• **Image Enhancement:**
  - No Explicit definition

• **Methods**
  - **Spatial Domain:**
    • Linear
    • Nonlinear
  - **Frequency Domain:**
    • Linear
    • Nonlinear
Digital Image Processing

Intensity Transformations and Spatial Filtering

- Spatial Domain Process

\[ g(x, y) = T(f(x, y)) \]
Digital Image Processing

Intensity Transformations and Spatial Filtering

• For 1×1 neighborhood: \( s = T(r) \)
  - Contrast Enhancement/Stretching/Point Process

• For \( w \times w \) neighborhood:
  - Filtering/Mask/Kernel/Window/Template Processing

\[
s = T(r) \]

\[
s_0 = T(r_0)
\]

\[
T(r)
\]

\[
T(r)
\]

**FIGURE 3.2**
Intensity transformation functions.
(a) Contrast-stretching function.
(b) Thresholding function.

E. Fatemizadeh, Sharif University of Technology, 2012
Intensity Transformation Functions:

- **Negative**: Reflects the input intensity levels across the midpoint.
- **Log**: Applies a logarithmic transformation, amplifying small intensities and compressing large ones.
- **Identity**: No transformation applied, maintains the original intensity levels.
- **Inverse Log**: The inverse of the log transformation, compressing small intensities and amplifying large ones.
- **nth Root**: A general exponentiation transformation, where the root of the input intensity level is taken.
- **nth Power**: A general exponentiation transformation, where the input intensity level is raised to the nth power.

**FIGURE 3.3** Some basic intensity transformation functions. All curves were scaled to fit in the range shown.
Digital Image Processing

Intensity Transformations and Spatial Filtering

- Image Negatives:

\[ s = L - 1 - r \]

FIGURE 3.4
(a) Original digital mammogram.
(b) Negative image obtained using the negative transformation in Eq. (3.2-1). (Courtesy of G.E. Medical Systems.)
Log Transform:

\[ s = c \log (1 + r) \]

**FIGURE 3.5**
(a) Fourier spectrum. (b) Result of applying the log transformation in Eq. (3.2-2) with \( c = 1 \).
• Power-Law Transform:

\[ S = cr^\gamma \]

**FIGURE 3.6** Plots of the equation \( S = cr^\gamma \) for various values of \( \gamma \) (\( c = 1 \) in all cases). All curves were scaled to fit in the range shown.
**Gamma Correction:**

\[ r^{1.8 \cdots 2.5} \]

(Figure 3.7)
(a) Intensity ramp image. (b) Image as viewed on a simulated monitor with a gamma of 2.5. (c) Gamma-corrected image. (d) Corrected image as viewed on the same monitor. Compare (d) and (a).
Gamma Correction
- Too Dark Image

Original

\[ \gamma = 0.6 \]

\[ \gamma = 0.4 \]

\[ \gamma = 0.3 \]
• Gamma Correction
  – Too Bright Image
Digital Image Processing

Intensity Transformations and Spatial Filtering

- Contrast Stretching

![Graph showing contrast stretching](image)

Original

C. S.

THR.
Digital Image Processing

Intensity Transformations and Spatial Filtering

- Gray Level Slicing

![Graph showing grayscale level slicing with intervals](image)
Digital Image Processing

Intensity Transformations and Spatial Filtering

• Example

Using Fig 3.11 (a)
Using Fig 3.11 (b)

FIGURE 3.12 (a) Aortic angiogram. (b) Result of using a slicing transformation of the type illustrated in Fig. 3.11(a), with the range of intensities of interest selected in the upper end of the gray scale. (c) Result of using the transformation in Fig. 3.11(b), with the selected area set to black, so that grays in the area of the blood vessels and kidneys were preserved. (Original image courtesy of Dr. Thomas R. Gest, University of Michigan Medical School.)
Bit-Plane Slicing:
- Highlighting effect of a single bit!
Example:

