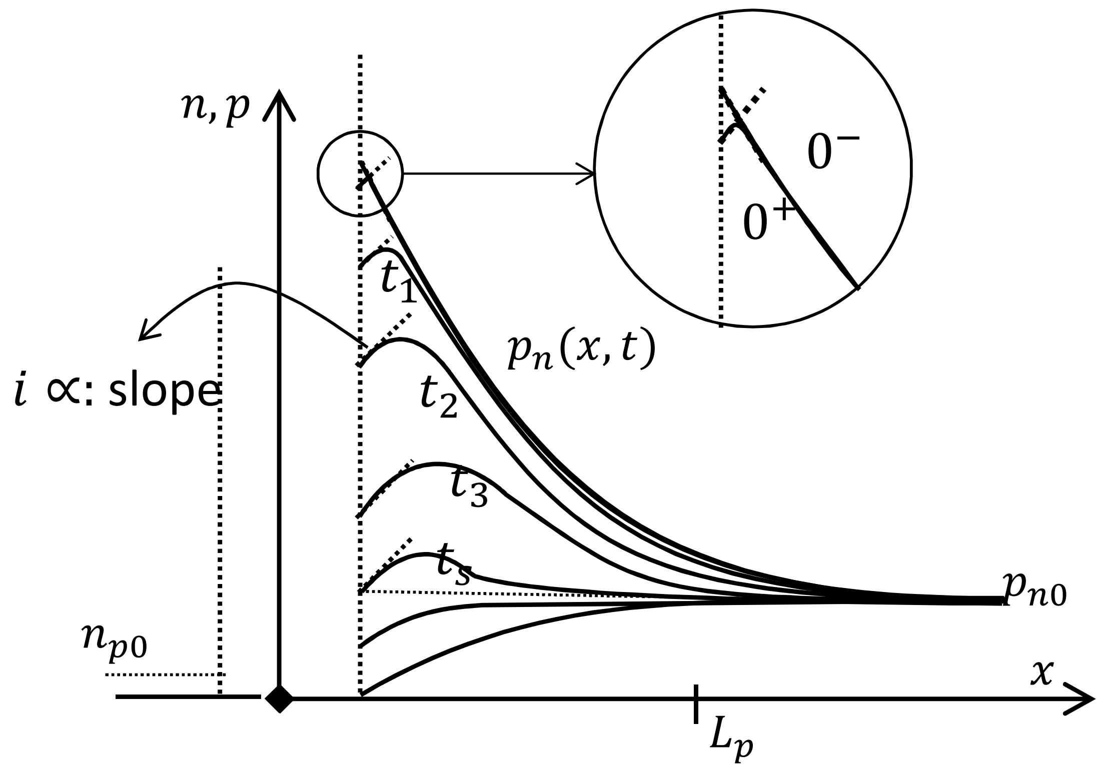


$$@\ t = 0^- \ (V_F \gg V_A) \ \rightarrow \ I_F = \frac{V_F}{R_F} \quad (as\ V_A < 0.7V)$$

$$@\ t = 0^+ \quad I_R = \frac{V_R + V_A}{R_R} \sim \frac{V_R}{R_R}$$







pn Junction Transient Response

charge control for p+n diode

$$\frac{dQ_p(t)}{dt} = i(t) - \frac{Q_p(t)}{\tau_p}$$

for $0 < t < t_s$: $i(t) = -I_R$

$$\frac{dQ_p(t)}{dt} = -I_R - \frac{Q_p(t)}{\tau_p} \rightarrow \int_{Q_p(0^+)}^{Q_p(t_s)=0} \frac{dQ_p(t)}{I_R + \frac{Q_p(t)}{\tau_p}} = -\int_0^{t_s} dt = -t_s = -\tau_p \ln\left(1 + \frac{Q_p(0^+)}{I_R \tau_p}\right)$$

But for $t = 0^-$:

