RF Communication Circuits

Lecture 2: Transmission Lines

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Waveguiding Structures

A wave guiding structure is one that carries a signal (or power) from one point to another.

There are three common types:

- Transmission lines
- Fiber-optic guides
- Waveguides

Transmission Line

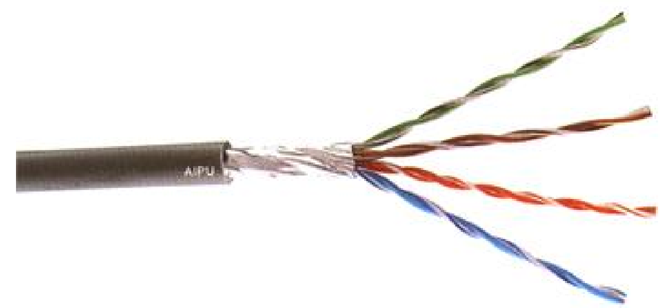
Properties

- Has two conductors running parallel
- Can propagate a signal at any frequency (in theory)
- Becomes lossy at high frequency
- Can handle low or moderate amounts of power
- Does not have signal distortion, unless there is loss
- May or may not be immune to interference
- Does not have E_z or H_z components of the fields (TEM_z)

Coaxial cable (coax)

Twin lead
(shown connected to a 4:1 impedance-transforming balun)

Transmission Line (cont.)

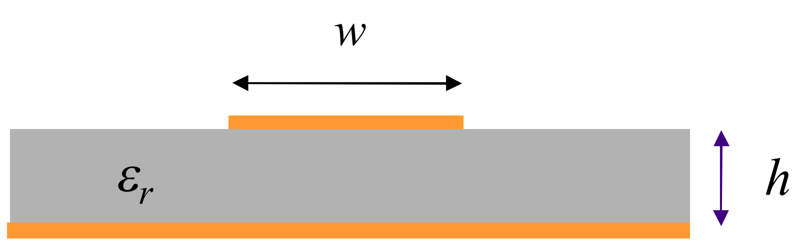


CAT 5 cable
(twisted pair)

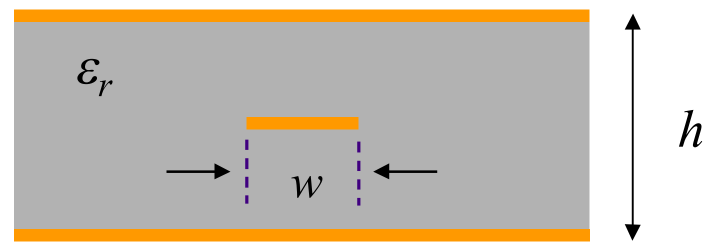
The two wires of the transmission line are twisted to reduce interference and radiation from discontinuities.

Transmission Line (cont.)

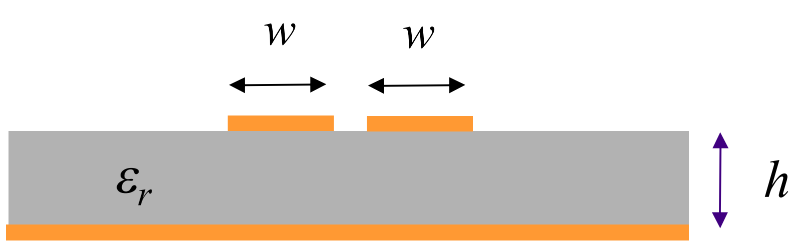
Transmission lines commonly met on printed-circuit boards



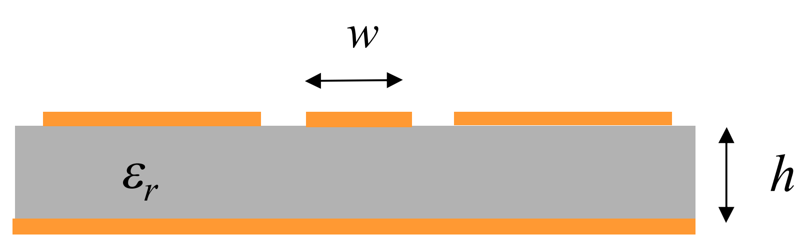
Microstrip



Stripline



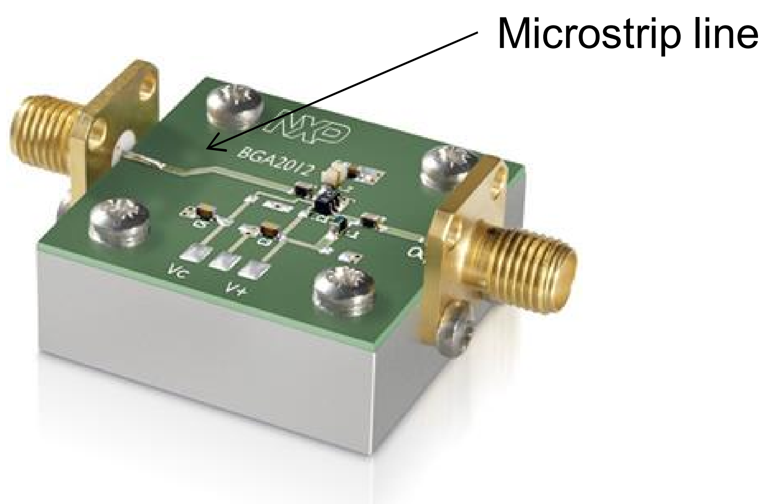
Coplanar strips



Coplanar waveguide (CPW)

Transmission Line (cont.)

Transmission lines are commonly met on printed-circuit boards.

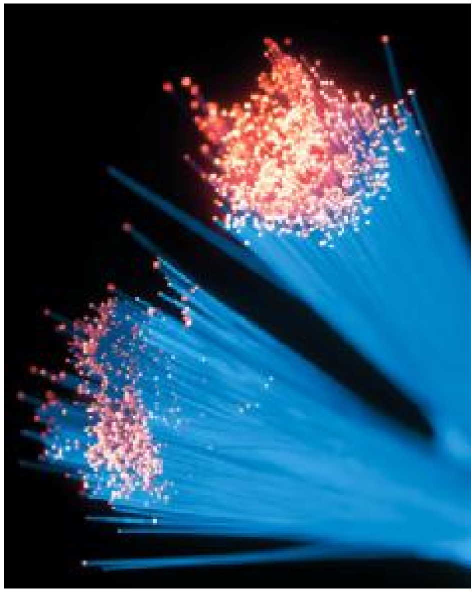


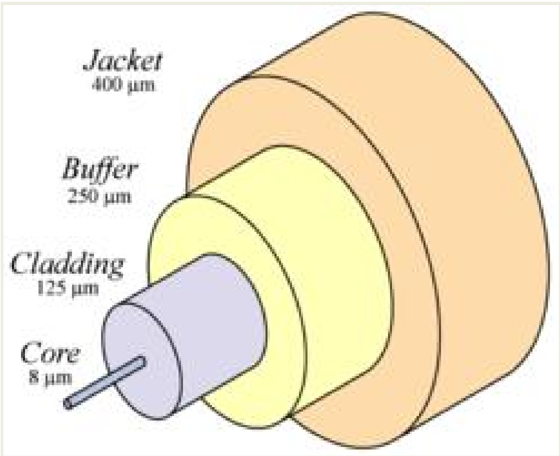
A microwave integrated circuit

Fiber-Optic Guide

Properties

- Uses a dielectric rod
- Can propagate a signal at any frequency (in theory)
- Can be made very low loss
- Has minimal signal distortion
- Very immune to interference
- Not suitable for high power
- Has both E_z and H_z components of the fields





Fiber-Optic Guide (cont.)

Two types of fiber-optic guides:

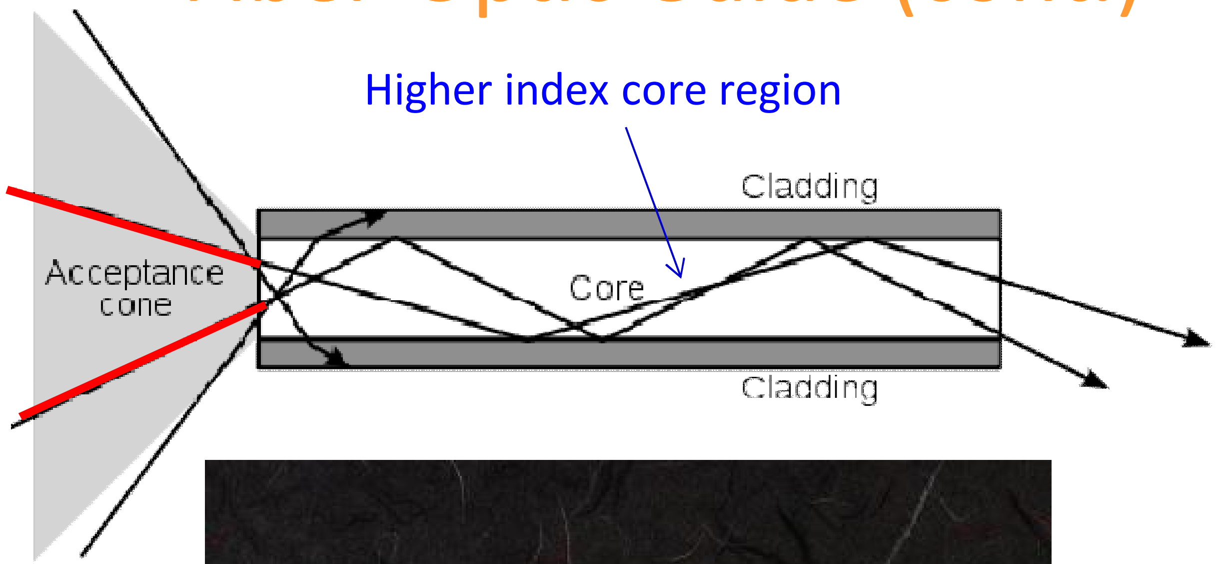
1) Single-mode fiber

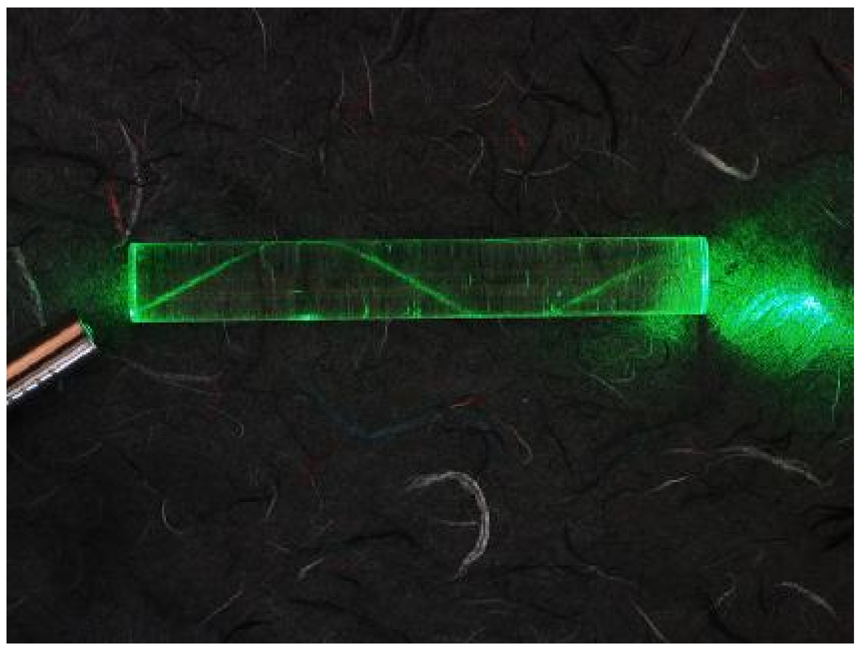
Carries a single mode, as with the mode on a transmission line or waveguide. Requires the fiber diameter to be small relative to a wavelength.

2) Multi-mode fiber

Has a fiber diameter that is large relative to a wavelength. It operates on the principle of total internal reflection (critical angle effect).

Fiber-Optic Guide (cont.)



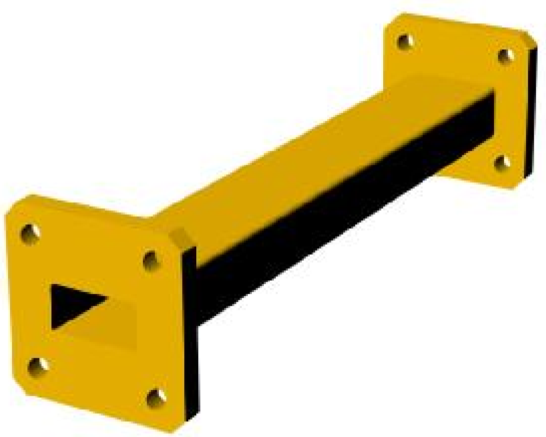


http://en.wikipedia.org/wiki/Optical_fiber

Waveguides

Properties

- Has a single hollow metal pipe
- Can propagate a signal only at high frequency: $\omega > \omega_c$
- The width must be at least one-half of a wavelength
- Has signal distortion, even in the lossless case
- Immune to interference
- Can handle large amounts of power
- Has low loss (compared with a transmission line)
- Has either E_z or H_z component of the fields (TM$_z$ or TE$_z$)



http://en.wikipedia.org/wiki/Waveguide_(electromagnetism)

Transmission-Line Theory

- Lumped circuits: resistors, capacitors, inductors

  neglect time delays (phase)

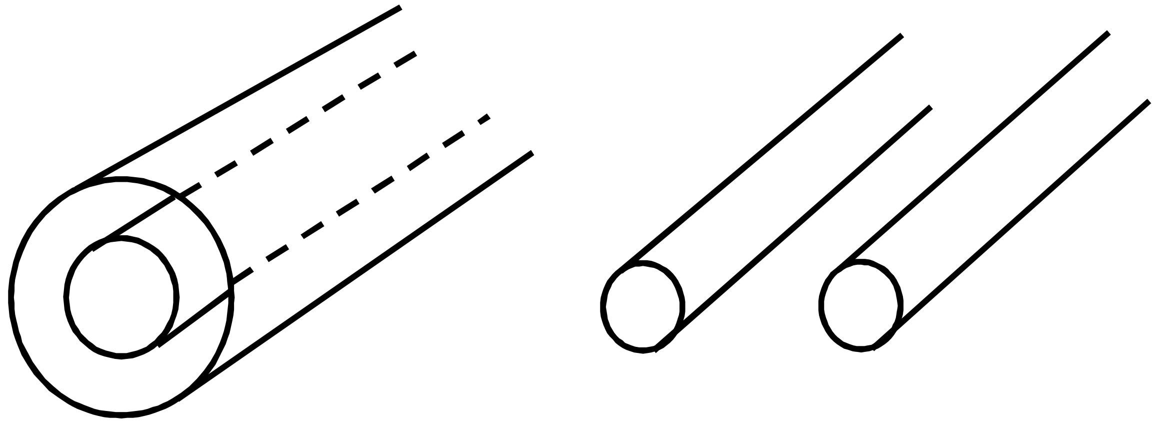
- Distributed circuit elements: transmission lines

  account for propagation and time delays (phase change)

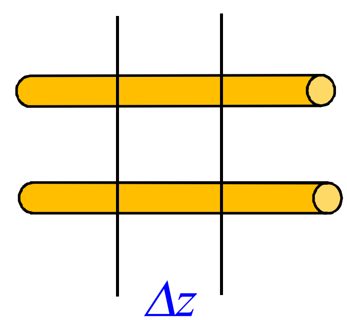
We need transmission-line theory whenever the length of a line is significant compared with a wavelength.

