Digital Image Processing

Digital Image Fundamental

- Digital Image Fundamentals:
  - Elements of Visual Perception
  - Light and the Electromagnetic Spectrum
  - Image Sensing and Acquisition
  - Image Sampling and Quantization
  - Some Basic Relationships between Pixels
  - An Introduction to the Mathematical Tools Used in Digital Image Processing
Digital Image Processing

Digital Image Fundamental

• Eye Physiology:
Optical Sensors in retina:

- Cones: Highly Sensitive to Color \(\approx (6-7) \times 10^6\)
- Rods: Highly Sensitive to Low Levels of Illumination \(\approx (75-150) \times 10^6\)
Digital Image Processing

Digital Image Fundamental

• Image Formation in the Eye:
• Brightness Adaptation:
  - Eyes can adapt to a large dynamic range of intensity ($10^{10}$) but not simultaneously.
Digital Image Processing

Digital Image Fundamental

- Brightness Discrimination:

\[ I + \Delta I \]
• Weber Ratio:

\[ \Delta I_C = \text{Increment of illumination discriminable 50% times} \]
• Match Band Effect:

Actual intensity

Perceived intensity
• Simultaneous Contrast:

Appear Darker
Digital Image Processing

Digital Image Fundamental

- Eye illusions:
Digital Image Fundamental

- 2.2: Light and the Electromagnetic Spectrum
- 2.3: Image Sensing and Acquisition
- Ignored!
Digital Image Fundamental

- A Simple Image Formation:
  \[ f(x, y) = i(x, y) r(x, y) \]
  - \( 0 < i(x, y) < \infty \): Illumination
  - \( 0 < r(x, y) < 1 \): Reflection

- Gray Level: Intensity of monochrome images.
Digital Image Processing

Digital Image Fundamental

- Image Sampling and Quantization (1):

Scan Line
Digital Image Processing

Digital Image Fundamental

- Image Sampling and Quantization (2):
Digital Image Fundamental

- Image Sampling and Quantization:
  - Spatial and Gray Level Resolution
  - How to determine the sampling rate?
  - Nyquist sampling theorem
    - If input is a **band-limited signal** with maximum frequency $\Omega_N$
    - The input can be **uniquely determined** if sampling rate $\Omega_S > 2\Omega_N$
      - Nyquist frequency : $\Omega_N$
      - Nyquist rate : $\Omega_S$
Digital Image Processing

Digital Image Fundamental

- Digital Image Representation:
Digital Image, Mathematical Definition:

- \( I = f(x,y) \)
- \( I \): intensity (or color)
- \( (x,y) \): Position or Coordination
- When \( (x,y) \) and \( I \) are finite and discrete quantities \( \rightarrow \) digital image
- pixels, picture elements, image elements
Digital Image Processing

Digital Image Fundamental

• Mathematical Representation:

\[
f(x, y) = \begin{bmatrix}
    f(0, 0) & f(0, 1) & \cdots & f(0, N - 1) \\
    f(1, 0) & f(1, 1) & \cdots & f(1, N - 1) \\
    \vdots & \vdots & \ddots & \vdots \\
    f(M - 1, 0) & f(M - 1, 1) & \cdots & f(M - 1, N - 1)
\end{bmatrix}
\]

bits to store the image = \(M \times N \times k\)

gray level = \(L = 2^k\)
Digital Image Fundamental

- **L-level digital image of size M×N**
  - Means: A digital image having:
    - A spatial resolution M×N pixels
    - A gray-level resolution of L levels (0 ... L-1)

- **Spatial resolution in real-world space**

  ![Resolution Diagram](image)

  - Resolution = \(1/2W\) (line/cm)
Digital Image Fundamental

- $L = 2^k$ gray levels, gray scales $[0,\ldots,L-1]$
- **The dynamic range of an image**
  - $[\text{min(image)} \ \text{max(image)}]$
  - If the dynamic range of an image spans a significant portion of the gray scale $\rightarrow$ **high contrast**
  - Otherwise, **low dynamic range** results in a washed out gray look.
Digital Image Processing

Digital Image Fundamental

- Saturation and Noise:
Digital Image Processing

Digital Image Fundamental

- Number of Storage bits (M=N):

