# Sources Indentification : A Solution Based on the Cumulants

**J.L. Lacoume* and P. Ruiz****

* CEPHAG, UA346 CNRS, ENSIEG, BP46, F-38402 St Martin d'Hères Cedex
** Société Techniphone, BP 22, F-13610 Le Puy Sainte Réparade

## Abstract

In a great number of situations (array processing, seismic sounding...), the signal received on an array of sensors is the sum of several contributions (sources). In general, it is realistic to assume that the sources emit independent stochastic signals. In this case, several methods using the second order statistic of the signal have been developed in order to detect the number of sources and to identify the sources. It is known that the source identification is then not feasible in the general case and some hypotheses (plane waves...) must be added.

When the signals emitted by the sources are not gaussian, the second order statistic does not contain a complete description of the statistical properties of the signals. We show that, in the non gaussian case, it is possible to identify the sources using the third and fourth order cumulants. We illustrate the possibilities of this method in simulated situations.

## 1. Introduction

In a great number of situations (array processing : submarine acoustics, seismic sounding...), the signal received on an array of sensors is the sum of several contributions that are called *sources*. In order to study the effect of each source on the whole signals, it is necessary to separate the sources by a filtering procedure. To achieve this separation, a partial (in general not total) identification of the sources must be done. A large number of works on that topic have been done using the second order properties of the signal which are resumed in the spectral matrix [1,2,3].

However, it has been shown that in the interesting case of more than one source, the information contained in the spectral matrix are not sufficient for source separation and that supplementary relations must be introduced [4]. It is well known, particularly in the case of a linear antenna with equispaced sensors, that the plane wave hypothesis is sufficient. This procedure leads to the large amount of works concerned with high resolution [5]. In a large class of real situations (near field measurement for example) the plane wave hypothesis is not valid. Other procedures have been proposed in this context : use of large band signal [6] or other kinds of hypothesis on the signal emitted by each source [7] not so restrictive as the plane wave model. Nevertheless all these approaches must use at the same time the information contained in the received signals <u>and</u> extra information not contained in them.

It will be useful to develop identification methods that are <u>only based</u> on the information contained in the received signals. This task is not feasible in the very common case (common essentially in theoretical works, but perhaps not so common in real life !) of gaussian signal. In this situation, the spectral matrix <u>contains all the information</u> and so the introduction of an a priori information is unavoidable. On the contrary, if the signals emitted by the sources are not gaussian, the spectral matrix related to the second order moments of the signals does not contain all the information and extra information are given by the moments of order greater than two. This idea is not new and a lot of works have shown the interest of the use of the higher order cumulants [8] in particular it is now well known that the multispectra issuing from these cumulants make feasible the indification of a non minimum phase filter knowing only its output [8].

In this communication we will show that in non gaussian situations, the use of higher order cumulants allows the multiple source identification without any supplementary hypothesis. We anticipate that this result will be of interest in a great number of real situations in which the pecularities of the propagation does not justify the plane wave hypothesis. As for the non gaussian hypothesis, it is well known that it is the most probable issue and a great number of models are developed for this situation [9]. These models have been proved useful for example in underwater acoustics.

After a short presentation of the cumulants we will study the simplest case of two sources identification. We will show that in the case of two sources and two sensors, the model is described by two parameters. We will then present the identification of these two parameters with the cumulants up to order 4. The potentialities of this new algorithm of sources identification will be illustrated on simulated data.

## 2. The cumulants

**2.1. Definition** [10]. If $(Y_1,...,Y_r)$ is a r variate random variable, the $r^{th}$ order joint cumulant $C[Y_1,...,Y_r]$ of $(Y_1,...,Y_r)$ is given by

$$C[Y_1,...,Y_r] = \sum (-1)^{p-1}\,(p-1)!\,\Big(E\Big\{\prod_{j\in v_1} Y_j\Big\}\Big)...\Big(E\Big\{\prod_{j\in v_p} Y_j\Big\}\Big) \quad (1)$$

where the summation extends over all partitions $(v_1,...,v_p)$, of $p = 1,...,r$, of $(1,...,r)$. For example, if $X_1$ and $X_2$ are two random variables, two partitions may occur : one with $v_1=\{1\}$, $v_2=\{2\}$, one with $v_1=\{1,2\}$. So $C[X_1,X_2] = E\{X_1X_2\} - E\{X_1\}\,E\{X_2\}$ is the cross-covariance function of $X_1$ and $X_2$.