Transmission Line

2 conductors



4 per-unit-length parameters:



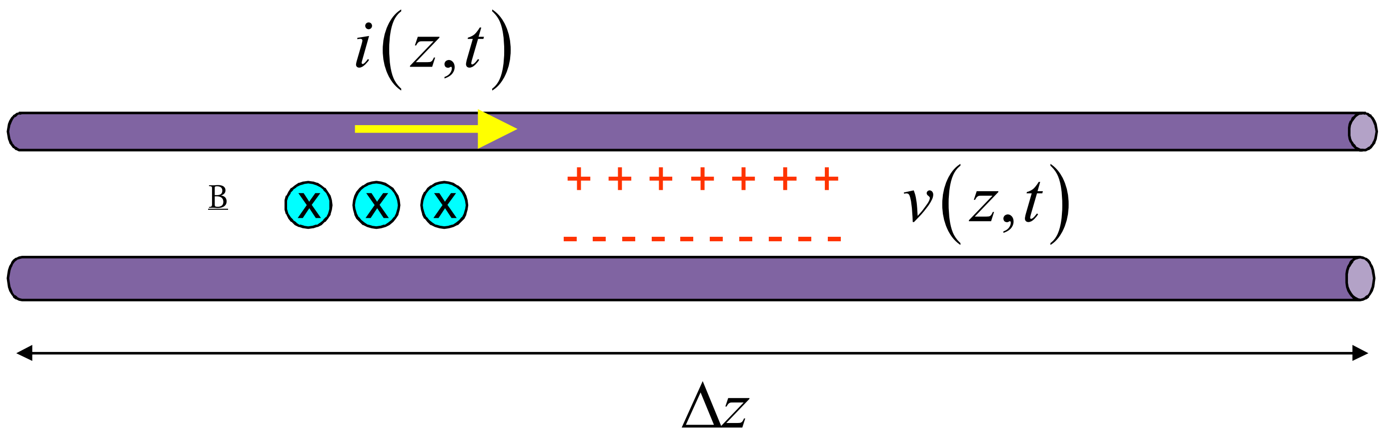
C = capacitance/length [F/m]

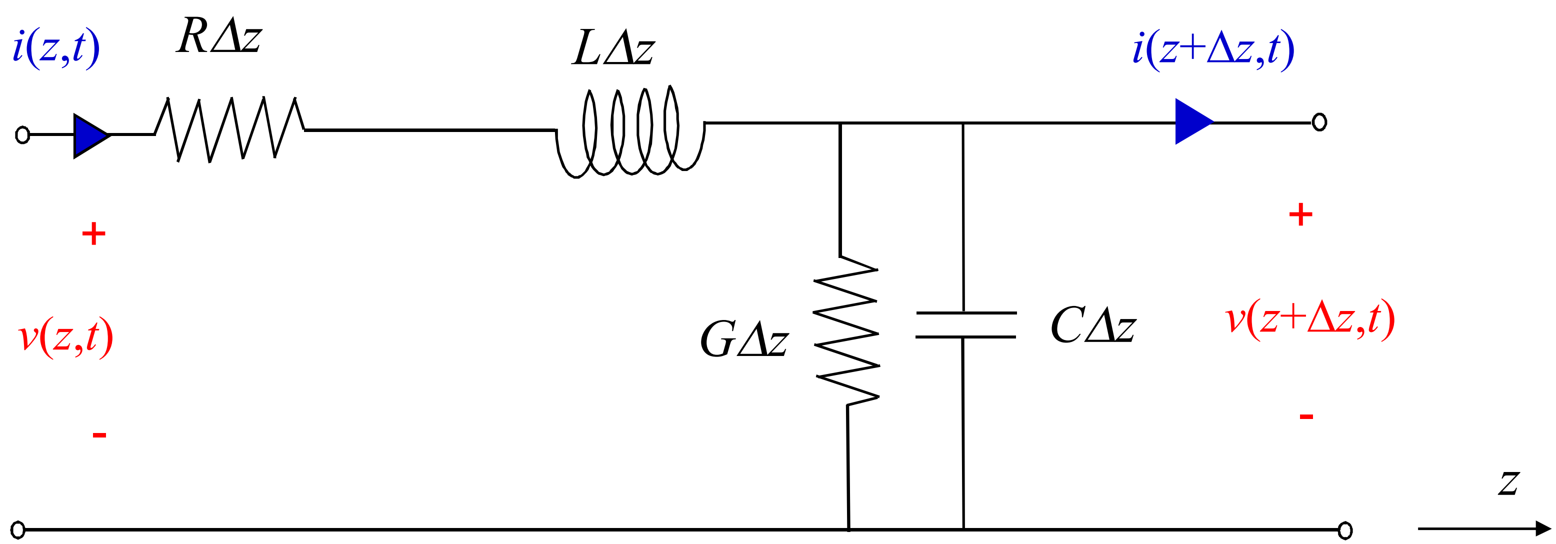
L = inductance/length [H/m]

R = resistance/length [Ω/m]

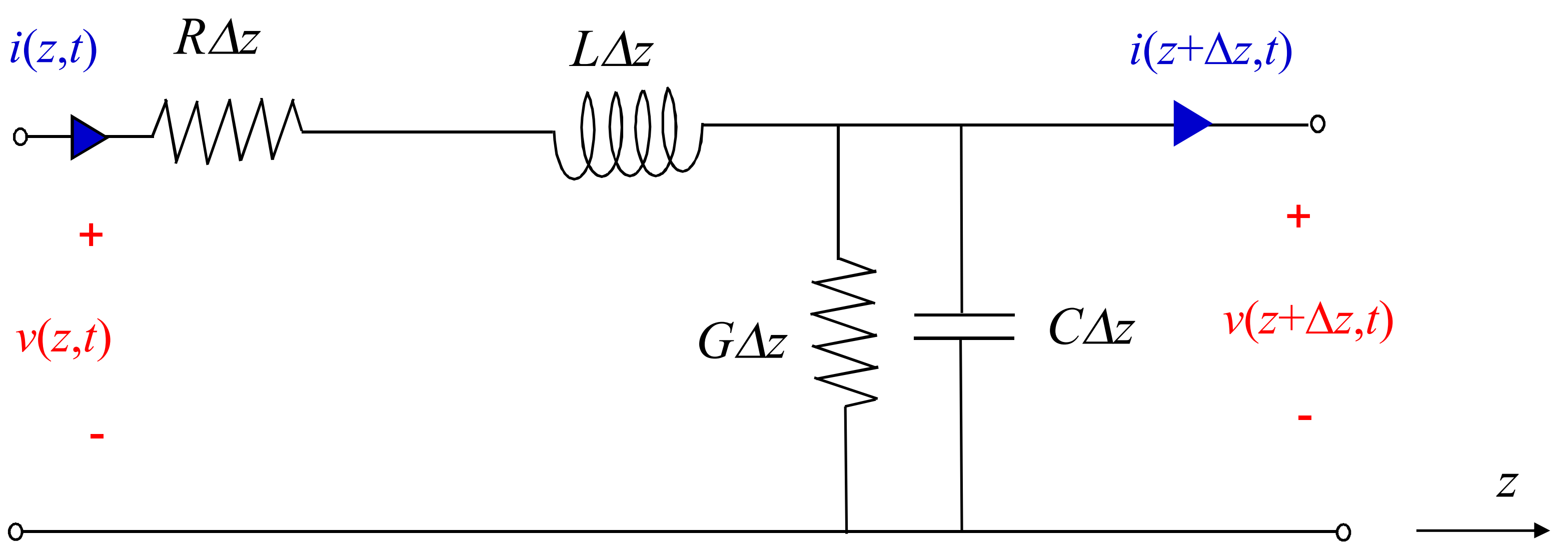
G = conductance/length [℧/m or S/m]

Transmission Line (cont.)





Transmission Line (cont.)



$$v(z,t) = v(z+\Delta z,t) + i(z,t)R\Delta z + L\Delta z \frac{\partial i(z,t)}{\partial t}$$

$$i(z,t) = i(z+\Delta z,t) + v(z+\Delta z,t)G\Delta z + C\Delta z \frac{\partial v(z+\Delta z,t)}{\partial t}$$

TEM Transmission Line (cont.)

Hence

$$\frac{v(z+\Delta z,t)-v(z,t)}{\Delta z} = -Ri(z,t) - L\frac{\partial i(z,t)}{\partial t}$$

$$\frac{i(z+\Delta z,t)-i(z,t)}{\Delta z} = -Gv(z+\Delta z,t) - C\frac{\partial v(z+\Delta z,t)}{\partial t}$$

Now let $\Delta z \to 0$:

$$\frac{\partial v}{\partial z} = -Ri - L\frac{\partial i}{\partial t}$$

$$\frac{\partial i}{\partial z} = -Gv - C\frac{\partial v}{\partial t}$$

"Telegrapher's Equations"

TEM Transmission Line (cont.)

To combine these, take the derivative of the first one with respect to z:

$$\frac{\partial^2 v}{\partial z^2} = -R\frac{\partial i}{\partial z} - L\frac{\partial}{\partial z}\left(\frac{\partial i}{\partial t}\right)$$

$$= -R\frac{\partial i}{\partial z} - L\frac{\partial}{\partial t}\left(\frac{\partial i}{\partial z}\right)$$

$$= -R\left[-Gv - C\frac{\partial v}{\partial t}\right]$$

$$- L\left[-G\frac{\partial v}{\partial t} - C\frac{\partial^2 v}{\partial t^2}\right]$$

Switch the order of the derivatives.

TEM Transmission Line (cont.)

$$\frac{\partial^2 v}{\partial z^2} = -R\left[-Gv - C\frac{\partial v}{\partial t}\right] - L\left[-G\frac{\partial v}{\partial t} - C\frac{\partial^2 v}{\partial t^2}\right]$$

Hence, we have:

$$\frac{\partial^2 v}{\partial z^2} - (RG)v - (RC + LG)\frac{\partial v}{\partial t} - LC\left(\frac{\partial^2 v}{\partial t^2}\right) = 0$$

The same equation also holds for i.

TEM Transmission Line (cont.)

Time-Harmonic Waves:

$$\frac{\partial^2 v}{\partial z^2} - (RG)v - (RC + LG)\frac{\partial v}{\partial t} - LC\left(\frac{\partial^2 v}{\partial t^2}\right) = 0$$

$$\Downarrow$$

$$\frac{d^2 V}{dz^2} - (RG)V - (RC + LG)\,j\omega V - LC(-\omega^2)V = 0$$

TEM Transmission Line (cont.)

$$\frac{d^2V}{dz^2} = \left(RG\right)V + j\omega(RC + LG)V - \left(\omega^2 LC\right)V$$

Note that

$$RG + j\omega(RC + LG) - \omega^2 LC = (R + j\omega L)(G + j\omega C)$$

$Z = R + j\omega L$ = series impedance/length

$Y = G + j\omega C$ = parallel admittance/length

Then we can write:

$$\frac{d^2V}{dz^2} = (ZY)V$$

TEM Transmission Line (cont.)

Let $\gamma^2 = ZY$ Then $\dfrac{d^2V}{dz^2} = (\gamma^2)V$

Solution: $V(z) = Ae^{-\gamma z} + Be^{+\gamma z}$

γ is called the "propagation constant."

Convention: $\gamma = \left[(R + j\omega L)(G + j\omega C)\right]^{1/2}$

$=$ principal square root

$$\sqrt{z} = \sqrt{|z|}\, e^{j\theta/2}$$

$$-\pi < \theta < \pi$$

$$\gamma = \alpha + j\beta$$

$$\alpha \geq 0,\ \beta \geq 0$$

$\alpha =$ attenuation contant

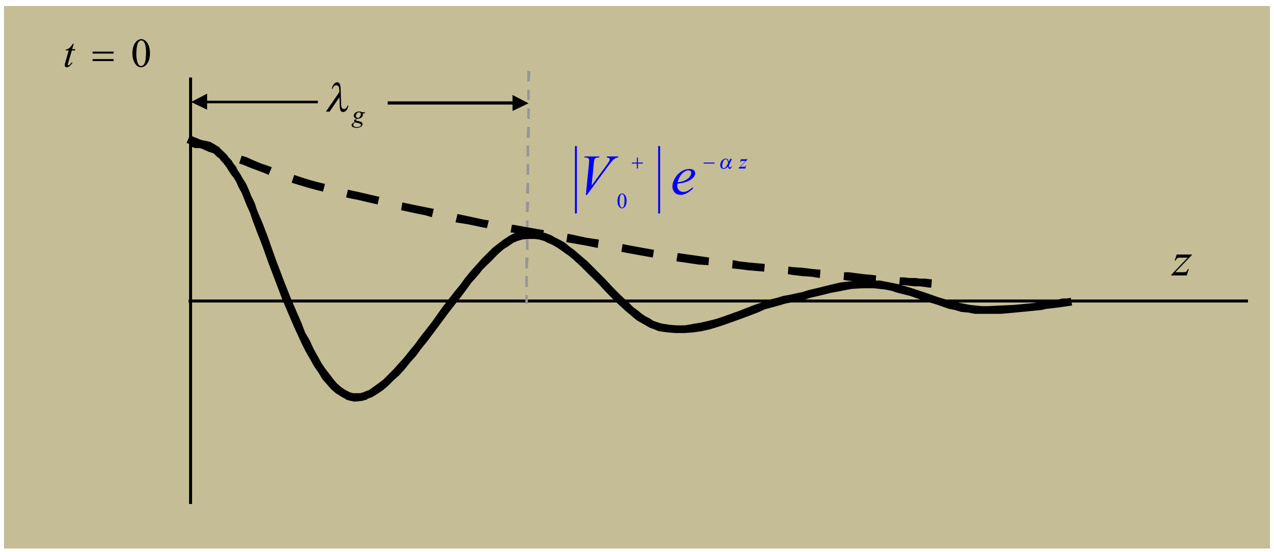
$\beta =$ phase constant

TEM Transmission Line (cont.)

Forward travelling wave (a wave traveling in the positive z direction):

$$V^+(z) = V_0^+ e^{-\gamma z} = V_0^+ e^{-\alpha z} e^{-j\beta z}$$

$$\begin{aligned} v^+(z,t) &= \mathrm{Re}\left\{\left(V_0^+ e^{-\alpha z} e^{-j\beta z}\right) e^{j\omega t}\right\} \\ &= \mathrm{Re}\left\{\left(\left|V_0^+\right| e^{j\phi} e^{-\alpha z} e^{-j\beta z}\right) e^{j\omega t}\right\} \\ &= \left|V_0^+\right| e^{-\alpha z} \cos\left(\omega t - \beta z + \phi\right) \end{aligned}$$



The wave "repeats" when:

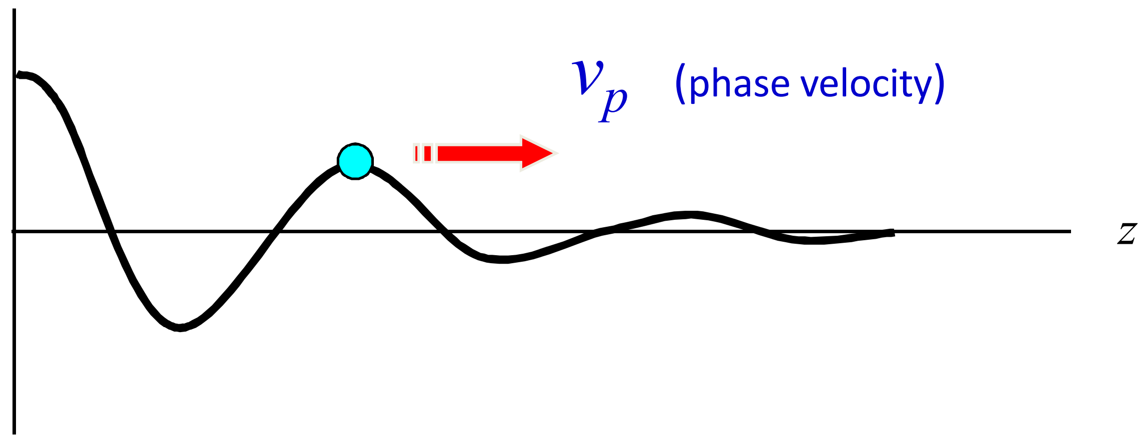
$$\beta \lambda_g = 2\pi$$

Hence:

$$\beta = \frac{2\pi}{\lambda_g}$$

Phase Velocity

Track the velocity of a fixed point on the wave (a point of constant phase), e.g., the crest.



$$v^+(z,t) = \left|V_0^+\right| e^{-\alpha z} \cos(\omega t - \beta z + \phi)$$

Phase Velocity (cont.)

Set $\omega t - \beta z = \text{constant}$

$$\omega - \beta \frac{dz}{dt} = 0$$

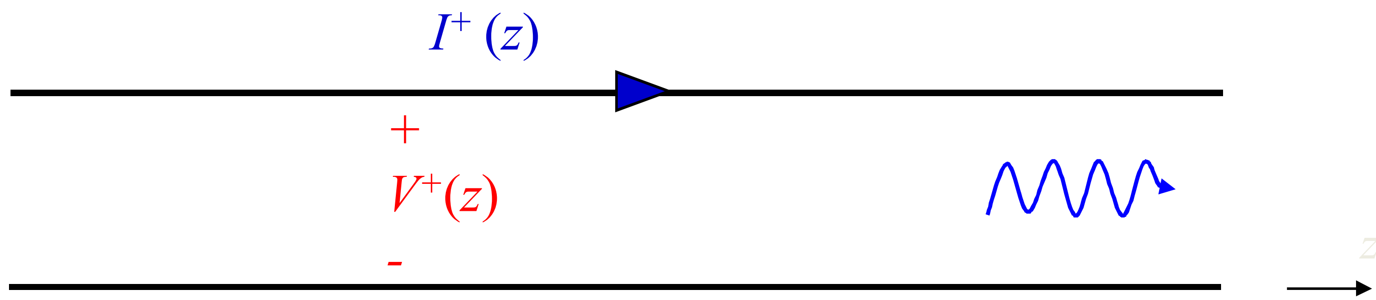
$$\frac{dz}{dt} = \frac{\omega}{\beta}$$

Hence $$v_p = \frac{\omega}{\beta}$$

In expanded form:

$$v_p = \frac{\omega}{\text{Im}\left\{\left[(R + j\omega L)(G + j\omega C)\right]^{1/2}\right\}}$$

Characteristic Impedance Z_0



A wave is traveling in the positive z direction.

$$Z_0 \equiv \frac{V^+(z)}{I^+(z)}$$

$$V^+(z) = V_0^+ e^{-\gamma z}$$

$$I^+(z) = I_0^+ e^{-\gamma z}$$

so $$Z_0 = \frac{V_0^+}{I_0^+}$$

(Z_0 is a number, not a function of z.)