<table>
<thead>
<tr>
<th>N/k</th>
<th>1 (L = 2)</th>
<th>2 (L = 4)</th>
<th>3 (L = 8)</th>
<th>4 (L = 16)</th>
<th>5 (L = 32)</th>
<th>6 (L = 64)</th>
<th>7 (L = 128)</th>
<th>8 (L = 256)</th>
</tr>
</thead>
<tbody>
<tr>
<td>32</td>
<td>1,024</td>
<td>2,048</td>
<td>3,072</td>
<td>4,096</td>
<td>5,120</td>
<td>6,144</td>
<td>7,168</td>
<td>8,192</td>
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<td>16,384</td>
<td>20,480</td>
<td>24,576</td>
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<td>32,768</td>
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<td>128</td>
<td>16,384</td>
<td>32,768</td>
<td>49,152</td>
<td>65,536</td>
<td>81,920</td>
<td>98,304</td>
<td>114,688</td>
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</tr>
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<td>327,680</td>
<td>393,216</td>
<td>458,752</td>
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<td>512</td>
<td>262,144</td>
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<td>786,432</td>
<td>1,048,576</td>
<td>1,310,720</td>
<td>1,572,864</td>
<td>1,835,008</td>
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<td>4,194,304</td>
<td>5,242,880</td>
<td>6,291,456</td>
<td>7,340,032</td>
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<td>16,777,216</td>
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<td>25,165,824</td>
<td>29,369,128</td>
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<td>8192</td>
<td>67,108,864</td>
<td>134,217,728</td>
<td>201,326,592</td>
<td>268,435,456</td>
<td>335,544,320</td>
<td>402,653,184</td>
<td>469,762,048</td>
<td>536,870,912</td>
</tr>
</tbody>
</table>
Digital Image Processing

Digital Image Fundamental

- Spatial and Intensity Resolution:

1250 dpi  300 dpi  150 dpi  75 dpi
• Bits Reduction (More Quantization):
  - 8 bits to 1 bits (Left to Right-Top to Down)
Digital Image Fundamental

- Three types of image (Low/Medium/High Details):
Digital Image Processing

Digital Image Fundamental

- Sampling-Quantization Tradeoff:
Digital Image Processing

Digital Image Fundamental

- **Image Interpolation:**
  - Nearest Neighbor (NN)
  - Bilinear (BL) using 4 nearest neighbor:
    \[ f(x, y) = ax + by + cxy + d \]
  - Bicubic (BC) using 16 nearest neighbor:
    \[ f(x, y) = \sum_{i=0}^{3} \sum_{j=0}^{3} a_{ij}x^i y^j \]
• Image Interpolation (Example):
  - Reduced to 72 dpi
  - NN, BL, BC
  - Reduced to 150 dpi
  - NN, BL, BC
Digital Image Processing

Digital Image Fundamental

- Basic Relationships Between Pixels:
  - 4-Neighbors
    \[ N_4(p) : \{(x+1, y), (x-1, y), (x, y+1), (x, y-1)\} \]
  - Diagonal Neighbors
    \[ N_D(p) : \{(x+1, y+1), (x+1, y-1), (x-1, y+1), (x-1, y-1)\} \]
  - 8-Neighbors:
    \[ N_8(p) : N_4(p) \cup N_D(p) \]
Digital Image Processing

Digital Image Fundamental

• Basic Relationships Between Pixels:

\[
N_4 = \begin{bmatrix}
0 & 1 & 0 \\
1 & 1 & 1 \\
0 & 1 & 0
\end{bmatrix} \quad
N_D = \begin{bmatrix}
1 & 0 & 1 \\
0 & 1 & 0 \\
1 & 0 & 1
\end{bmatrix} \quad
N_8 = \begin{bmatrix}
1 & 1 & 1 \\
1 & 1 & 1 \\
1 & 1 & 1
\end{bmatrix}
\]
• Adjacency:
  - p and q are 4-adjacent: \( q \in N_4(p) \)
  - p and q are 8-adjacent: \( q \in N_8(p) \)
  - p and q are m-adjacent:

\[
\{ q \in N_4(p) \} \text{ or } \{ q \in N_D(p) \} \text{ and } \{ N_4(p) \cap N_4(q) \} = \emptyset
\]
Digital Image Fundamental

**Distance Measure:**

a. \( D(p,q) \geq 0 \quad D(p,q) = 0 \quad \text{iff} \quad p = q \)

b. \( D(p,q) = D(q,p) \)

b. \( D(p,q) \leq D(p,r) + D(r,q) \)

**Examples:**

- **Euclidean:** \( D_e(p,q) = \sqrt{(x-s)^2 + (y-t)^2} \)
- **\( D_4 \) (City Block or Manhattan):** \( D_4(p,q) = |x-s| + |y-t| \)
- **\( D_8 \) (Chessboard):** \( D_8(p,q) = \max\{|x-s|, |y-t|\} \)
Digital Image Fundamental

- Constant Distance Contour:
  - D4 (Left)
  - D8 (Right)

```
  2  1  2  2
2  1  2  1  1  2
2  1  0  1  2
2  1  2  1  1  2
  2
```

```
  2  2  2  2  2
2  1  1  1  2
2  1  0  1  2
2  1  1  1  2
  2  2  2  2  2
```
Mathematical Tools:

- Array and Matrix Operations
- Linear and nonlinear Operation
  - Fourier Filtering, Ordered Statistics Filtering, ...
- Arithmetic Operation (+, -, *, /)
  - Averaging, Subtraction, ...
- Set and Logical Operations
  - Fuzzy or Crisp Sets
• Image Averaging:
  – Consider an additive noise condition:
    \[ g(x,y) = f(x,y) + \eta(x,y) \]
  – Conditions:
    • \textit{Noise, }\eta(x,y):
      – Uncorrelated
      – i.i.d
      – Zero Mean
    • \textit{Subject, }f(x,y):
      – Physical Stationary
      – Repeatable Experiments
Digital Image Processing

Digital Image Fundamental

- Image Averaging:

\[ \bar{g}(x, y) = \frac{1}{N} \sum_{i=1}^{N} g_i(x, y) \Rightarrow \begin{cases} E\{\bar{g}(x, y)\} = f(x, y) \\ \sigma_{\bar{g}(x,y)}^2 = \frac{1}{N} \sigma_{\eta(x,y)}^2 \end{cases} \]
Digital Image Processing

Digital Image Fundamental

- Example:
  - 5, 10, 20, 50, and 100 averaging.
Digital Image Processing

Digital Image Fundamental

- **Image Subtraction:**
  - Original (Left), LSB set to zero (Center), Difference (Right)
• Digital Subtraction Radiography (DSA):
  – Pre and Post Imaging

**FIGURE 2.28**
Digital subtraction angiography.
(a) Mask image.
(b) A live image.
(c) Difference between (a) and (b).
(d) Enhanced difference image.
(Figures (a) and (b) courtesy of The Image Sciences Institute, University Medical Center, Utrecht, The Netherlands.)
Image Multiplication/Division:

- Shading Correction

**FIGURE 2.29** Shading correction. (a) Shaded SEM image of a tungsten filament and support, magnified approximately 130 times. (b) The shading pattern. (c) Product of (a) by the reciprocal of (b). (Original image courtesy of Michael Shaffer, Department of Geological Sciences, University of Oregon, Eugene.)
Image Multiplication/Division:

- ROI Masking

**FIGURE 2.30** (a) Digital dental X-ray image. (b) ROI mask for isolating teeth with fillings (white corresponds to 1 and black corresponds to 0). (c) Product of (a) and (b).
Digital Image Processing

Digital Image Fundamental

• Set and Logical Operation:
Digital Image Processing

Digital Image Fundamental

• Example
  - Original (Left), Negative (Center), Right (union with Constant)

\[ A \cup B = \{\max(A, B)\} \]
Digital Image Processing

Digital Image Fundamental

- Logical Operation:
Mathematical Tools:

- Spatial Operations
  - Single pixel
  - Neighborhood
• Single Pixel:

\[ s = T(z) \]
Digital Image Processing

Digital Image Fundamental

- Neighborhood Operation:

\[
g(x, y) = \frac{1}{mn} \sum_{(r,c) \in S_{xy}} f(r,c)
\]

**FIGURE 2.35**
Local averaging using neighborhood processing. The procedure is illustrated in (a) and (b) for a rectangular neighborhood. (c) The aortic angiogram discussed in Section 1.3.2. (d) The result of using Eq. (2.6-21) with \( m = n = 41 \). The images are of size 790 \( \times \) 686 pixels.
Digital Image Processing

Digital Image Fundamental

- **Geometric Spatial Transform**
  - General Formulation:
    \[
    (x, y) = T \{(u, v)\}
    \]
  - Example (Affine Transform)
    \[
    \begin{pmatrix}
    x \\
    y \\
    1
    \end{pmatrix} =
    \begin{bmatrix}
    a & b & c \\
    d & e & f \\
    0 & 0 & 1
    \end{bmatrix}
    \begin{pmatrix}
    u \\
    v \\
    1
    \end{pmatrix}
    \]
### Digital Image Processing