**2.2. Main properties of cumulants** [10].

* If any group of the Y's are independent of the remaining Y's, then $C[Y_1,...,Y_r] = 0$. Particularly, if two "sources" $S_1$ and $S_2$ are independent :

$$C[\underbrace{S_1,...,S_1}_{p},\underbrace{S_2,...,S_2}_{n}] = C[S_1^p, S_2^n] = 0,\ \forall\,(p,n) \in N^2 \quad (2)$$

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* If $S_1$ and $S_2$ are two zero mean Gaussian processes,

$$C[S_1^p] = 0 \qquad \text{with } p > 2$$

$$C[S_1^p, S_2^n] = 0 \qquad \text{with } n+p > 2$$

So that all the statistical properties of 2 Gaussian processes are defined by their first and second order joint cumulant (or mean and variance function). Moreover, in this case, uncorrelation of the two processes implies independence. Yet if the processes are not Gaussian, we can have

$C[S_1, S_2] = 0$ (uncorrelation), but not necessarily

$C[S_1^p, S_2^n] = 0, \ \forall \ n > 2$ (independence)

* We use the fourth order joint cumulant to distinguish uncorrelated "sources" ( $C[S_1,S_2] = 0$ but $C[S_1^p,S_2^n] \neq 0$ for $n+p = 4$) and independent "sources ( $C[S_1^p, S_2^n] = 0$ ).

By this way, the identification of independent "sources" will be possible, by taking into account the information given by the value of the fourth order cumulant.

## 3. The model and its description

In this presentation, we limit ourself to 2 sources ($s_1(n)$,$s_2(n)$) and 2 measurements ($x_1(n)$,$x_2(n)$). The measurements are *linearly* related to the sources. And the sources emit random signal *which are statistically independent.* These two hypotheses: linearity and independency are the only done. We observe the two measured signals $x_1(n)$ and $x_2(n)$, each of them depends on the two sources. We want to construct a linear filter giving two outputs u(n) and v(n) such that *each of these output depends only on one source.*

We must be aware of the fact that with this modelization, it is impossible to obtain exactly the signals $s_1(n)$ (or $s_2(n)$) emitted by the sources. A simple way to see that is to apply a linear transformation at one source (say $s_1(n)$ gives $F(s_1(n) = w_1(n))$ and to see that the problem is exactly the same if we replace $s_1(n)$by $w_1(n)$. Moreover, it will also not be possible to associate each output u(n) and v(n) to one specific source. In order to do that, we must use other kind of information.

Let us go to the frequency domain. In this domain, we will note the signal by capital letters :

$$A(\lambda) \triangleq TZ\{a(n)\}_{z=e^{2\pi j\lambda}}$$

TZ : z transform

$\lambda$ : z transform restricted to the unit circle. $\lambda$ is a parameter similar to the frequency. We can obtain a frequency with the sampling period $T_E$ through :

$$\nu = \lambda / T_E$$

In the following, we will omit the variable $\lambda$.