Characteristic Impedance Z_0 (cont.)

Use Telegrapher's Equation:

$$\frac{\partial v}{\partial z} = -Ri - L\frac{\partial i}{\partial t}$$

so $$\frac{dV}{dz} = -RI - j\omega LI$$

$$= -ZI$$

Hence $$-\gamma V_0^{+} e^{-\gamma z} = -Z I_0^{+} e^{-\gamma z}$$

Characteristic Impedance Z_0 (cont.)

From this we have:

$$Z_0 = \frac{V_0^+}{I_0^+} = \frac{Z}{\gamma} = \left(\frac{Z}{Y}\right)^{1/2}$$

Using

$$Z = R + j\omega L$$

$$Y = G + j\omega C$$

We have

$$Z_0 = \left(\frac{R + j\omega L}{G + j\omega C}\right)^{1/2}$$

Note: The principal branch of the square root is chosen, so that Re $(Z_0) > 0$.

General Case (Waves in Both Directions)

$$V(z) = V_0^+ e^{+\gamma z} + V_0^- e^{+\gamma z}$$

$$= \left|V_0^+\right| e^{j\phi^+} e^{+\alpha z} e^{-j\beta z} + \left|V_0^-\right| e^{j\phi^-} e^{+\alpha z} e^{+j\beta z}$$

Note:

wave in $+z$ direction

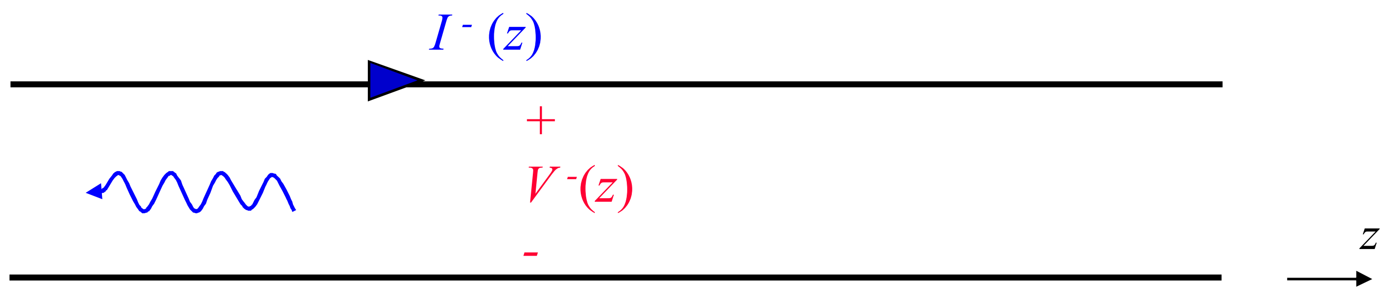
wave in -z direction

$$v(z,t) = \mathrm{Re}\left\{V(z) e^{j\omega t}\right\}$$

$$= \left|V_0^+\right| e^{-\alpha z} \cos\left(\omega t - \beta z + \phi^+\right)$$

$$+ \left|V_0^-\right| e^{+\alpha z} \cos\left(\omega t + \beta z + \phi^-\right)$$

Backward-Traveling Wave

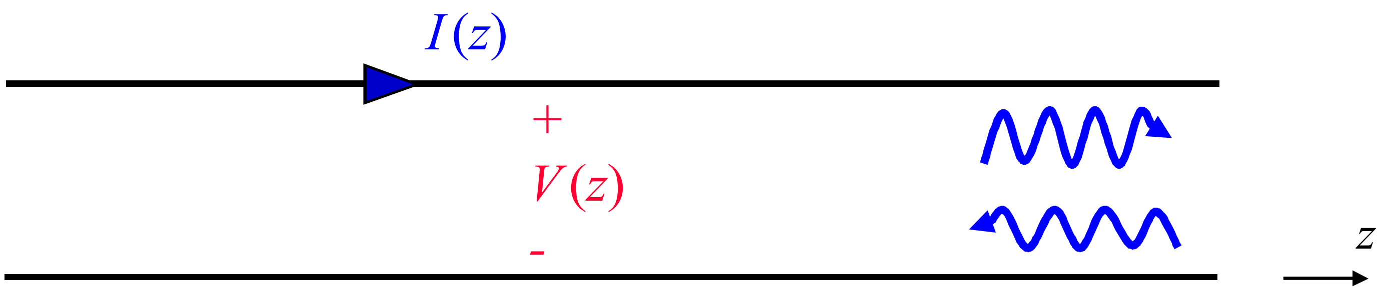


A wave is traveling in the negative z direction.

$$\frac{V^-(z)}{-I^-(z)} = Z_0 \qquad \text{so} \qquad \frac{V^-(z)}{I^-(z)} = -Z_0$$

Note: The reference directions for voltage and current are the same as for the forward wave.

General Case



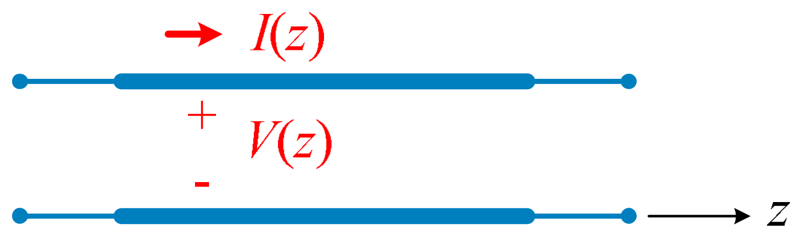
Most general case: A general superposition of forward and backward traveling waves:

$$V(z) = V_0^+ e^{-\gamma z} + V_0^- e^{+\gamma z}$$

$$I(z) = \frac{1}{Z_0}\left[V_0^+ e^{-\gamma z} - V_0^- e^{+\gamma z}\right]$$

Note: The reference directions for voltage and current are the same for forward and backward waves.

Summary of Basic TL formulas



$$V(z) = V_0^+ e^{-\gamma z} + V_0^- e^{+\gamma z}$$

$$I(z) = \frac{V_0^+}{Z_0} e^{-\gamma z} - \frac{V_0^-}{Z_0} e^{+\gamma z}$$

$$\gamma = \alpha + j\beta = \left[(R + j\omega L)(G + j\omega C) \right]^{1/2}$$

$$Z_0 = \left(\frac{R + j\omega L}{G + j\omega C} \right)^{1/2}$$

guided wavelength $\equiv \lambda_g$

$$\lambda_g = \frac{2\pi}{\beta} \, [\mathrm{m}]$$

phase velocity $\equiv v_p$

$$v_p = \frac{\omega}{\beta} \, [\mathrm{m/s}]$$

Lossless Case

$$R = 0,\ \ G = 0$$

$$\gamma = \alpha + j\beta = \left[(\not{R} + j\omega L)(\not{G} + j\omega C)\right]^{1/2}$$
$$= j\omega\sqrt{LC}$$

so

$$\alpha = 0$$
$$\beta = \omega\sqrt{LC}$$

$$v_p = \frac{\omega}{\beta}$$

$$Z_0 = \left(\frac{\not{R} + j\omega L}{\not{G} + j\omega C}\right)^{1/2}$$

$$Z_0 = \sqrt{\frac{L}{C}}$$

(real and indep. of freq.)

$$v_p = \frac{1}{\sqrt{LC}}$$

(indep. of freq.)

Lossless Case (cont.)

$$v_p = \frac{1}{\sqrt{LC}}$$

In the medium between the two conductors is homogeneous (uniform) and is characterized by (ε, μ), then we have that

$$LC = \mu\varepsilon$$

The speed of light in a dielectric medium is $$c_d = \frac{1}{\sqrt{\mu\varepsilon}}$$

Hence, we have that $$v_p = c_d$$

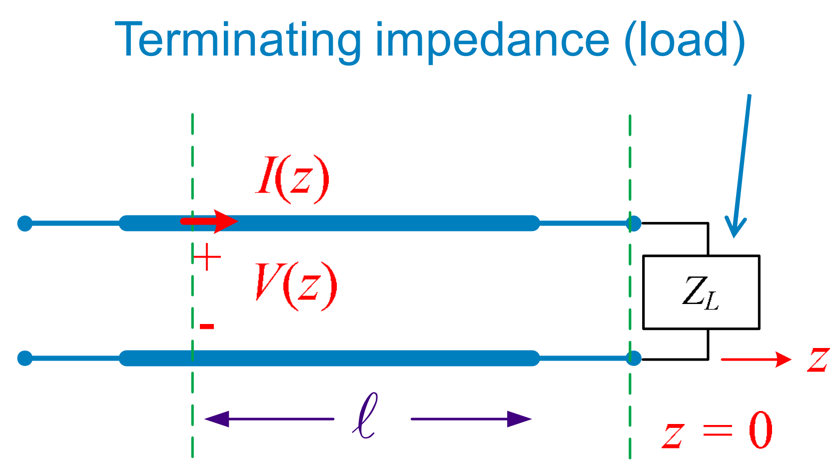
The phase velocity does not depend on the frequency, and it is always the speed of light (in the material).

Terminated Transmission Line

$$V(z) = V_0^+ e^{-\gamma z} + V_0^- e^{+\gamma z}$$

Ampl. of voltage wave propagating in positive z direction at $z = 0$.

Ampl. of voltage wave propagating in negative z direction at $z = 0$.



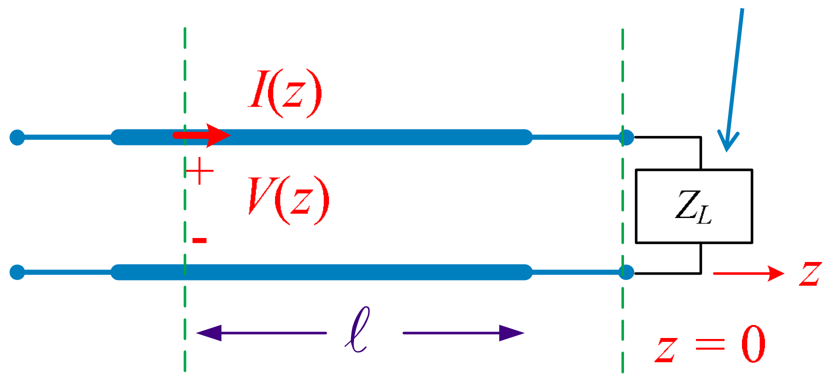
Where do we assign $z = 0$?

The usual choice is at the load.

Note: The length ℓ measures distance from the load: $\ell = -z$

Terminated Transmission Line (cont.)

$$V(z) = V_0^+ e^{-\gamma z} + V_0^- e^{+\gamma z}$$



What if we know

V^+ and V^- @ $z = -\ell$

Can we use $z = -\ell$ as a reference plane?

$$V_0^+ = V^+(0) = V^+(-\ell)e^{-\gamma\ell}$$

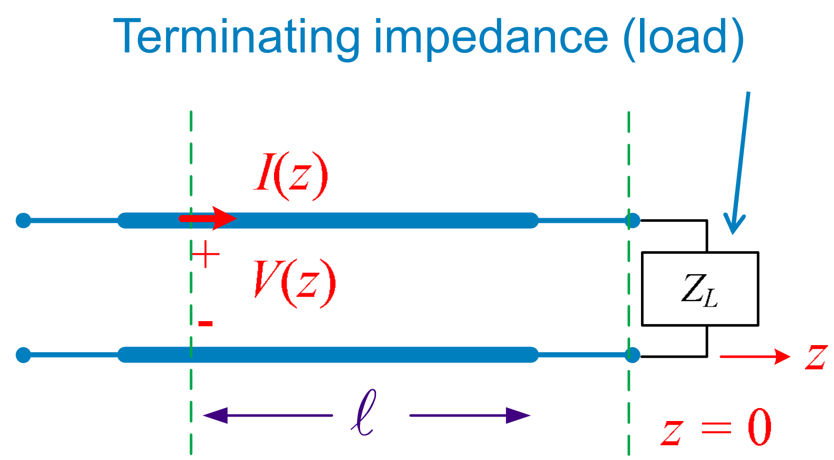
$$V^-(-\ell) = V^-(0)e^{-\gamma\ell}$$

$$\Rightarrow V_0^- = V^-(0) = V^-(-\ell)e^{\gamma\ell}$$

Hence

$$V(z) = V^+(-\ell)e^{-\gamma(z+\ell)} + V^-(-\ell)e^{\gamma(z+\ell)}$$

Terminated Transmission Line (cont.)



Compare:

$$V(z) = V^{+}(0)e^{-\gamma z} + V^{-}(0)e^{+\gamma z}$$

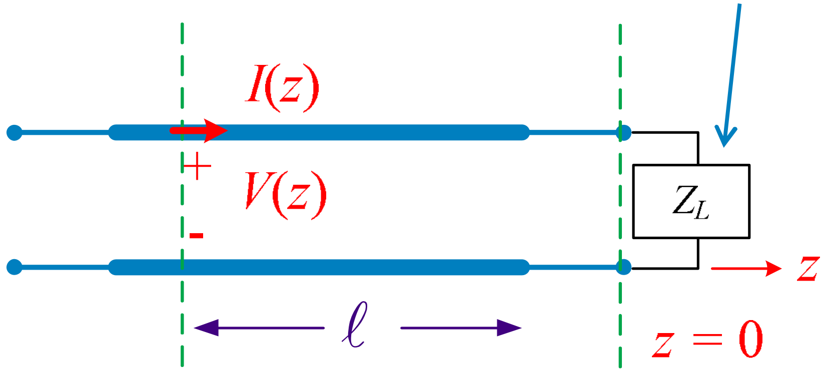
$$V(z) = V^{+}(-\ell)e^{-\gamma(z-(-\ell))} + V^{-}(-\ell)e^{\gamma(z-(-\ell))}$$

Note: This is simply a change of reference plane, from $z = 0$ to $z = -\ell$.

Terminated Transmission Line (cont.)

$$V(z) = V_0^+ e^{-\gamma z} + V_0^- e^{+\gamma z}$$

Terminating impedance (load)



What is $V(-\ell)$?

$$V(-\ell) = V_0^+ e^{\gamma \ell} + V_0^- e^{-\gamma \ell}$$

propagating forwards

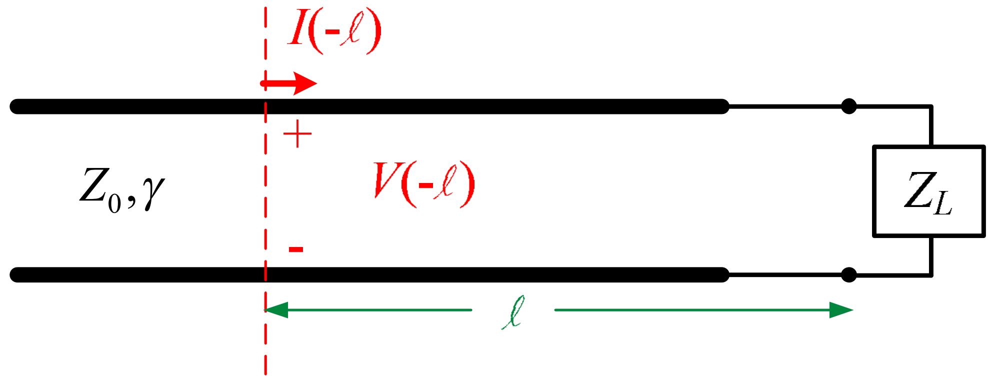
propagating backwards

The current at $z = -\ell$ is then

$$I(-\ell) = \frac{V_0^+}{Z_0} e^{\gamma \ell} - \frac{V_0^-}{Z_0} e^{-\gamma \ell}$$

$\ell \equiv$ distance away from load

Terminated Transmission Line (cont.)