#### Digital Image Fundamental

- **Geometric Transform (1):**

<table>
<thead>
<tr>
<th>Transformation Name</th>
<th>Affine Matrix, $T$</th>
<th>Coordinate Equations</th>
<th>Example</th>
</tr>
</thead>
<tbody>
<tr>
<td>Identity</td>
<td>$\begin{bmatrix} 1 &amp; 0 &amp; 0 \ 0 &amp; 1 &amp; 0 \ 0 &amp; 0 &amp; 1 \end{bmatrix}$</td>
<td>$x = v \quad y = w$</td>
<td>$T$</td>
</tr>
<tr>
<td>Scaling</td>
<td>$\begin{bmatrix} c_x &amp; 0 &amp; 0 \ 0 &amp; c_y &amp; 0 \ 0 &amp; 0 &amp; 1 \end{bmatrix}$</td>
<td>$x = c_x v \quad y = c_y w$</td>
<td>$T$</td>
</tr>
<tr>
<td>Rotation</td>
<td>$\begin{bmatrix} \cos \theta &amp; \sin \theta &amp; 0 \ -\sin \theta &amp; \cos \theta &amp; 0 \ 0 &amp; 0 &amp; 1 \end{bmatrix}$</td>
<td>$x = v \cos \theta - w \sin \theta \quad y = v \cos \theta + w \sin \theta$</td>
<td>$\text{Rotation}$</td>
</tr>
</tbody>
</table>
- Geometric Transform (2):

Translation
\[
\begin{bmatrix}
1 & 0 & 0 \\
0 & 1 & 0 \\
t_x & t_y & 1
\end{bmatrix}
\]
\[x = v + t_x\]
\[y = w + t_y\]

Shear (vertical)
\[
\begin{bmatrix}
1 & 0 & 0 \\
s_v & 1 & 0 \\
0 & 0 & 1
\end{bmatrix}
\]
\[x = v + s_v w\]
\[y = w\]

Shear (horizontal)
\[
\begin{bmatrix}
1 & s_h & 0 \\
0 & 1 & 0 \\
0 & 0 & 1
\end{bmatrix}
\]
\[x = v\]
\[y = s_h v + w\]
Image Rotation:

**FIGURE 2.36** (a) A 300 dpi image of the letter T. (b) Image rotated 21° using nearest neighbor interpolation to assign intensity values to the spatially transformed pixels. (c) Image rotated 21° using bilinear interpolation. (d) Image rotated 21° using bicubic interpolation. The enlarged sections show edge detail for the three interpolation approaches.
Digital Image Processing

Digital Image Fundamental

Image Registration

- General Formulation:

\[(x, y) = T\{(u, v)\}\]

- Example (Bilinear Transform)

\[x = a_1 u + b_1 v + c_1 uv + d_1\]
\[y = a_2 u + b_2 v + c_2 uv + d_2\]
• Image Registration (Example):
  – Using tie points

FIGURE 2.37
Image registration. 
(a) Reference image. (b) Input (geometrically distorted image). 
Corresponding tie points are shown as small white squares near the corners.
(c) Registered image (note the errors in the border).
(d) Difference between (a) and (c), showing more registration errors.
Digital Image Processing

Digital Image Fundamental

- Vector and Matrix Operation:
  - Multispectral Image Processing
  - Image Transform (Fourier and etc.)
  - Probabilistic Methods
Digital Image Processing

Digital Image Fundamental

• Multispectral Image Processing:

\[ D(z, a) = [(z - a)^T(z - a)]^{\frac{1}{2}} \]

\[ = [(z_1 - a_1)^2 + (z_2 - a_2)^2 + \cdots + (z_n - a_n)^2]^{\frac{1}{2}} \]
Digital Image Processing

Digital Image Fundamental

- Image Transform (Fourier and etc.):

$$T(u, v) = \sum_{x=0}^{M-1} \sum_{y=0}^{N-1} f(x, y) r(x, y, u, v)$$

$$f(x, y) = \sum_{u=0}^{M-1} \sum_{v=0}^{N-1} T(u, v) s(x, y, u, v)$$
Digital Image Processing

Digital Image Fundamental

- Fourier Filtration:

![Fourier Filtration Example](image-url)
• Probabilistic Approaches:
  – Low-Medium-High contrast (From Left to Right)

\[
m_L = 14.3, \quad m_M = 31.6, \quad m_H = 49.2
\]
\[
\sigma_L^2 = 204.3, \quad \sigma_M^2 = 997.8, \quad \sigma_H^2 = 22424.9
\]
Digital Image Processing

Digital Image Fundamental

• Paradigm of image processing:
  – Low-level processing
    • Inputs and outputs are images
    • Primitive operations: de-noise, enhancement, sharpening, ...
  – Mid-level processing
    • Inputs are images, outputs are attributes extracted from images
    • Segmentation, classification,...
  – High-level processing
    • “Make sense” of an ensemble of recognized objects by machines
Matlab Image Processing Read/Write:

- imformats
- imfinfo, imread, imwrite
- dicominfo, dicomread, dicomwrite
- analyze75info, analyze75read (Mayo Clinic)
- interfileinfo, interfileread
Matlab Image Processing *Display*:
- `image`, `imagesc`, `imshow`, `imtool`, `subimage`
- `colorbar`, `montage`
Matlab Image Processing *Type Conversion*:

- `double`, `ind2gray`, `im2double`
- `uint16`, `uint8`, `gray2ind`