The relation between the measurements ($X_i$) and the sources ($S_j$) can be written :

$$\left.\begin{aligned} X_1 &= H_{11} S_1 + H_{12} S_2 \\ X_2 &= H_{21} S_1 + H_{22} S_2 \end{aligned}\right\} \qquad (4)$$

$H_{ij}$ are the transfer functions of the linear filters relating X and S. The relation (4) contains 8 real unknowns but, as we have seen before, all of them are not identifiable. We have noticed that we can replace each source by a linear transform i.e. , in frequency, multiply $S_1$ and $S_2$ by arbitraries variables $\alpha$ and $\beta$. This leads to :

$$\left.\begin{aligned} X_1 &= H'_{11} u' + H'_{12} v' \\ X_2 &= H'_{21} u' + H'_{22} v' \end{aligned}\right\} \qquad (5)$$

$$\text{with : } \ u' = \alpha S_1 \ ; \ v' = \beta S_2 \ ; \ H'_{i1} = \frac{H_{i1}}{\alpha} \ ; \ H'_{i2} = \frac{H_{i2}}{\beta}$$

$\alpha$ and $\beta$ are arbitrary complex numbers. First of all, we can fix the phase of $\alpha$ and $\beta$ in order to get u'and v'real. The relations (5) split in two groups of real relation.

$$\left.\begin{aligned} R_e(X_1) &= R_e(H'_{11})\, u' + R_e(H'_{12})\, v' \\ R_e(X_2) &= R_e(H'_{21})\, u' + R_e(H'_{22})\, v' \end{aligned}\right\} \qquad (6)$$

$$\left.\begin{aligned} I_m(X_1) &= I_m(H'_{11})\, u' + I_m(H'_{12})\, v' \\ I_m(X_2) &= I_m(H'_{21})\, u' + I_m(H'_{22})\, v' \end{aligned}\right\} \qquad (7)$$

We continue with (6) but the same is valid for (7). Inverting (6) we get (we suppose that this system has one solution) :

$$\left.\begin{aligned} u' &= a\, R_e(X_1) + b\, R_e(X_2) \\ v' &= c\, R_e(X_1) + d\, R_e(X_2) \end{aligned}\right\} \qquad (8)$$

a,b,c,d are real parameters. We normalize by :

$$u = \frac{u'}{a^2+b^2} \qquad v = \frac{v'}{a^2+b^2}$$

giving finally :

$$\left.\begin{aligned} u &= \cos(\phi_1) R_e(X_1) + \sin(\phi_1) R_e(X_2) \\ v &= \cos(\phi_2) R_e(X_1) + \sin(\phi_2) R_e(X_2) \end{aligned}\right\} \qquad (9)$$

The only identifiable parameters are the two angles $\phi_1$ and $\phi_2$ that appear in (9). We will adopt the model (9) in the following.

Going back to our objective, we see that knowing $\phi_1$ and $\phi_2$ it will be possible to obtain u and v ; each of them is linearly related to only one source.

## 4. The algorithm

The model is given by (9). Let us call $\phi_{1T}$ and $\phi_{2T}$ the presumed value of $\phi_1$ and $\phi_2$ giving the "presumed" sources :

$$\left.\begin{aligned} u_T &= \cos(\phi_{1T}) R_e(X_1) + \sin(\phi_{1T}) R_e(X_2) \\ v_T &= \cos(\phi_{2T}) R_e(X_1) + \sin(\phi_{2T}) R_e(X_2) \end{aligned}\right\} \qquad (10)$$

We will use the fact that for the good value of $\phi_{1T}$ and $\phi_{2T}$, $u_T$ and $v_T$ are statistically independent and so their *cross-cumulants are null.*

The second order cross-cumulants resume all the information contained in the spectral matrix. From :

$$C(u_T, v_T) = 0 \qquad (11)$$

we obtain :

$$tg(\phi_{1T}) = \frac{\cos(\phi_{2T})\, E\{(R_eX_1)^2\} + \sin(\phi_{2T})\, E\{(R_eX_1)(R_eX_2)\}}{-\sin(\phi_{2T})\, E\{(R_eX_1)^2\} - \cos(\phi_{2T})\, E\{(R_eX_1)(R_eX_2)\}} \quad (12)$$