Total volt. at distance ℓ from the load

$$V(-\ell) = V_0^+ e^{\gamma\ell} + V_0^- e^{-\gamma\ell} = V_0^+ e^{\gamma\ell}\left(1 + \frac{V_0^-}{V_0^+} e^{-2\gamma\ell}\right)$$

Ampl. of volt. wave prop. towards load, at the load position ($z = 0$).

Ampl. of volt. wave prop. away from load, at the load position ($z = 0$).

$\Gamma_L \equiv$ Load reflection coefficient

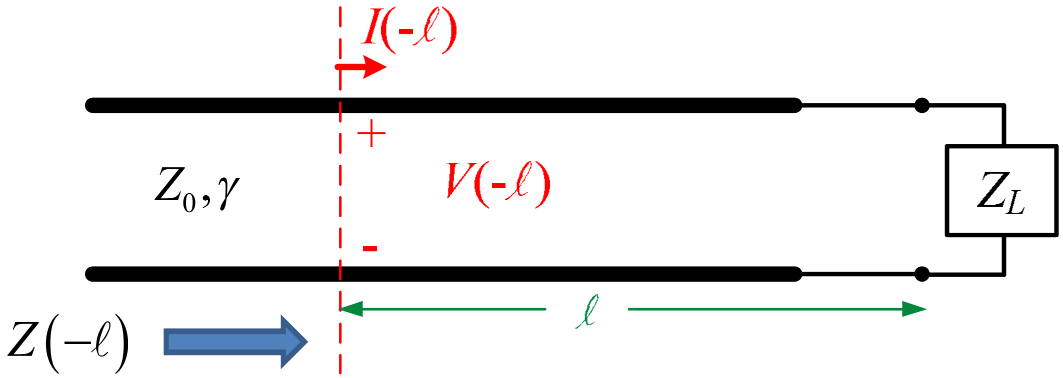
$\Gamma_\ell \equiv$ Reflection coefficient at $z = -\ell$

$$= V_0^+ e^{\gamma\ell}\left(1 + \Gamma_L e^{-2\gamma\ell}\right)$$

Similarly,

$$I(-\ell) = \frac{V_0^+}{Z_0} e^{\gamma\ell}\left(1 - \Gamma_L e^{-2\gamma\ell}\right)$$

Terminated Transmission Line (cont.)



$$V(-\ell) = V_0^+ e^{\gamma \ell}\left(1 + \Gamma_L e^{-2\gamma \ell}\right)$$

$$I(-\ell) = \frac{V_0^+}{Z_0} e^{\gamma \ell}\left(1 - \Gamma_L e^{-2\gamma \ell}\right)$$

$$Z(-\ell) = \frac{V(-\ell)}{I(-\ell)} = Z_0\left(\frac{1 + \Gamma_L e^{-2\gamma \ell}}{1 - \Gamma_L e^{-2\gamma \ell}}\right)$$

Input impedance seen "looking" towards load at $z = -\ell$.

Terminated Transmission Line (cont.)

At the load ($\ell = 0$):

$$Z(0) = Z_0\left(\frac{1+\Gamma_L}{1-\Gamma_L}\right) \equiv Z_L$$

$$\Rightarrow \Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0}$$

Recall $$Z(-\ell) = Z_0\left(\frac{1+\Gamma_L e^{-2\gamma\ell}}{1-\Gamma_L e^{-2\gamma\ell}}\right)$$

Thus,

$$Z(-\ell) = Z_0\left(\frac{1+\left(\frac{Z_L - Z_0}{Z_L + Z_0}\right)e^{-2\gamma\ell}}{1-\left(\frac{Z_L - Z_0}{Z_L + Z_0}\right)e^{-2\gamma\ell}}\right)$$

Terminated Transmission Line (cont.)

Simplifying, we have

$$Z(-\ell) = Z_0 \left(\frac{1 + \left(\dfrac{Z_L - Z_0}{Z_L + Z_0} \right) e^{-2\gamma\ell}}{1 - \left(\dfrac{Z_L - Z_0}{Z_L + Z_0} \right) e^{-2\gamma\ell}} \right) = Z_0 \left(\frac{(Z_L + Z_0) + (Z_L - Z_0) e^{-2\gamma\ell}}{(Z_L + Z_0) - (Z_L - Z_0) e^{-2\gamma\ell}} \right)$$

$$= Z_0 \left(\frac{(Z_L + Z_0) e^{+\gamma\ell} + (Z_L - Z_0) e^{-\gamma\ell}}{(Z_L + Z_0) e^{+\gamma\ell} - (Z_L - Z_0) e^{-\gamma\ell}} \right)$$

$$= Z_0 \left(\frac{Z_L \cosh(\gamma\ell) + Z_0 \sinh(\gamma\ell)}{Z_0 \cosh(\gamma\ell) + Z_L \sinh(\gamma\ell)} \right)$$

Hence, we have

$$Z(-\ell) = Z_0 \left(\frac{Z_L + Z_0 \tanh(\gamma\ell)}{Z_0 + Z_L \tanh(\gamma\ell)} \right)$$

Terminated Lossless Transmission Line

$$\gamma = \cancel{\alpha} + j\beta = j\beta$$

$$V(-\ell) = V_0^+ e^{j\beta\ell}\left(1 + \Gamma_L e^{-2j\beta\ell}\right)$$

$$I(-\ell) = \frac{V_0^+}{Z_0} e^{j\beta\ell}\left(1 - \Gamma_L e^{-2j\beta\ell}\right)$$

$$Z(-\ell) = Z_0\left(\frac{1 + \Gamma_L e^{-2j\beta\ell}}{1 - \Gamma_L e^{-2j\beta\ell}}\right)$$

$$Z(-\ell) = Z_0\left(\frac{Z_L + jZ_0 \tan(\beta\ell)}{Z_0 + jZ_L \tan(\beta\ell)}\right)$$

Note: $\tanh(\gamma\ell) = \tanh(j\beta\ell) = j\tan(\beta\ell)$

Impedance is periodic with period $\lambda_g/2$

tan repeats when

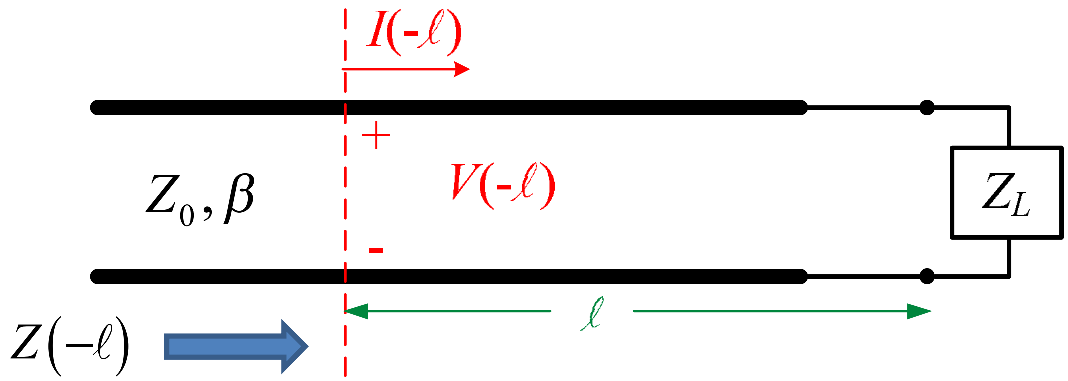
$$\beta\ell = \pi$$

$$\frac{2\pi}{\lambda_g}\ell = \pi$$

$$\Rightarrow \ell = \lambda_g / 2$$

Terminated Lossless Transmission Line

For the remainder of our transmission line discussion we will assume that the transmission line is lossless.



$$V(-\ell) = V_0^+ e^{j\beta\ell}\left(1 + \Gamma_L e^{-2j\beta\ell}\right)$$

$$I(-\ell) = \frac{V_0^+}{Z_0} e^{j\beta\ell}\left(1 - \Gamma_L e^{-2j\beta\ell}\right)$$

$$Z(-\ell) = \frac{V(-\ell)}{I(-\ell)} = Z_0\left(\frac{1 + \Gamma_L e^{-2j\beta\ell}}{1 - \Gamma_L e^{-2j\beta\ell}}\right)$$

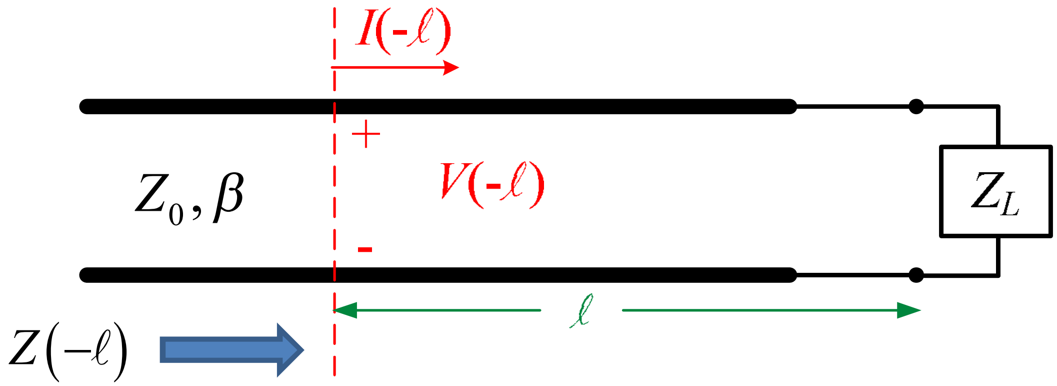
$$= Z_0\left(\frac{Z_L + jZ_0 \tan(\beta\ell)}{Z_0 + jZ_L \tan(\beta\ell)}\right)$$

$$\Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0}$$

$$\lambda_g = \frac{2\pi}{\beta}$$

$$v_p = \frac{\omega}{\beta}$$

Matched Load



Ⓐ Matched load: $(Z_L = Z_0)$

$$\Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0} = 0$$

No reflection from the load

$$\Rightarrow V(-\ell) = V_0^+ e^{+j\beta\ell}$$

$$I(-\ell) = \frac{V_0^+}{Z_0} e^{+j\beta\ell}$$

$$\Rightarrow Z(-\ell) = Z_0$$

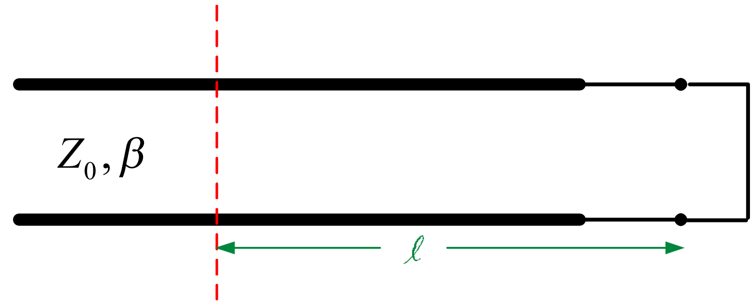
For any ℓ

Short-Circuit Load

(B) Short circuit load: ($Z_L = 0$)

$$\Gamma_L = \frac{0 - Z_0}{0 + Z_0} = -1$$

$$\Rightarrow Z(-\ell) = jZ_0 \tan(\beta\ell)$$

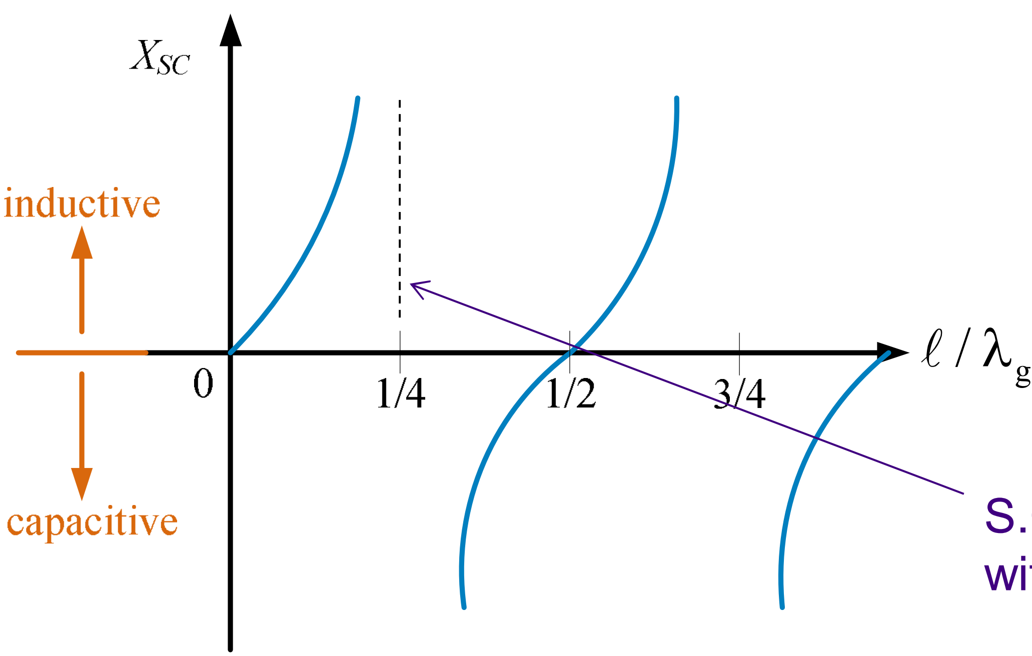


Note: $\beta\ell = 2\pi \dfrac{\ell}{\lambda_g}$

Always imaginary!

$$\Rightarrow Z(-\ell) = jX_{sc}$$

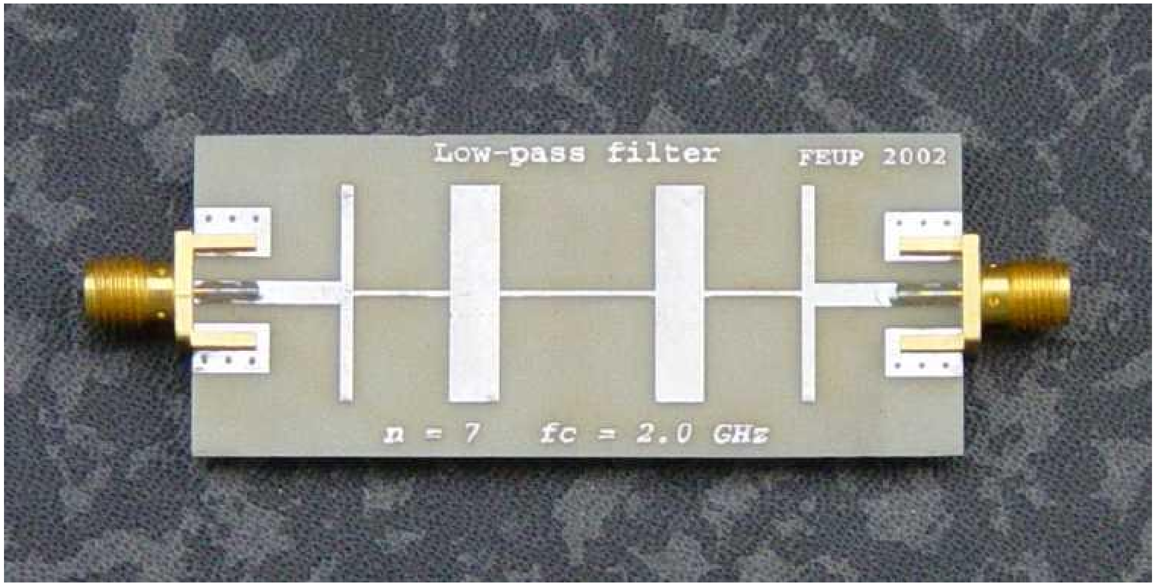
$$X_{sc} = Z_0 \tan(\beta\ell)$$



S.C. can become an O.C. with a $\lambda_g/4$ trans. line

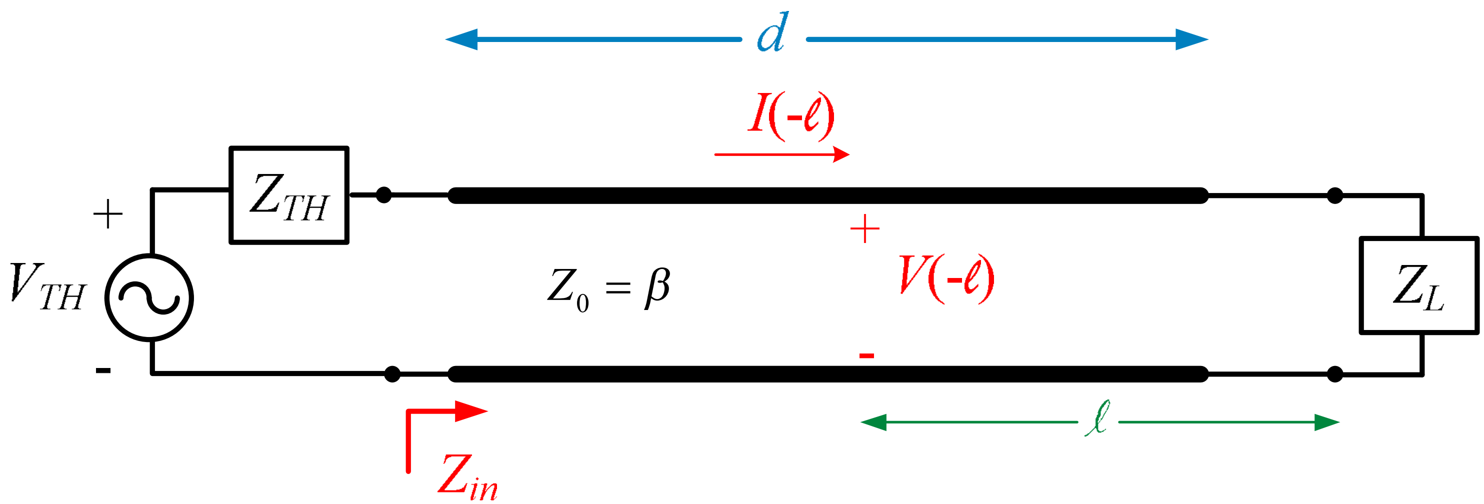
Using Transmission Lines to Synthesize Loads

This is very useful is microwave engineering.



