تکلیف سری پنجم فیلترهای وفقی

Problem 1: In the method of steepest descent, the acceptable range of values for the step size to insure convergence is

$$0 < \alpha < 2/\lambda_{max}$$

It is of more interest, however, to determine the value of α that gives the fastest rate of convergence. For a fixed α the speed of convergence of the weight vector $\mathbf{w}(n)$ is dominated by the slowest converging mode in

$$\mathbf{w}(n) = \mathbf{w}_0 + \sum_{k=1}^{M} \mathbf{q}_k(1 - \alpha\lambda_k)^n v_k(0)$$

(a) Find the value for α that maximizes the rate of convergence.

(b) At what rate does the slowest mode decrease for the optimum step size found in (a)?

Problem 2: Newton's method applied to the minimization of the error function

$$J(\mathbf{w}) = E\{e^2(n)\}$$

where $e(n) = d(n) - \mathbf{w}^T\mathbf{x}(n)$ is given by

$$\mathbf{w}(n+1) = \mathbf{w}(n) - \tfrac{1}{2}\mathbf{R}^{-1}\nabla_{\mathbf{w}} J(\mathbf{w})$$

where $\mathbf{R}$ is the autocorrelation matrix for $x(n)$. If we add a step-size parameter μ to the Newton algorithm we have

$$\mathbf{w}(n+1) = \mathbf{w}(n) - \tfrac{1}{2}\mu\mathbf{R}^{-1}\nabla_{\mathbf{w}} J(\mathbf{w}) \tag{1}$$

Comparing (1) to a steepest descent algorithm we see that the step size parameter μ has been replaced with a matrix, $\mu\mathbf{R}^{-1}$.

(a) For what values of μ is the Newton algorithm (1) stable?

(b) What is the optimum value of μ, i.e., for what value of μ is the convergence the fastest?

(c) Suppose that we form an LMS-type algorithm by replacing the gradient with the gradient estimate

$$\hat{\nabla}(n) = \nabla_{\mathbf{w}} e^2(n)$$

Evaluate the gradient and find the resulting coefficient update equation. How does this differ from the LMS algorithm?

(d) Derive an expression that defines the time evolution of $E\{\mathbf{w}(n)\}$ using the LMS-type Newton algorithm derived in (c).

Problem 3: In this problem we explore another way of deriving the steepest descent algorithm that is used to adjust the tap weights in a tranversal filter. The inverse of a positive definite matrix may be expanded in a power series as follows

$$\mathbf{R}^{-1} = \alpha \sum_{k=0}^{\infty} (\mathbf{I} - \alpha \mathbf{R})^k$$

where $\mathbf{I}$ is the identity matrix and α is a positive constant. To ensure convergence of the series, the constant α must lie inside the range

$$0 < \alpha < 2/\lambda_{\max}$$

where $\lambda_{\max}$ is the largest eigenvalue of the matrix $\mathbf{R}$. By using this series expansion for the inverse of the correlation matrix in the normal equations, develop the recursion

$$\mathbf{w}(n+1) = \mathbf{w}(n) - \alpha[\mathbf{R}\mathbf{w}(n) - \mathbf{p}]$$

where $\mathbf{w}(n)$ is the approximation to the Wiener solution for the tap-weight vector

$$\mathbf{w}(n) = \alpha \sum_{k=0}^{n-1} (\mathbf{I} - \alpha \mathbf{R})^k \mathbf{p}$$

Problem 4:

In some applications, it may be necessary to delay the update of the filter coefficients for a short period of time. For example, in decision-directed feedback equalization, if a sophisticated algorithm such as Viterbi decoding is used to improve the decisions, then the desired signal and thus the error is not available until a number of sample periods later. Therefore, consider the delayed LMS algorithm which has a filter coefficient update equation given by

$$\mathbf{w}(n+1) = \mathbf{w}(n) + \mu e(n - n_0)\mathbf{u}(n - n_0)$$

where

$$e(n - n_0) = d(n - n_0) - y(n - n_0)$$

Note that if the delay, n_0, is equal to zero then we have the conventional LMS algorithm.

(a) For $n_0 = 1$, determine the values of μ for which the delayed LMS algorithm converges in the mean.

(b) If $\lambda_k = 1$, for $k = 1, \ldots, N$ and if the step size $\mu = 0.1$, find the time constant, τ_L for the LMS adaptive filter ($n_0 = 0$) and the time constant τ_D for the delayed LMS adaptive filter with $n_0 = 1$ (You may assume that the signal and weight vectors are uncorrelated).