At that point, we see that the spectral matrix is not sufficient for $\phi_1$ and $\phi_2$ identification. We propose then to use higher order cross-cumulants. It is clear that this procedure will not succeed if the two sources are Gaussian, so we must assume that no more than one source is Gaussian (this condition is also necessary for more than 2 sources). We think that it will be more general to use the fourth order cumulants. We propose to solve :

$$\left.\begin{aligned} c_{31} &= C(u_T,u_T,u_T,v_T) = 0 \\ c_{22} &= C(u_T,u_T,v_T,v_T) = 0 \\ c_{13} &= C(u_T,v_T,v_T,v_T) = 0 \end{aligned}\right\} \quad (13)$$

With $X_1$ and $X_2$ zero mean, we have :

$$\left.\begin{aligned} c_{31} &= E(u_T^3 v_T) - 3E(u_T^2)\, E(u_T v_T) \\ c_{13} &= E(v_T^3 u_T) - 3E(v_T^2)\, E(u_T v_T) \\ c_{22} &= E(v_T^2 u_T^2) - E(v_T^2)\, E(u_T^2) - 2E^2(u_T v_T) \end{aligned}\right\} \quad (14)$$

From (14) we see that the only remaining unknown parameter is $\phi_{2T}$. So one of the relations (13) will be sufficient. However, the values of the cumulants will be estimated with errors and so it is more precise to use a combination of the three cumulants. As a first candidate, we have chosen to maximize :

$$d^2 = \frac{1}{c_{13}^2 + c_{12}^2 + c_{31}^2} \quad (15)$$

Finally our algorithm works in the following way (Fig. 1) :

1. Calculation of the cumulants ($c_{11}$, $c_{13}$, $c_{31}$, $c_{22}$) of the real part of the Fourier transform of the observations. We take $\phi_{2T}$ as a parameter that will vary from 0 to $\Pi$.
2. For each value of $\phi_{2T}$ calculation of $\phi_{1T}$ and $d^2(\phi_{2T})$ from the relations (12), (14), (15).
3. The maximum value of $d^2$ gives $\phi_{2T}$.

Let us notice that $d^2$ will have at least two maxima associated with $\phi_{2T} = \phi_1$ or $\phi_2$ but this leads to the same solution for the couple ($\phi_{1T}$, $\phi_{2T}$). This is related to the fact that we cannot relate u and v and a specific source.

## 5. Simulation results

We define, from (10) :

$$\left.\begin{aligned} R_e(X_1) &= \frac{1}{\sin(\phi_2-\phi_1)}(\sin\phi_2\, u - \sin\phi_1\, v) \\ R_e(X_2) &= \frac{1}{\sin(\phi_2-\phi_1)}(-\cos\phi_2\, u + \cos\phi_1\, v) \end{aligned}\right\} \quad (16)$$

where u and v are zero mean generated white noises, statistically independent, with $\sigma_u^2 = \sigma_v^2 = 1$ and $\phi_1$, $\phi_2$ are chosen by the user.

u is Gaussian white noise ; v is a non Gaussian noise obtained from the Gaussian zero mean white noise $v_1(n)$ through the non linear transformation :

$$\begin{cases} v(n) = a\, v_1(n) & \text{if } |v_1(n)| > b \quad a \neq 1 \\ v(n) = v_1(n) & \text{else} \end{cases}$$

The Gaussian processes u(n) and $v_1(n)$ are statistically independent and thus u(n) and v(n) are the independent sources.

The purpose of the simulation is to identify the independent sources from the family ($u_T$,$v_T$) defined by (10) and (12).

From (16), obtained with known parameters $\phi_1$ and $\phi_2$, we implant the algorithm described in Figure 1.