A microwave filter constructed from microstrip.

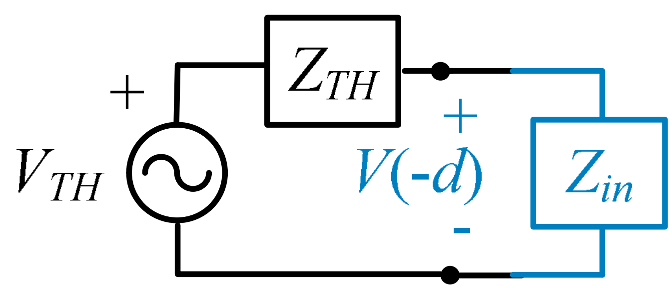
Example



Find the voltage at any point on the line.

$$Z_{in} = Z(-d) = Z_0 \left(\frac{Z_L + jZ_0 \tan(\beta d)}{Z_0 + jZ_L \tan(\beta d)} \right)$$

$$\Rightarrow \quad V(-d) = V_{TH} \left(\frac{Z_{in}}{Z_{in} + Z_{TH}} \right)$$



Example (cont.)

Note: $$V(-\ell) = V_0^+ e^{j\beta\ell}\left(1+\Gamma_L e^{-2j\beta\ell}\right)$$

$$\Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0}$$

At $\ell = d$:

$$V(-d) = V_0^+ e^{j\beta d}\left(1+\Gamma_L e^{-j2\beta d}\right) = V_{TH}\left(\frac{Z_{in}}{Z_{in}+Z_{TH}}\right)$$

$$\Rightarrow \; V_0^+ = V_{TH}\left(\frac{Z_{in}}{Z_{in}+Z_{TH}}\right) e^{-j\beta d}\left(\frac{1}{1+\Gamma_L e^{-j2\beta d}}\right)$$

Hence

$$V(-\ell) = V_{TH}\left(\frac{Z_{in}}{Z_m+Z_{TH}}\right) e^{-j\beta(d-\ell)}\left(\frac{1+\Gamma_L e^{-j2\beta\ell}}{1+\Gamma_L e^{-j2\beta d}}\right)$$

Example (cont.)

Some algebra: $$Z_{in} = Z(-d) = Z_0\left(\frac{1+\Gamma_L e^{-j2\beta d}}{1-\Gamma_L e^{-j2\beta d}}\right)$$

$$\Rightarrow \frac{Z_{in}}{Z_{in}+Z_{TH}} = \frac{Z_0\left(\frac{1+\Gamma_L e^{-j2\beta d}}{1-\Gamma_L e^{-j2\beta d}}\right)}{Z_0\left(\frac{1+\Gamma_L e^{-j2\beta d}}{1-\Gamma_L e^{-j2\beta d}}\right)+Z_{TH}} = \frac{Z_0\left(1+\Gamma_L e^{-j2\beta d}\right)}{Z_0\left(1+\Gamma_L e^{-j2\beta d}\right)+Z_{TH}\left(1-\Gamma_L e^{-j2\beta d}\right)}$$

$$= \frac{Z_0\left(1+\Gamma_L e^{-j2\beta d}\right)}{\left(Z_{TH}+Z_0\right)+\Gamma_L e^{-j2\beta d}\left(Z_0-Z_{TH}\right)}$$

$$= \left(\frac{Z_0}{Z_{TH}+Z_0}\right)\frac{\left(1+\Gamma_L e^{-j2\beta d}\right)}{1+\Gamma_L e^{-j2\beta d}\left(\frac{Z_0-Z_{TH}}{Z_{TH}+Z_0}\right)}$$

$$= \left(\frac{Z_0}{Z_{TH}+Z_0}\right)\frac{\left(1+\Gamma_L e^{-j2\beta d}\right)}{1-\Gamma_L e^{-j2\beta d}\left(\frac{Z_{TH}-Z_0}{Z_{TH}+Z_0}\right)}$$

Example (cont.)

Hence, we have

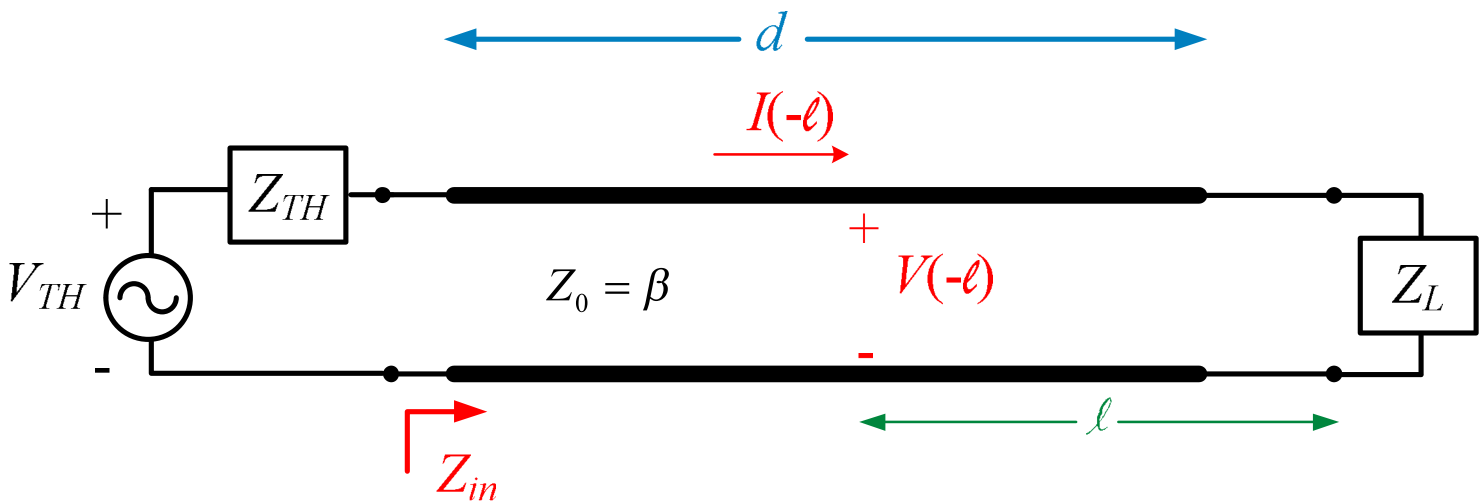
$$\frac{Z_{in}}{Z_{in}+Z_{TH}} = \left(\frac{Z_0}{Z_0+Z_{TH}}\right)\left(\frac{1+\Gamma_L e^{-j2\beta d}}{1-\Gamma_S\Gamma_L e^{-j2\beta d}}\right)$$

where $\Gamma_S = \dfrac{Z_{TH}-Z_0}{Z_{TH}+Z_0}$

Therefore, we have the following alternative form for the result:

$$V(-\ell) = V_{TH}\left(\frac{Z_0}{Z_0+Z_{TH}}\right)e^{-j\beta(d-\ell)}\left(\frac{1+\Gamma_L e^{-j2\beta\ell}}{1-\Gamma_S\Gamma_L e^{-j2\beta d}}\right)$$

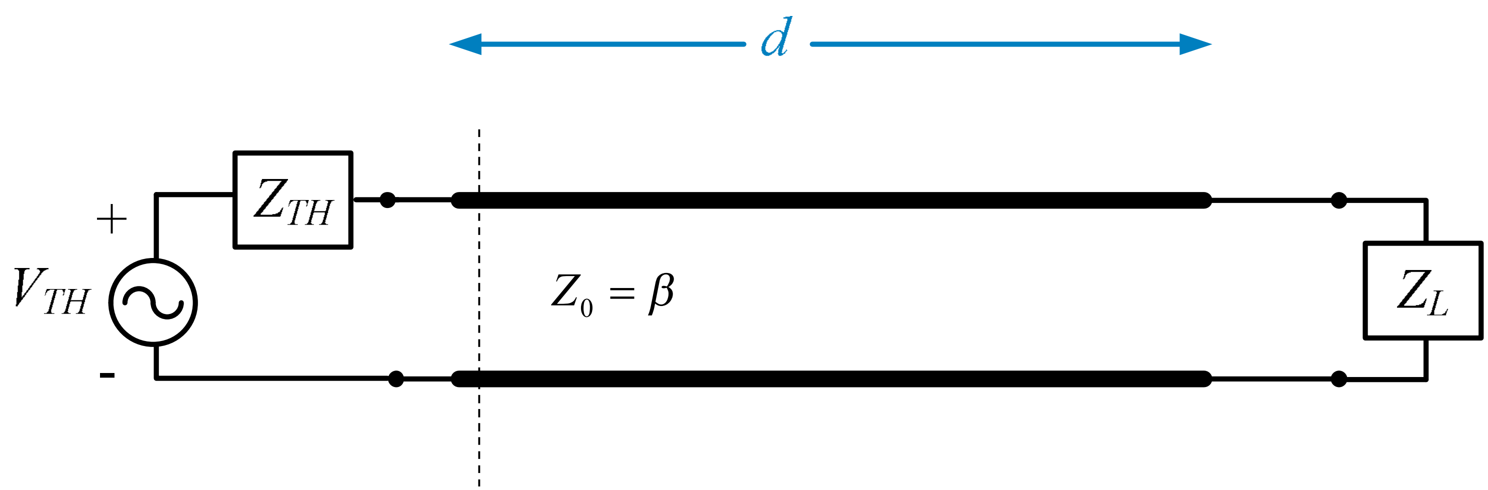
Example (cont.)



$$V(-\ell) = V_{TH}\left(\frac{Z_0}{Z_0 + Z_{TH}}\right)e^{-j\beta(d-\ell)}\left(\frac{1+\Gamma_L e^{-j2\beta\ell}}{1-\Gamma_S\Gamma_L e^{-j2\beta d}}\right)$$

Voltage wave that would exist if there were no reflections from the load (a semi-infinite transmission line or a matched load).

Example (cont.)



Wave-bounce method (illustrated for $\ell = d$):

$$V(-d) = V_{TH}\left(\frac{Z_0}{Z_0 + Z_{TH}}\right)\begin{bmatrix} 1 + \Gamma_L e^{-j2\beta d} + \left(\Gamma_L e^{-j2\beta d}\right)\Gamma_S \\ + \left[\left(\Gamma_L e^{-j2\beta d}\right)\Gamma_S\right]\left(\Gamma_L e^{-j2\beta d}\right) + \left[\left(\Gamma_L e^{-j2\beta d}\right)\Gamma_S\left(\Gamma_L e^{-j2\beta d}\right)\right]\Gamma_S \\ + \ldots \end{bmatrix}$$

Example (cont.)

$$V(-d) = V_{TH}\left(\frac{Z_0}{Z_0 + Z_{TH}}\right)\left[\begin{array}{l} 1 + \Gamma_L e^{-j2\beta d} + \left(\Gamma_L e^{-j2\beta d}\right)\Gamma_S \\ \quad + \left[\left(\Gamma_L e^{-j2\beta d}\right)\Gamma_S\right]\left(\Gamma_L e^{-j2\beta d}\right) + \left[\left(\Gamma_L e^{-j2\beta d}\right)\Gamma_S\left(\Gamma_L e^{-j2\beta d}\right)\right]\Gamma_S \\ \qquad + \ldots \end{array}\right]$$

$$V(-d) = V_{TH}\left(\frac{Z_0}{Z_0 + Z_{TH}}\right)\left[\begin{array}{l} 1 + \left(\Gamma_L \Gamma_S e^{-j2\beta d}\right) + \left(\Gamma_L \Gamma_S e^{-j2\beta d}\right)^2 + \ldots \\ \quad + \Gamma_L e^{-j2\beta d}\left[1 + \left(\Gamma_L \Gamma_S e^{-j2\beta d}\right) + \left(\Gamma_L \Gamma_S e^{-j2\beta d}\right)^2 + \ldots\right] \\ \qquad + \ldots \end{array}\right]$$

Geometric series:

$$\sum_{n=0}^{\infty} z^n = 1 + z + z^2 + \ldots = \frac{1}{1-z}, \quad |z| < 1 \qquad\qquad z = \Gamma_L \Gamma_S e^{-j2\beta d}$$

Example (cont.)

Hence

$$V(-d) = V_{TH}\left(\frac{Z_0}{Z_0 + Z_{TH}}\right)\left[\frac{1}{1-\Gamma_L \Gamma_s e^{-j2\beta d}} + \Gamma_L e^{-j2\beta d}\left(\frac{1}{1-\Gamma_L \Gamma_s e^{-j2\beta d}}\right)\right]$$

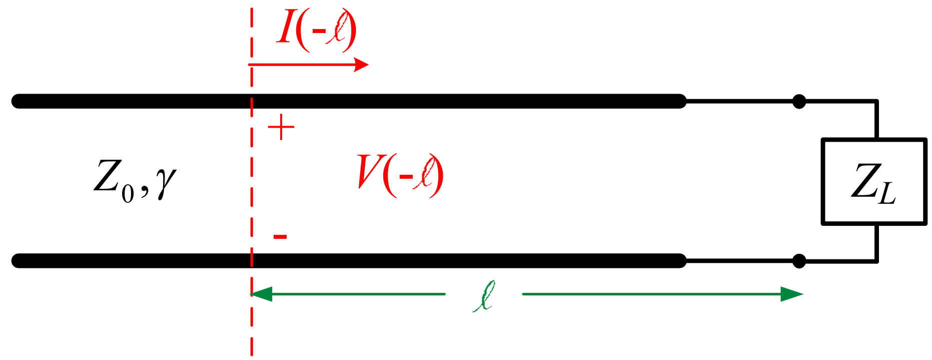
or

$$V(-d) = V_{TH}\left(\frac{Z_0}{Z_0 + Z_{TH}}\right)\left[\frac{1+\Gamma_L e^{-j2\beta d}}{1-\Gamma_L \Gamma_s e^{-j2\beta d}}\right]$$

This agrees with the previous result (setting $\ell = d$).

Note: This is a very tedious method – not recommended.

