Adaptive Filters
HW Solutions
#6

Problem 1:

In order to insure that the RLS adaptive filter has linear phase, all that one needs to do is to define a new set of input signals. For a 5-tap filter one would use

$$\mathbf{z}(n) = \begin{bmatrix} u(n) + u(n-4) \\ u(n-1) + u(n-3) \\ u(n-2) \end{bmatrix}$$

Except for using $\mathbf{z}(n)$ as the input signal, the RLS equations are unchanged.

Problem 2:

(a) For convergence in mean with the LMS algorithm

$$0 < \mu < 2/\lambda_{max} = 2$$

With $\mu = \mu_{max}/10 = 1/5$, the convergence time is

$$\tau = \frac{1}{\mu\lambda_{min}} = \frac{5}{\lambda_{min}} = 500$$

For an adaptive filter with M tap weights, the number of multiplications required per update is $2M + 1$ so, for convergence, we require $1000M + 500$ multiplications.

The RLS algorithm converges in $2M$ iterations and the number of multiplications required per iteration (using $\lambda = 1$) is $2M^2 + 4M$. Therefore, if

$$1000M + 500 = 4M^3 + 8M^2$$

the RLS and LMS adaptive filters are approximately equal in terms of the computational requirements necessary to reach convergence. Ignoring the constant term and solving for M we find that the two are approximately equivalent when $M = 15$ (hence, ignoring the constant is justified).

(b) Reasons why one may want to consider using RLS instead of LMS are

1. Convergence of RLS is not dependent on the eigenvalues of the autocorrelation matrix of $u(n)$.
2. There is no misadjustment if the growing window RLS algorithm is used.

Problem 3:

(a) Beginning with the RLS coefficient update equation

$$\mathbf{w}(n) = \mathbf{w}(n-1) + \alpha(n)\mathbf{k}(n)$$

we have

$$\mathbf{x}^T(n)\mathbf{w}(n) = \mathbf{x}^T(n)\mathbf{w}(n-1) + \alpha(n)\mathbf{x}^T(n)\mathbf{k}(n)$$

Therefore,

$$\begin{aligned}
\epsilon(n) &= d(n) - \mathbf{u}^T(n)\mathbf{w}(n) \\
&= d(n) - \mathbf{u}^T(n)\mathbf{w}(n-1) + \alpha(n)\mathbf{u}^T(n)\mathbf{k}(n) \\
&= \alpha(n) - \alpha(n)\mathbf{u}^T(n)\mathbf{k}(n) \\
&= \alpha(n)\left[1 - \mu(n)\mathbf{u}^T(n)\mathbf{R}^{-1}(n-1)\mathbf{u}(n)\right] \\
&= \alpha(n)\mu(n)
\end{aligned}$$

(b) Since

$$\begin{aligned}
\mu(n) &= d(n) - \mathbf{u}^T(n)\mathbf{k}(n) \\
&= d(n) - \mu(n)\mathbf{r}^T(n)\mathbf{R}^{-1}(n-1)\mathbf{u}(n) \\
&= d(n) - \left[1 - \mu(n)\right]
\end{aligned}$$

then

$$d(n) = 1$$