HW #6

Adaptive Filters

Problem 1

Modify the RLS algorithm so that the adaptive filter, $W(z)$, has linear phase. Note that a linear phase filter has symmetric unit sample response. Thus, for example, if we had a five-tap filter then the coefficient vector, $\mathbf{w}$, would be of the form

$$\mathbf{w}^T(n) = [w_0(n), w_1(n), w_2(n), w_1(n), w_0(n)]$$

Problem 2

There are many different ways that one may compare the performance of adaptive filtering algorithms. Suppose, for example, that we are interested in adaptive linear prediction and our measure of performance is the number of arithmetic operations required for the adaptive filter to "converge".

(a) If the eigenvalues of the $M \times M$ correlation matrix for $u(n)$ are

$$\lambda_1 = 1.0 \quad \text{and} \quad \lambda_2 = \cdots = \lambda_M = 0.01$$

and if we choose the step size μ for the LMS algorithm so that it is one tenth the largest possible value for convergence in the mean, for approximately what order filter, M, are the RLS and LMS adaptive filters equal in terms of their computational requirements to reach convergence? Assume that you are using the conventional RLS algorithm as opposed to a *fast RLS* algorithm.

(b) For high order filters, $M >> 1$, the computational requirements of the RLS filter become extensive and the LMS algorithm becomes an attractive alternative. For what reasons might you prefer to use the RLS algorithm in spite of its computational cost?

(c) Based on our experience with the RLS algorithm, consider the following modification to the LMS algorithm

$$\mathbf{w}(n+1) = \mathbf{w}(n) + \mu e(n)\mathbf{R}^{-1}(n)\mathbf{u}(n)$$

where $\mathbf{R}^{-1}(n)$ is recursively updated using the matrix inversion lemma. Compare this to the LMS and RLS algorithms and describe qualitatively how its performance compares with LMS and RLS. For what types of applications would this modification be worth the extra computation.

Problem 3

Let $\alpha(n)$ be the a priori error and $e(n)$ the a posteriori error in the RLS algorithm and let

$$\mu(n) = \frac{1}{1 + \mathbf{u}^T(n)\mathbf{P}(n-1)\mathbf{u}(n)}$$

be the scaler used in the calculation of the gain vector $\mathbf{k}(n)$.

(a) Show that $e(n)$ may be written as a function of $\alpha(n)$ and $\mu(n)$

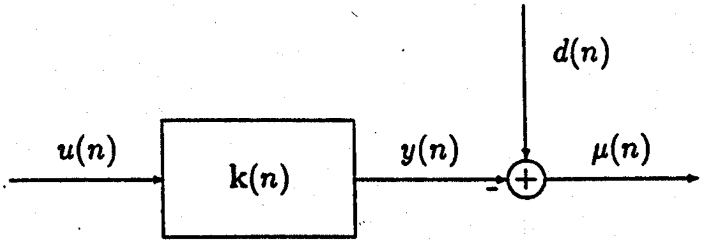
$$e(n) = f[\alpha(n), \mu(n)]$$

by finding the explicit relation between $e(n)$, $\alpha(n)$, and $\mu(n)$.

(b) Suppose that we have an RLS adaptive filter that is filtering a signal $u(n)$. Let $\mathbf{k}(n) = \mu(n)\mathbf{P}(n-1)\mathbf{u}(n)$ be the gain vector of the adaptive filter. Now consider the time-varying filter that has coefficients $\mathbf{k}(n)$ and an input $u(n)$, i.e.,

$$y(n) = \sum_{i=0}^{N-1} k_i(n)u(n-i)$$

where $u(n)$ is the same signal that is used in the RLS adaptive filter. For what signal, $d(n)$ shown in the figure below



is the difference signal $d(n) - y(n)$ equal $\mu(n)$? (Show your work! It is not sufficient to simply write down an answer.)