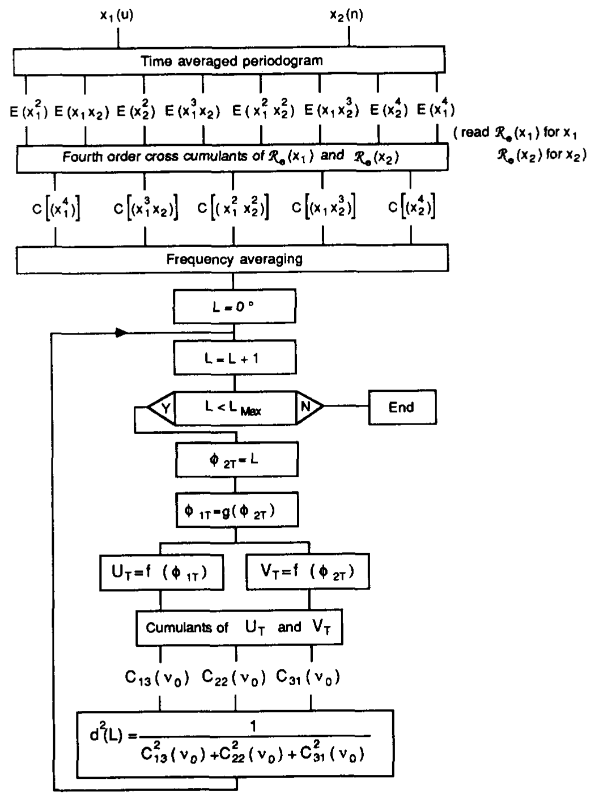


**Figure 1**

Notations :

$c_{11} = E\{(R_eX_1)^2\}$ $\quad c_{12} = E\{(R_eX_1)(R_eX_2)\}$ $\quad c_{22} = E\{(R_eX_2)^2\}$

$c_{1111} = C\,[R_eX_1, R_eX_1, R_eX_1, R_eX_1]$,

$c_{1112} = C\,[R_eX_1, R_eX_1, R_eX_1, R_eX_2]$, etc...

The following Table gives the coefficients of $c_{13}$, $c_{22}$, $c_{31}$ linear functions of $c_{1111}$, $c_{1112}$, $c_{1122}$, ...

| | $c_{31}$ | $c_{22}$ | $c_{13}$ |
|---|---|---|---|
| $c_{1111}$ | $\cos^3\phi_{1T}\ \cos\phi_{2T}$ | $\cos^2\phi_{2T}\ \cos^2\phi_{1T}$ | $\cos^3\phi_{2T}\ \cos\phi_{1T}$ |
| $c_{1112}$ | $\cos^3\phi_{1T}\sin\phi_{2T}$ $+ 3\cos^2\phi_{1T}\sin\phi_{1T}\cos\phi_{2T}$ | $2\cos^2\phi_{1T}\cos\phi_{2T}\sin\phi_{2T}$ $+ 2\cos\phi_{1T}\sin\phi_{1T}\cos^2\phi_{2T}$ | $\cos^3\phi_{2T}\sin\phi_{1T}$ $+ 3\cos^2\phi_{2T}\sin\phi_{2T}\cos\phi_{1T}$ |
| $c_{1122}$ | $3\cos^2\phi_{1T}\sin\phi_{1T}\sin\phi_{2T}$ $+ 3\sin^2\phi_{1T}\cos\phi_{1T}\cos\phi_{2T}$ | $\cos^2\phi_{1T}\sin^2\phi_{2T}$ $+ 4\cos\phi_{1T}\sin\phi_{1T}\cos\phi_{2T}\sin\phi_{2T}$ $+ \sin^2\phi_{1T}\cos^2\phi_{2T}$ | $3\cos^2\phi_{2T}\sin\phi_{2T}\sin\phi_{1T}$ $+ 3\sin^2\phi_{2T}\cos\phi_2\cos\phi_{1T}$ |
| $c_{1222}$ | $\sin^3\phi_{1T}\ \cos\phi_{2T}$ $+ 3\sin^2\phi_{1T}\cos\phi_{1T}\sin\phi_{2T}$ | $2\sin^2\phi_{1T}\cos\phi_{2T}\sin\phi_{2T}$ $+ 2\sin^2\phi_{2T}\cos\phi_{1T}\sin\phi_{1T}$ | $\sin^3\phi_{2T}\cos\phi_{1T}$ $+ 3\sin^2\phi_{2T}\cos\phi_{2T}\sin\phi_{1T}$ |
| $c_{2222}$ | $\sin^3\phi_{1T}\sin\phi_{2T}$ | $\sin^2\phi_{2T}\ \sin^2\phi_{1T}$ | $\sin^3\phi_{2T}\ \sin\phi_{1T}$ |