Time- Average Power Flow



At a distance ℓ from the load:

$$P(-\ell) = \frac{1}{2}\mathrm{Re}\left\{V(-\ell)\, I^*(-\ell)\right\}$$

$$= \frac{1}{2}\mathrm{Re}\left[\frac{\left|V_0^+\right|^2}{Z_0^*} e^{2\alpha\ell}\left(1+\Gamma_L e^{-2\gamma\ell}\right)\left(1-\Gamma_L^* e^{-2\gamma^*\ell}\right)\right]$$

$$V(-\ell) = V_0^+ e^{\gamma\ell}\left(1+\Gamma_L e^{-2\gamma\ell}\right)$$

$$I(-\ell) = \frac{V_0^+}{Z_0} e^{\gamma\ell}\left(1-\Gamma_L e^{-2\gamma\ell}\right)$$

$$\gamma = \alpha + j\beta$$

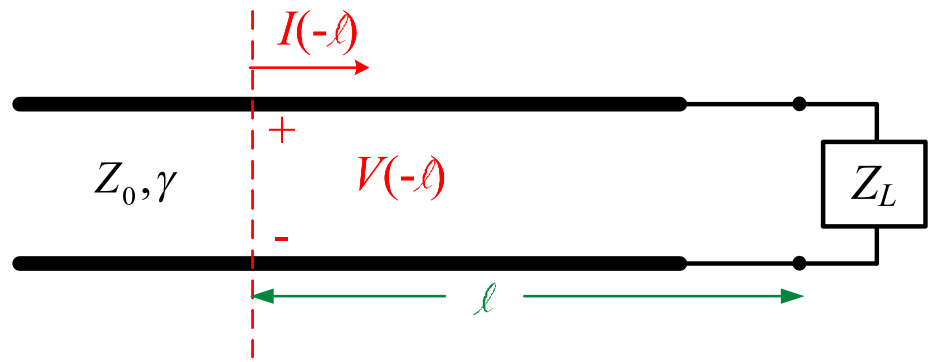
If $Z_0 \approx$ real (low-loss transmission line)

$$P(-\ell) \approx \frac{1}{2}\frac{\left|V_0^+\right|^2}{Z_0} e^{2\alpha\ell}\left(1-\left|\Gamma_L\right|^2 e^{-4\alpha\ell}\right)$$

Note:

$$\Gamma_L e^{-2\gamma\ell} - \Gamma_L^* e^{-2\gamma^*\ell}$$
$$= \Gamma_L e^{-2\gamma\ell} - \left(\Gamma_L e^{-2\gamma\ell}\right)^*$$
$=$ *pure imaginary*

Time- Average Power Flow



Low-loss line

$$P(-d) \approx \frac{1}{2}\frac{|V_0^+|^2}{Z_0} e^{2\alpha\ell}\left(1-|\Gamma_L|^2 e^{-4\alpha\ell}\right)$$

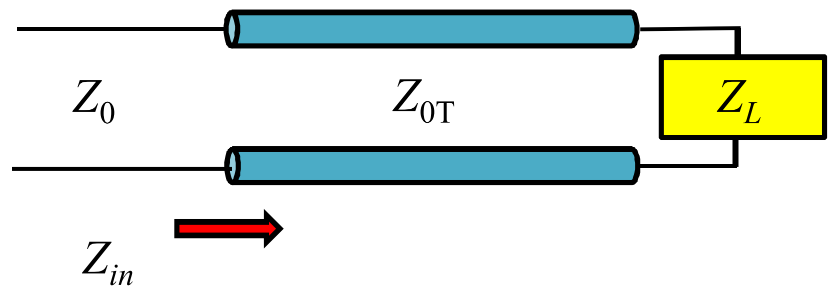
$$= \underbrace{\frac{1}{2}\frac{|V_0^+|^2}{Z_0^*} e^{2\alpha\ell}}_{\text{power in forward wave}} - \underbrace{\frac{1}{2}\frac{|V_0^+|^2}{Z_0^*}|\Gamma_L|^2 e^{-2\alpha\ell}}_{\text{power in backward wave}}$$

Lossless line $(\alpha = 0)$

$$P(-d) = \frac{1}{2}\frac{|V_0^+|^2}{Z_0}\left(1-|\Gamma_L|^2\right)$$

Quarter-Wave Transformer

$$Z_{in} = Z_{0T}\left(\frac{Z_L + jZ_{0T}\tan\beta\ell}{Z_{0T} + jZ_L\tan\beta\ell}\right)$$



$$\beta\ell = \beta\frac{\lambda_g}{4} = \frac{2\pi}{\lambda_g}\frac{\lambda_g}{4} = \frac{\pi}{2}$$

$$\Gamma_{in} = 0 \quad \Rightarrow \quad Z_{in} = Z_0$$

$$\Rightarrow Z_{in} = Z_{0T}\left(\frac{jZ_{0T}}{jZ_L}\right)$$

$$\Rightarrow \quad Z_0 = \frac{Z_{0T}^2}{Z_L}$$

This requires Z_L to be real.

so

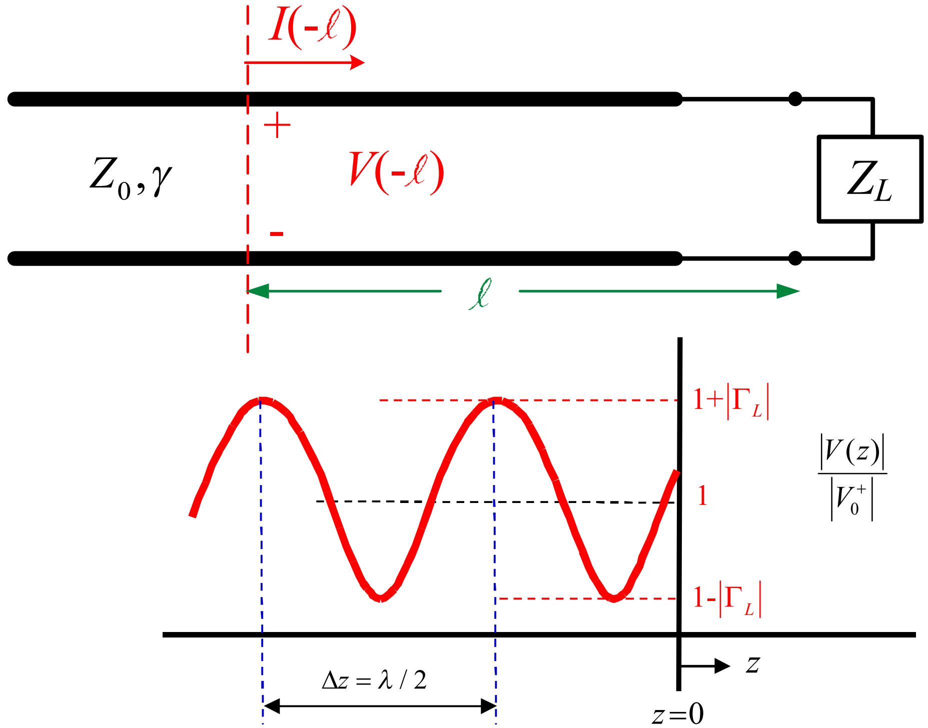
$$Z_{in} = \frac{Z_{0T}^2}{Z_L}$$

Hence

$$Z_{0T} = \left[Z_0 Z_L\right]^{1/2}$$

Voltage Standing Wave Ratio

$$V(-\ell)=V_0^+e^{j\beta\ell}\left(1+\Gamma_L e^{-2j\beta\ell}\right)$$
$$=V_0^+e^{j\beta\ell}\left(1+|\Gamma_L|e^{j\phi_L}e^{-2j\beta\ell}\right)$$



$$|V(-\ell)|=|V_0^+|\left|1+|\Gamma_L|e^{j\phi_L}e^{-j2\beta\ell}\right|$$

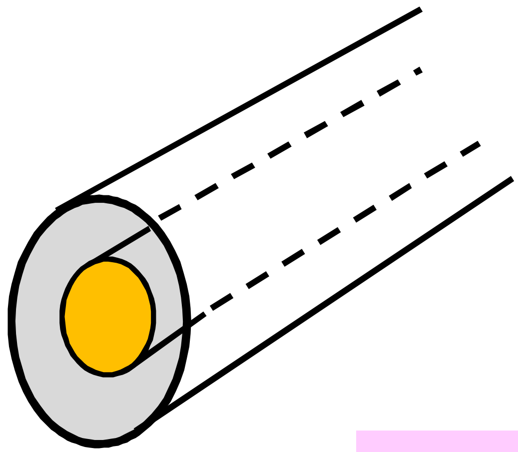
$$V_{\max}=|V_0^+|\left(1+|\Gamma_L|\right)$$
$$V_{\min}=|V_0^+|\left(1-|\Gamma_L|\right)$$

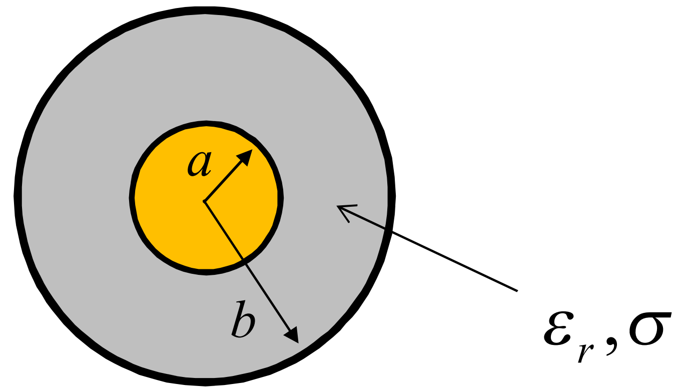
$$\text{Voltage Standing Wave Ratio (VSWR)}=\frac{V_{\max}}{V_{\min}}$$

$$\text{VSWR}=\frac{1+|\Gamma_L|}{1-|\Gamma_L|}$$

Coaxial Cable

Here we present a “case study” of one particular transmission line, the coaxial cable.



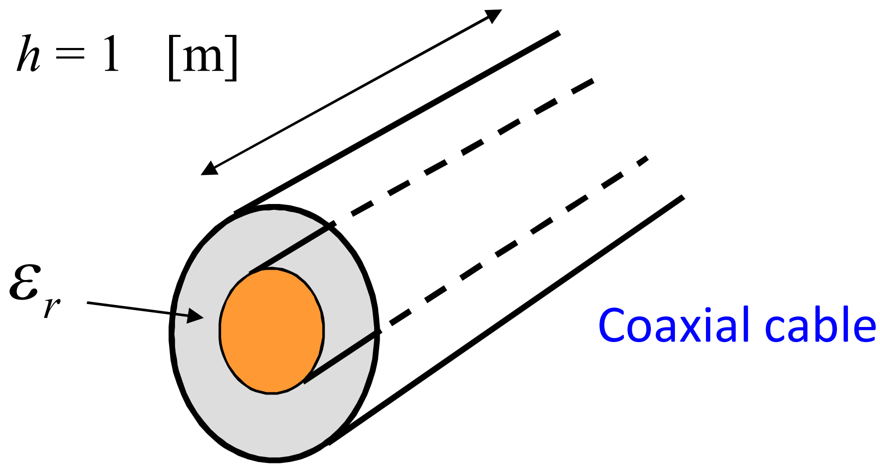


Find C, L, G, R

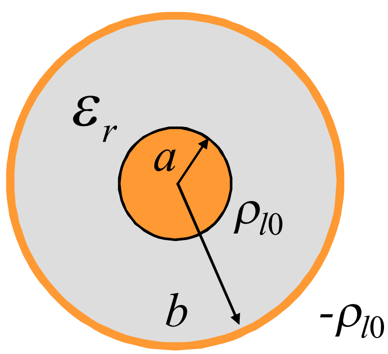
For a TEM_z mode, the shape of the fields is independent of frequency, and hence we can perform the calculation using electrostatics and magnetostatics.

We will assume no variation in the z direction, and take a length of one meter in the z direction in order top calculate the per-unit-length parameters.

Coaxial Cable (cont.)



Find C (capacitance / length)



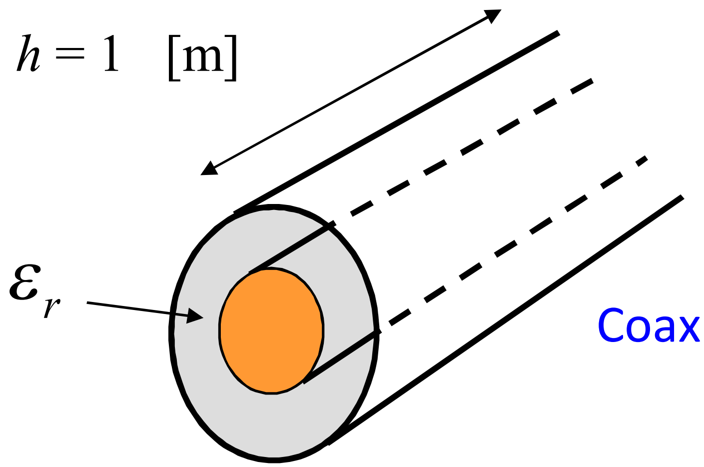
From Gauss's law:

$$\underline{E} = \underline{\hat{\rho}}\left(\frac{\rho_{\ell 0}}{2\pi\varepsilon\rho}\right) = \underline{\hat{\rho}}\left(\frac{\rho_{\ell 0}}{2\pi\varepsilon_0\varepsilon_r\rho}\right)$$

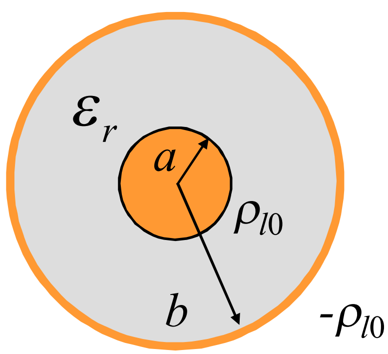
$$V = V_{AB} = \int_{\underline{A}}^{\underline{B}} \underline{E}\cdot\underline{dr}$$

$$= \int_a^b E_\rho\, d\rho = \frac{\rho_{\ell 0}}{2\pi\varepsilon_0\varepsilon_r}\ln\left(\frac{b}{a}\right)$$

Coaxial Cable (cont.)



Coaxial cable



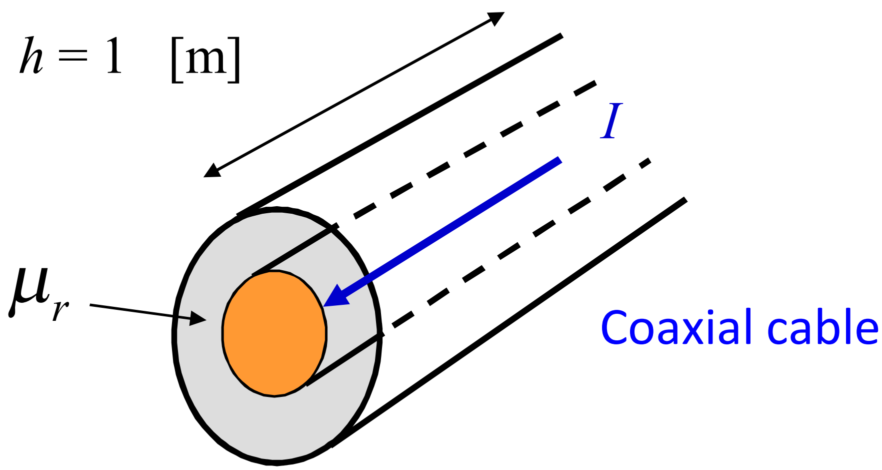
Hence

$$C = \frac{Q}{V} = \frac{\rho_{\ell 0}(1)}{\left(\frac{\rho_{\ell 0}}{2\pi\varepsilon_0\varepsilon_r}\right)\ln\left(\frac{b}{a}\right)}$$

We then have

$$C = \frac{2\pi\varepsilon_0\varepsilon_r}{\ln\left(\frac{b}{a}\right)} \quad [\text{F/m}]$$

Coaxial Cable (cont.)



Find L (inductance / length)

From Ampere's law:

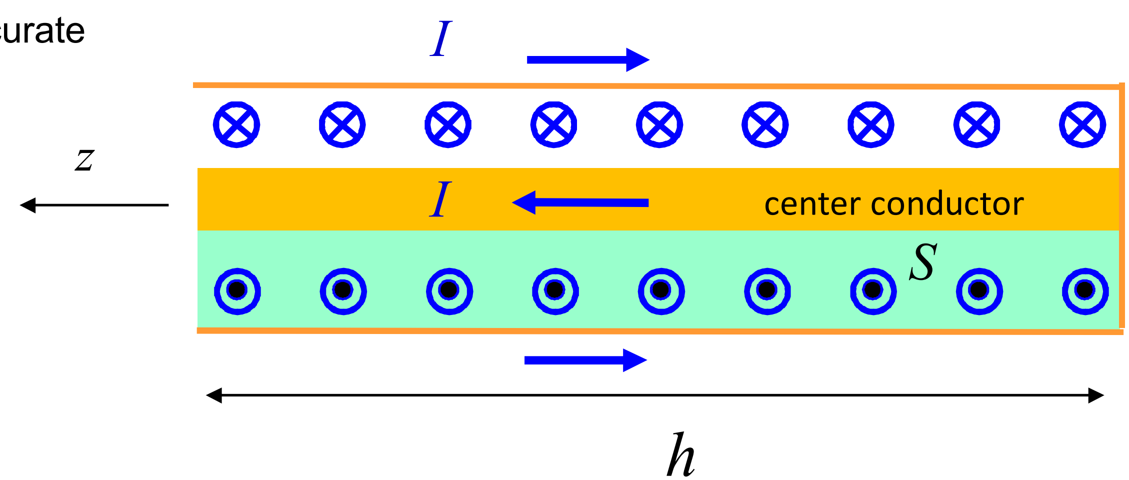
$$\underline{H} = \underline{\hat{\phi}} \left(\frac{I}{2\pi \rho} \right)$$

$$\underline{B} = \underline{\hat{\phi}} \left(\frac{I}{2\pi \rho} \right) \mu_0 \mu_r$$

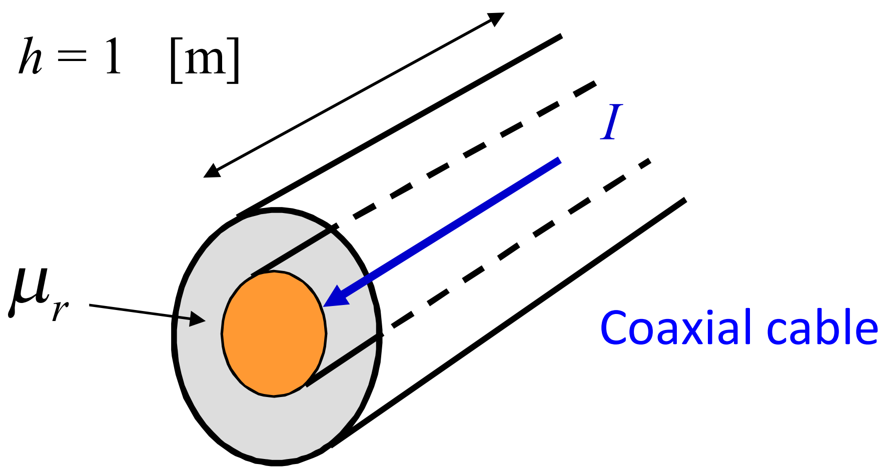
Note: We ignore "internal inductance" here, and only look at the magnetic field *between* the two conductors (accurate for high frequency.