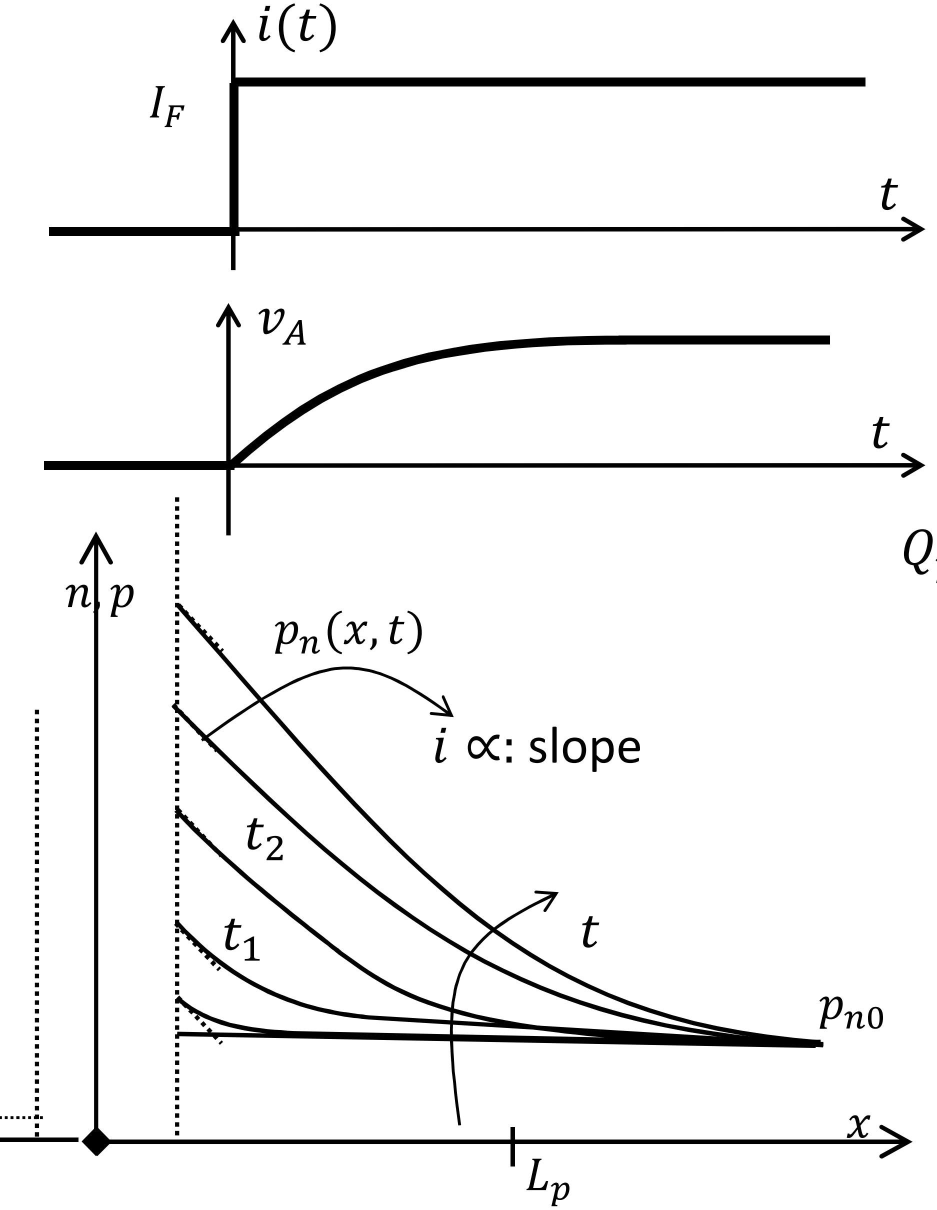
$$\frac{dQ_p}{dt} = 0 = I_F - \frac{Q_p(0^-)}{\tau_p} \quad \rightarrow \quad Q_p(0^-) = Q_p(0^-) = I_F \tau_p$$

$$t_s = \tau_p \ln\left(1 + \frac{I_F}{I_R}\right)$$

$$I_F \searrow, I_R \nearrow \Longrightarrow \quad t_s \searrow$$

pn Junction Transient Response

Turn-on transient



$$\frac{dQ_p(t)}{dt} = I_F - \frac{Q_p(t)}{\tau_p}$$

$$Q_p(t) = I_F \tau_p (1 - e^{-t/\tau_p})$$

$$p_n(x', t) = p_{n0} e^{qV/kT} e^{-x'/L_p}$$

$$Q_p(t) = I_F \tau_p \left(1 - e^{-t/\tau_p}\right) = qAp_{n0}L_P(e^{qV/kT} - 1)$$

$$v(t) = \frac{kT}{q} \ln\left(1 + \frac{I_F \tau_p (1 - e^{-t/\tau_p})}{qAp_{n0}L_P}\right)$$

If we define t_{ON} $(v\!:\;\; 0 \mapsto 0.9 v_\infty)$

$$\tau_F \searrow, I_F \searrow \;\Longrightarrow\; t_{ON} \searrow$$