The simulation of the sources and their identification has been conducted in different conditions. We give in Figure 2 the plot of the value of $\phi_{1T}$ versus $\phi_{2T}$ and of the parameter $d^2$ versus $\phi_{2T}$ in the following case.

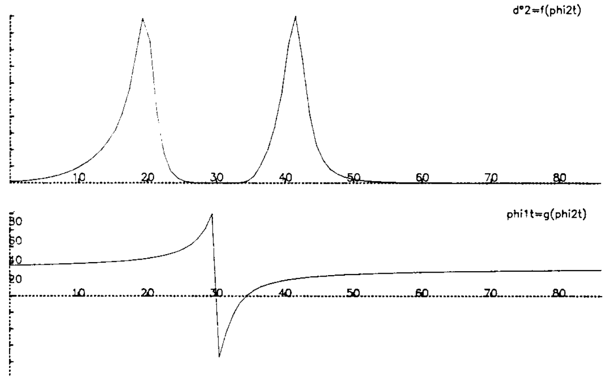


**Figure 2 :**
Plot of the value of $\phi_{1T}$ and of the parameter $d^2$ versus $\phi_{2T}$.

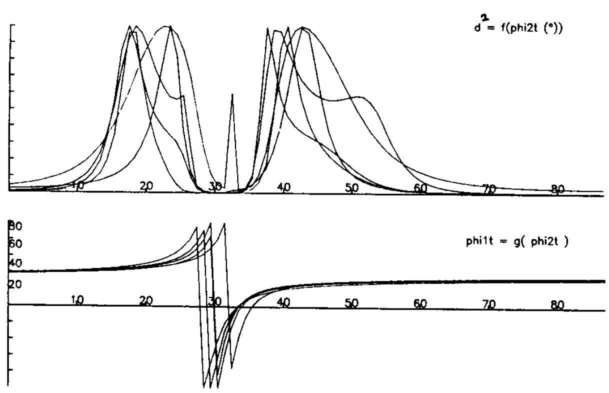


**Figure 3 :**
Five realizations of the simulation.

Simulated sources :

$v(n)$ : $a = 10$ $\quad b = 1$ $\quad \phi_{10} = 20°$ $\quad \phi_{20} = 40°$

Estimation of the cumulants :

Time averaged periodogram :
- total number of points of the signal in time : $M = 4096$
- number of points for each elementary slice : $N = 64$
- number of slices : $L = 64$ $(NL = M)$

Frequency averaging :
- number of averaged frequency channels : $p = 3$
- central frequency : $\nu_0 = 1/T_E$

$T_E$ : sampling period not explicited

The total number of average in thus : $LP = 64x3 = 192$

We see in Figure 2 that the two source are correctly obtained by the maxima of the parameter $d^2$. The twc maxima correspond to the same situation with the two commutated sources. The errors on the parameter values $\phi_1$ and $\phi_2$ can be related to the estimation errors of the different cumulants. It will be necessary to develop a more complete study of these estimation errors.

In Figure 3 we show 5 realizations of the same experiment giving a first visual approach to the statistical properties of the estimators.

## 6. Conclusion

The problem of source separation has no solution without a priori information when one uses only the spectral matrix.

We have developed a new algorithm using the fourth order cumulants that solve the problem of the two source separation without the necessity of a priori information. We have shown the potentialities of this method on simulated results.

To our mind, this work opens a new field of research in multidimensional signal processing. The open questions concern primarily the extension to more than two sources and the charaterization of the estimators (bias and variance).

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