Magnetic flux:

$$\psi = (1) \int_a^b B_\phi \, d\rho$$



Coaxial Cable (cont.)



$$\psi = (1)\,\mu_0\mu_r \int_a^b H_\phi \, d\rho$$

$$= \mu_0\mu_r \int_a^b \frac{I}{2\pi\rho}\, d\rho$$

$$= \mu_0\mu_r \frac{I}{2\pi} \ln\left(\frac{b}{a}\right)$$

$$L = \frac{\psi}{I} = \mu_0\mu_r \frac{1}{2\pi} \ln\left(\frac{b}{a}\right)$$

Hence

$$L = \frac{\mu_0\mu_r}{2\pi} \ln\left(\frac{b}{a}\right) \quad [\mathrm{H/m}]$$

Coaxial Cable (cont.)

Observation:

$$C = \frac{2\pi \varepsilon_0 \varepsilon_r}{\ln\left(\frac{b}{a}\right)} \quad \text{[F/m]}$$

$$L = \frac{\mu_0 \mu_r}{2\pi} \ln\left(\frac{b}{a}\right) \quad \text{[H/m]}$$

$$LC = \mu\varepsilon = \mu_0 \varepsilon_0 \left(\mu_r \varepsilon_r\right)$$

This result actually holds for any transmission line.

Coaxial Cable (cont.)

For a lossless cable: $$Z_0 = \sqrt{\frac{L}{C}}$$

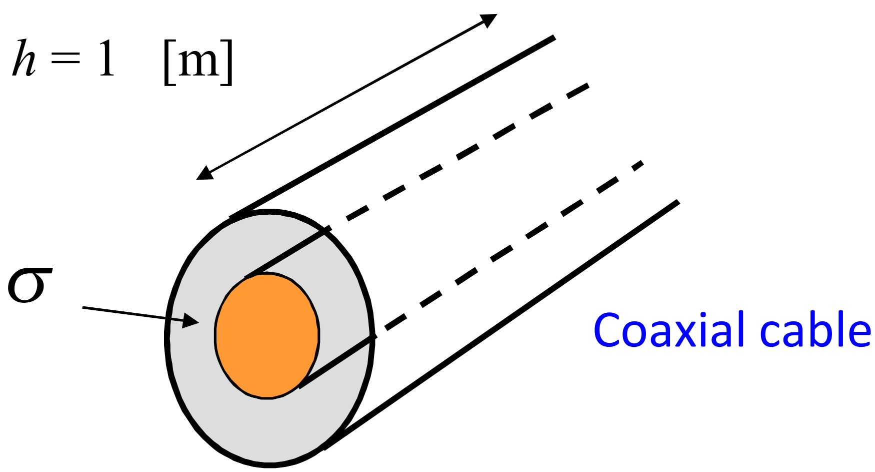
$$C = \frac{2\pi \varepsilon_0 \varepsilon_r}{\ln\left(\frac{b}{a}\right)} \quad \text{[F/m]}$$

$$L = \frac{\mu_0 \mu_r}{2\pi} \ln\left(\frac{b}{a}\right) \quad \text{[H/m]}$$

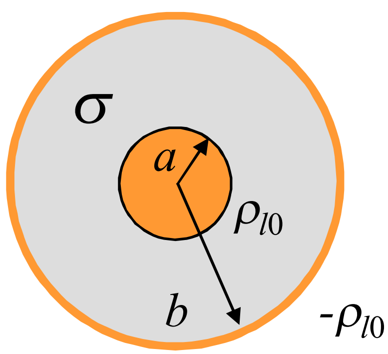
$$Z_0 = \eta_0 \sqrt{\frac{\mu_r}{\varepsilon_r}} \frac{1}{2\pi} \ln\left(\frac{b}{a}\right) \quad [\Omega]$$

$$\eta_0 = \sqrt{\frac{\mu_0}{\varepsilon_0}} = 376.7303 \ [\Omega]$$

Coaxial Cable (cont.)



Find G (conductance / length)



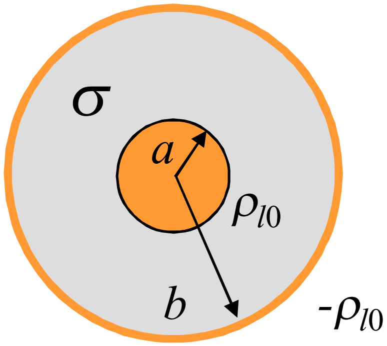
From Gauss's law:

$$\underline{E} = \underline{\hat{\rho}}\left(\frac{\rho_{\ell 0}}{2\pi\varepsilon\rho}\right) = \underline{\hat{\rho}}\left(\frac{\rho_{\ell 0}}{2\pi\varepsilon_0\varepsilon_r\rho}\right)$$

$$V = V_{AB} = \int_{\underline{A}}^{\underline{B}} \underline{E}\cdot\underline{dr}$$

$$= \int_a^b E_\rho\, d\rho = \frac{\rho_{\ell 0}}{2\pi\varepsilon_0\varepsilon_r}\ln\left(\frac{b}{a}\right)$$

Coaxial Cable (cont.)



$$\underline{J} = \sigma \underline{E}$$

$$\begin{aligned} I_{leak} &= J_\rho \big|_{\rho=a} \left[(1)\, 2\pi a\right] \\ &= 2\pi a \sigma\, E_\rho \big|_{\rho=a} \\ &= 2\pi a \sigma \left(\frac{\rho_{\ell 0}}{2\pi \varepsilon_0 \varepsilon_r a} \right) \end{aligned}$$

We then have $\quad G = \dfrac{I_{leak}}{V}$

$$G = \frac{2\pi a \sigma \left(\dfrac{\rho_{\ell 0}}{2\pi \varepsilon_0 \varepsilon_r a} \right)}{\dfrac{\rho_{\ell 0}}{2\pi \varepsilon_0 \varepsilon_r} \ln\left(\dfrac{b}{a} \right)}$$

or

$$G = \frac{2\pi\sigma}{\ln\left(\dfrac{b}{a}\right)} \quad [\mathrm{S/m}]$$

Coaxial Cable (cont.)

Observation:

$$C = \frac{2\pi\varepsilon}{\ln\left(\frac{b}{a}\right)} \quad \text{[F/m]} \qquad\qquad \varepsilon = \varepsilon_0 \varepsilon_r$$

$$G = \frac{2\pi\sigma}{\ln\left(\frac{b}{a}\right)} \quad \text{[S/m]}$$

$$G = C\left(\frac{\sigma}{\varepsilon}\right)$$

This result actually holds for any transmission line.

Coaxial Cable (cont.)

As just derived,

$$G = C\left(\frac{\sigma}{\varepsilon}\right)$$

To be more general:

$$\frac{G}{\omega C} = \left(\frac{\sigma}{\omega\varepsilon}\right) = \tan\delta$$

This is the loss tangent that would arise from conductivity effects.

$$\frac{G}{\omega C} = \tan\delta$$

The loss tangent actually arises from both conductivity loss and polarization loss (molecular friction loss), ingeneral.

Note: It is the loss tangent that is usually (approximately) constant for a material, over a wide range of frequencies.

Coaxial Cable (cont.)

General expression for loss tangent:

$$\varepsilon_c = \varepsilon - j\left(\frac{\sigma}{\omega}\right)$$ ← Effective permittivity that accounts for conductivity

$$= \left(\varepsilon' - j\varepsilon''\right) - j\left(\frac{\sigma}{\omega}\right)$$

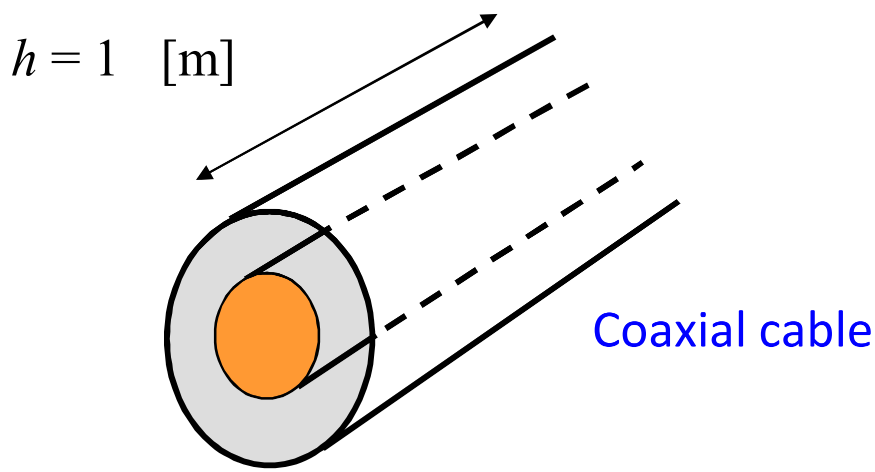
$$= \varepsilon_c' - j\varepsilon_c''$$

Loss due to molecular friction → ε''

Loss due to conductivity → $\left(\frac{\sigma}{\omega}\right)$

$$\tan\delta \equiv \frac{\varepsilon_c''}{\varepsilon_c'} = \frac{\varepsilon'' + \left(\frac{\sigma}{\omega}\right)}{\varepsilon'}$$

Coaxial Cable (cont.)



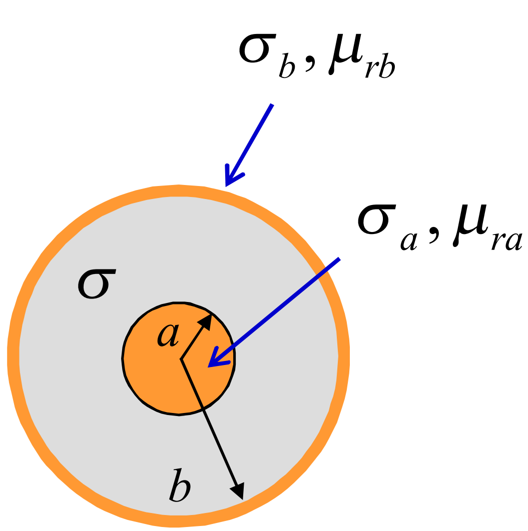
Find R (resistance / length)

$$R = R_a + R_b$$

$$R_a = R_{sa}\left(\frac{1}{2\pi a}\right)$$

$$R_b = R_{sb}\left(\frac{1}{2\pi b}\right)$$

R_s = surface resistance of metal



$$R_{sa} = \frac{1}{\sigma_a \delta_a}$$

$$R_{sb} = \frac{1}{\sigma_b \delta_b}$$

$$\delta_a = \sqrt{\frac{2}{\omega \mu_0 \mu_{ra} \sigma_a}}$$

$$\delta_b = \sqrt{\frac{2}{\omega \mu_0 \mu_{rb} \sigma_b}}$$

General Transmission Line Formulas

(1) $\sqrt{\dfrac{L}{C}} = Z_0^{lossless}$ = characteristic impedance of line (neglecting loss)

(2) $LC = \mu\varepsilon' = \mu_0\varepsilon_0\left(\mu_r\varepsilon_r'\right)$

(3) $\dfrac{G}{\omega C} = \tan\delta$

(4)
$$R = R_a + R_b$$
$$R_i = R_s\left[\frac{1}{|I|^2}\int_{C_i} |J_{sz}(l)|^2\, dl\right]$$
C_i = contour of conductor, $i = a, b$

Equations (1) and (2) can be used to find L and C if we know the material properties and the characteristic impedance of the lossless line.

Equation (3) can be used to find G if we know the material loss tangent.

Equation (4) can be used to find R (discussed later).

General Transmission Line Formulas (cont.)

Al four per-unit-length parameters can be found from $Z_0^{lossless}, \; R$

$$L = Z_0^{lossless} \sqrt{\mu \varepsilon'}$$

$$C = \sqrt{\mu \varepsilon'} \; / \; Z_0^{lossless}$$

$$G = (\omega C) \tan \delta$$

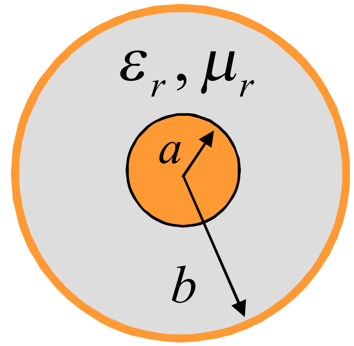
$$R = R$$

Common Transmission Lines

Coax

$$Z_0^{lossless} = \eta_0 \sqrt{\frac{\mu_r}{\varepsilon_r}} \frac{1}{2\pi} \ln\left(\frac{b}{a}\right) \quad [\Omega]$$

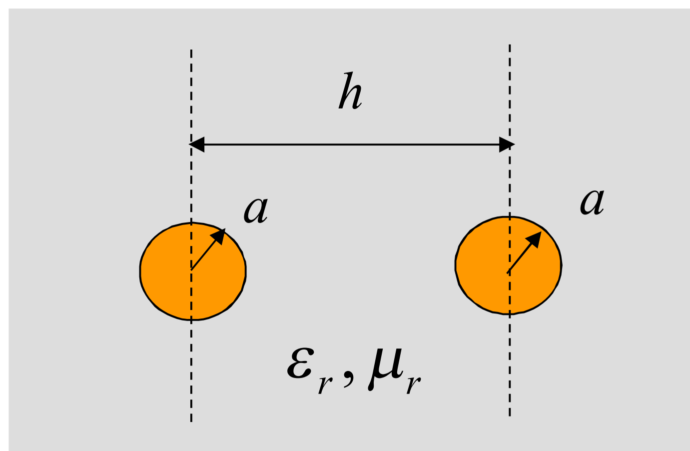
$$R = R_{sa}\left(\frac{1}{2\pi a}\right) + R_{sb}\left(\frac{1}{2\pi b}\right)$$



Twin-lead

$$Z_0^{lossless} = \frac{\eta_0}{\pi} \sqrt{\frac{\mu_r}{\varepsilon_r}} \cosh^{-1}\left(\frac{h}{2a}\right) \quad [\Omega]$$

$$R = R_s \left[\frac{1}{\pi a} \frac{\left(\frac{h}{2a}\right)}{\sqrt{\left(\frac{h}{2a}\right)^2 - 1}} \right]$$



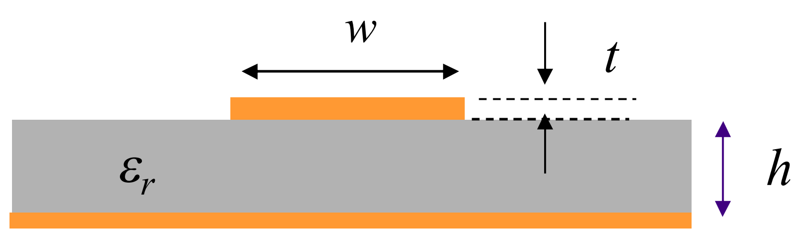
Common Transmission Lines (cont.)

Microstrip $(w/h \geq 1)$

$$Z_0(f) = Z_0(0)\left(\frac{\varepsilon_r^{eff}(f)-1}{\varepsilon_r^{eff}(0)-1}\right)\sqrt{\frac{\varepsilon_r^{eff}(0)}{\varepsilon_r^{eff}(f)}}$$

$$Z_0(0) = \frac{120\pi}{\sqrt{\varepsilon_r^{eff}(0)}\left[(w'/h)+1.393+0.667\ln\left((w'/h)+1.444\right)\right]}$$

$$w' = w + \frac{t}{\pi}\left(1+\ln\left(\frac{2h}{t}\right)\right)$$



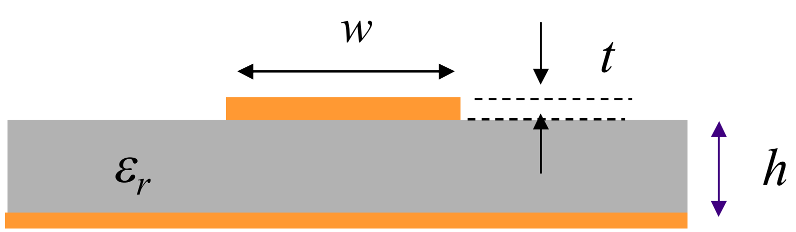
Common Transmission Lines (cont.)

Microstrip $(w/h \geq 1)$

$$\varepsilon_r^{eff}(f) = \left(\sqrt{\varepsilon_r^{eff}(0)} + \frac{\sqrt{\varepsilon_r} - \sqrt{\varepsilon_r^{eff}(0)}}{1 + 4F^{-1.5}} \right)^2$$

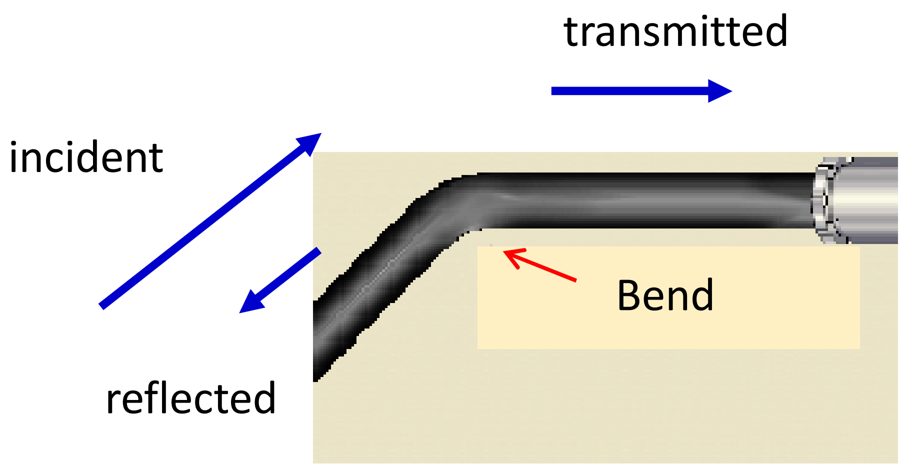
$$\varepsilon_r^{eff}(0) = \frac{\varepsilon_r + 1}{2} + \left(\frac{\varepsilon_r - 1}{2} \right) \left(\frac{1}{\sqrt{1 + 12(h/w)}} \right) - \left(\frac{\varepsilon_r - 1}{4.6} \right) \left(\frac{t/h}{\sqrt{w/h}} \right)$$

$$F = 4\left(\frac{h}{\lambda_0} \right) \sqrt{\varepsilon_r - 1} \left(0.5 + \left(1 + 0.868 \ln\left(1 + \frac{w}{h} \right) \right)^2 \right)$$

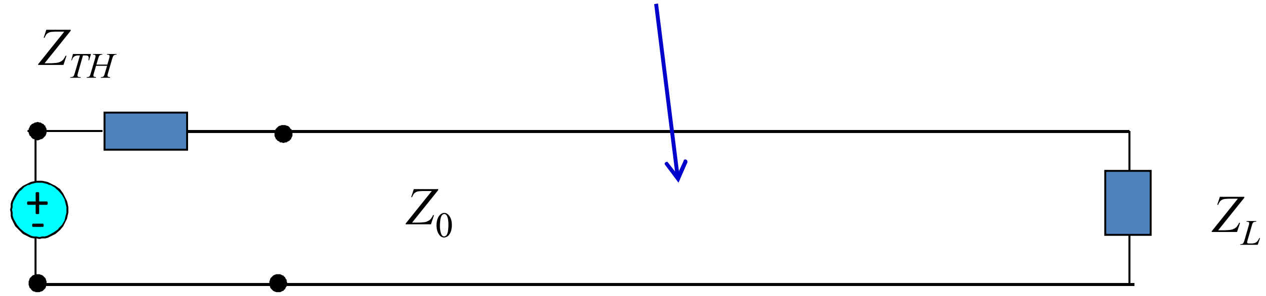


Limitations of Transmission-Line Theory

At high frequency, discontinuity effects can become important.



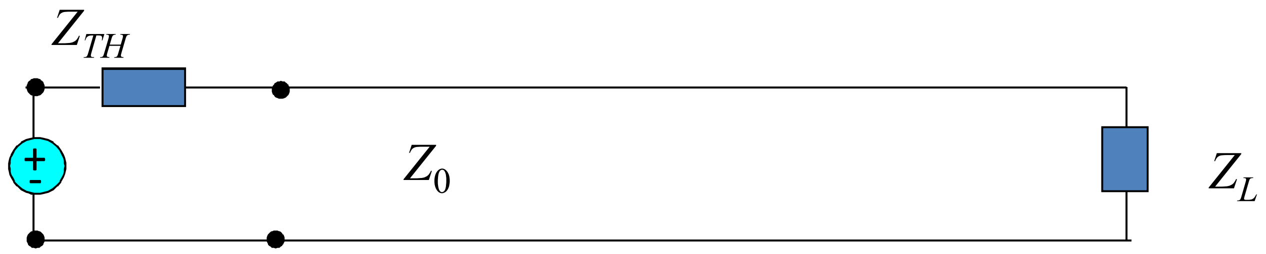
The simple TL model does not account for the bend.



Limitations of Transmission-Line Theory (cont.)

At high frequency, radiation effects can become important.

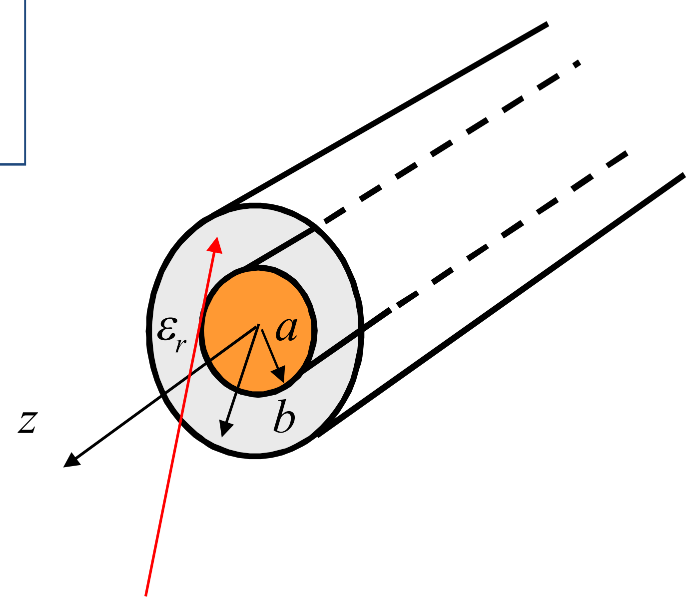
We want energy to travel from the generator to the load, without radiating.



When will radiation occur?

Limitations of Transmission-Line Theory (cont.)

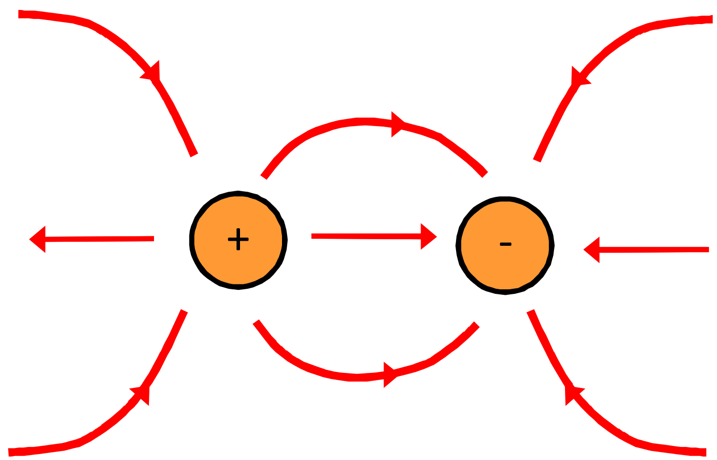
The coaxial cable is a perfectly shielded system – there is never any radiation at any frequency, or under any circumstances.

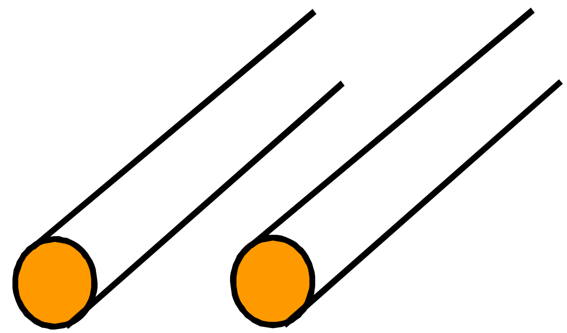


The fields are confined to the region between the two conductors.

Limitations of Transmission-Line Theory (cont.)

The twin lead is an open type of transmission line – the fields extend out to infinity.



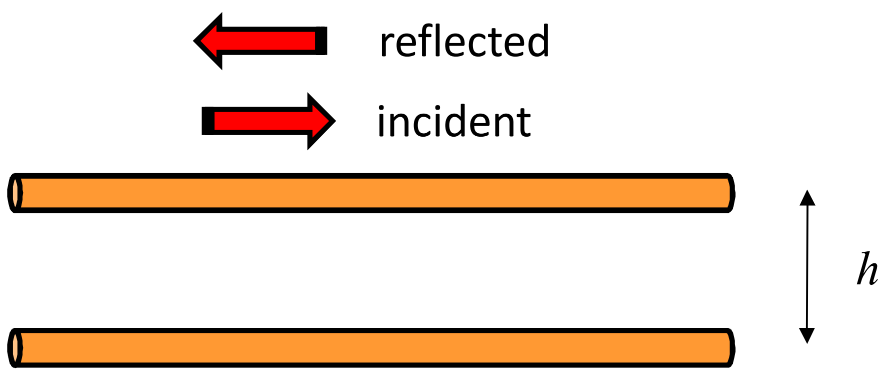


The extended fields may cause interference with nearby objects. (This may be improved by using "twisted pair.")

Having fields that extend to infinity is not the same thing as having radiation, however.

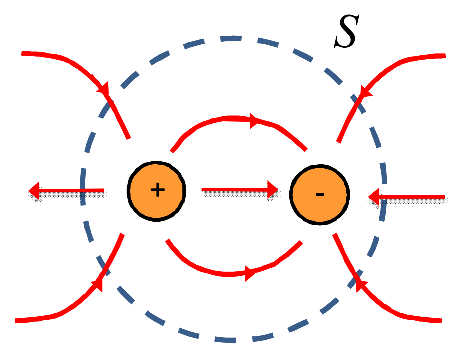
Limitations of Transmission-Line Theory (cont.)

The infinite twin lead will not radiate by itself, regardless of how far apart the lines are.



No attenuation on an infinite lossless line

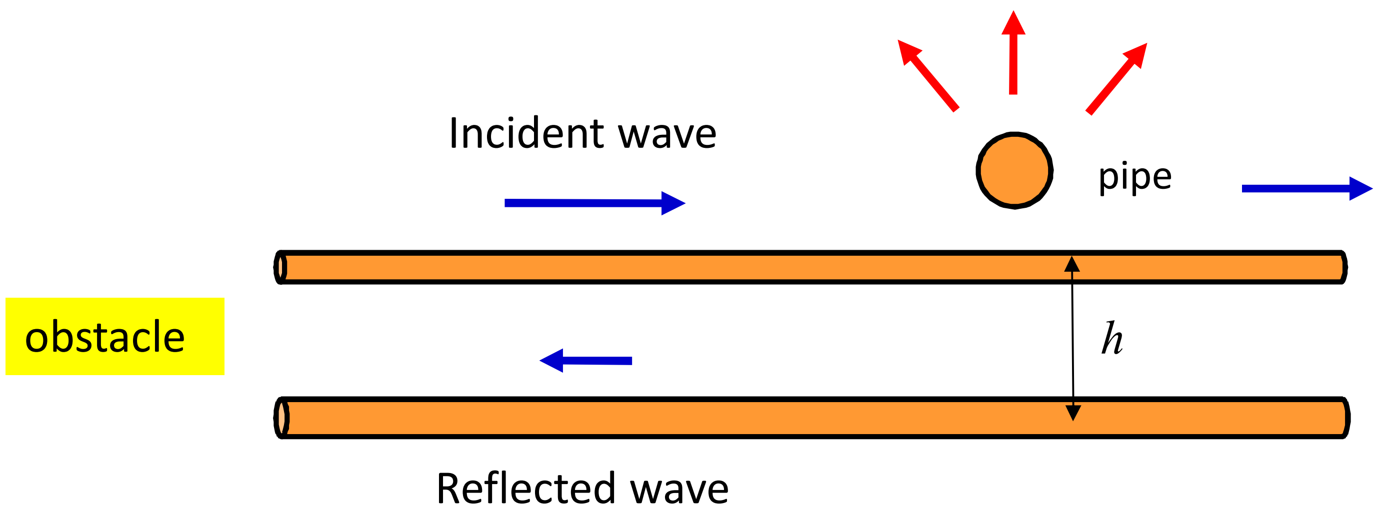
$$P_t = \int_S \mathrm{Re}\left(\frac{1}{2}\left(\underline{E}\times\underline{H}^*\right)\right)\cdot\underline{\hat{\rho}}\,dS = 0$$



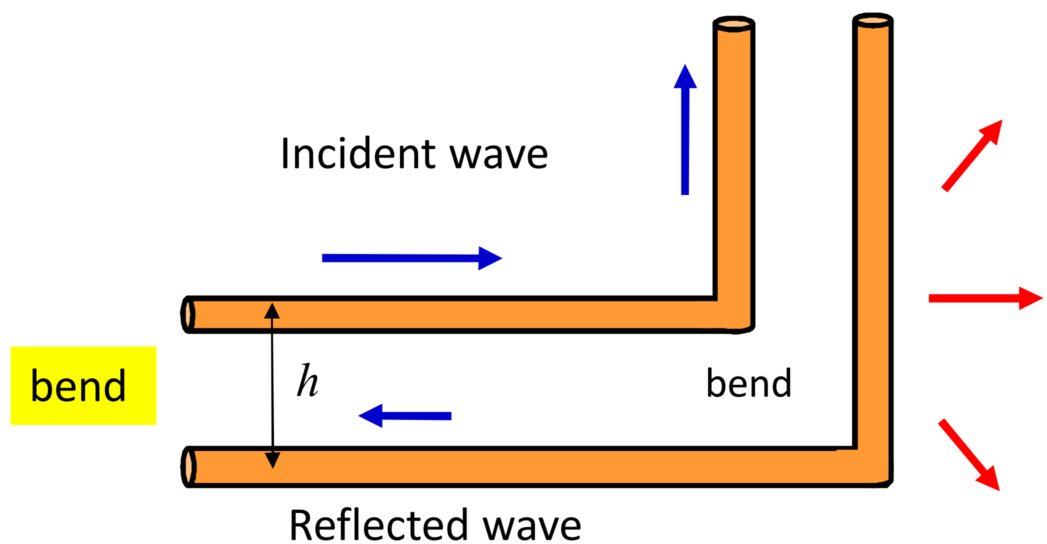
The incident and reflected waves represent an exact solution to Maxwell's equations on the infinite line, at any frequency.

Limitations of Transmission-Line Theory (cont.)

A discontinuity on the twin lead will cause radiation to occur.



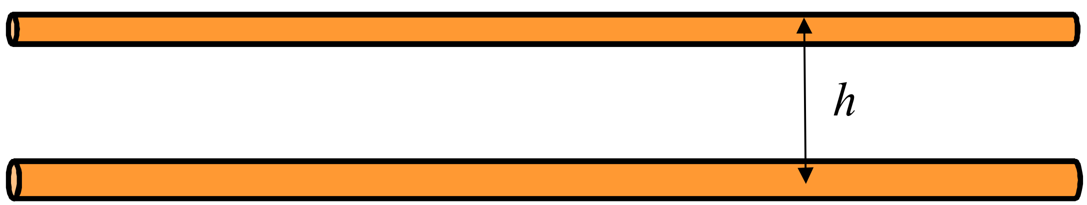
Note: Radiation effects increase as the frequency increases.



Limitations of Transmission-Line Theory (cont.)

To reduce radiation effects of the twin lead at discontinuities:

1) Reduce the separation distance h (keep $h << \lambda$).
2) Twist the lines (twisted pair).



CAT 5 cable
(twisted pair)