- LSB -> MSB (Left to Right and Top to Bottom)
Digital Image Processing

Intensity Transformations and Spatial Filtering

- Image Reconstruction from bit-planes

Bits (7,8)  Bits (6,7,8)  Bits (5,6,7,8)
Digital Image Processing

Intensity Transformations and Spatial Filtering

• Histogram Processing:
  – Enhancement based on statistical Properties:
    • Local
    • Global
  – Histogram Definition:

\[
h(r_k) = n_k, \quad r_k \in [0, L-1], \quad n_k \in [0, M \times N] \\
p(r_k) = \frac{n_k}{n} = \frac{1}{M \times N} n_k
\]
Digital Image Processing

Intensity Transformations and Spatial Filtering

- Histogram Visual Meaning:
  - Dark
  - Light
  - Low Contrast
  - High Contrast
• Histogram Equalization:
  – Continuous Case.
  – Seek for a suitable transform (Except for negative):
• Effect of Point Process on Histogram:

\[
\begin{align*}
  s &= T(r) \\
  r &= T^{-1}(s) \quad \Rightarrow \quad P_s(s) = P_r(r) \left| \frac{dr}{ds} \right|
\end{align*}
\]

• Special Case (CDF):

\[
s = T(r) = (L-1) \int_0^r p_r(w) \, dw \quad \Rightarrow \quad \frac{ds}{dr} = \frac{dT(r)}{dr} = (L-1) p_r(r)
\]

\[
\Rightarrow \quad p_s(s) = p_r(r) \left| \frac{dr}{ds} \right| = p_r(r) \left| \frac{1}{p_r(r)} \right| = \frac{1}{L-1}, \quad 0 \leq s \leq L-1
\]

\[\therefore\quad \text{Uniform Distribution on } [0, L-1]\]
Digital Image Processing

Intensity Transformations and Spatial Filtering

Concept Illustration:

\[ p_r(r) \]

\[ p_s(s) \]

\[ \frac{1}{L-1} \]

\[ 0 \quad L-1 \]

\[ 0 \quad L-1 \]

**FIGURE 3.18** (a) An arbitrary PDF. (b) Result of applying the transformation in Eq. (3.3-4) to all intensity levels, \( r \). The resulting intensities, \( s \), have a uniform PDF, independently of the form of the PDF of the \( r \)'s.
Inteivity Transformations and Spatial Filtering

• Discrete Case

\[ p_r (r_k) = \frac{n_k}{MN} = \frac{n_k}{n}, \quad k = 0, 1, 2, \ldots, L-1 \]

\[ S_k = T(r_k) = (L-1) \sum_{j=0}^{k} p_r (r_j) = \frac{L-1}{MN} \sum_{j=0}^{k} n_k, \quad k = 0, 1, \ldots, L-1 \]

\[ \hat{S}_k^1 = \left\lfloor S_k + 0.5 \right\rfloor = \text{round} (S_k) \]

\[ \hat{S}_k^2 = \left\lfloor \frac{S_k - S_k^{\min}}{L-1 - S_k^{\min}} (L-1) + 0.5 \right\rfloor \]

• Perfect equalization is NOT possible
Digital Image Processing

Intensity Transformations and Spatial Filtering

• Numerical Example:

\[
S_k = 7 \sum_{j=0}^{k} p_r(r_j)
\]

<table>
<thead>
<tr>
<th>( r_k )</th>
<th>( n_k )</th>
<th>( p_r(r_k) = n_k/MN )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( r_0 = 0 )</td>
<td>790</td>
<td>0.19</td>
</tr>
<tr>
<td>( r_1 = 1 )</td>
<td>1023</td>
<td>0.25</td>
</tr>
<tr>
<td>( r_2 = 2 )</td>
<td>850</td>
<td>0.21</td>
</tr>
<tr>
<td>( r_3 = 3 )</td>
<td>656</td>
<td>0.16</td>
</tr>
<tr>
<td>( r_4 = 4 )</td>
<td>329</td>
<td>0.08</td>
</tr>
<tr>
<td>( r_5 = 5 )</td>
<td>245</td>
<td>0.06</td>
</tr>
<tr>
<td>( r_6 = 6 )</td>
<td>122</td>
<td>0.03</td>
</tr>
<tr>
<td>( r_7 = 7 )</td>
<td>81</td>
<td>0.02</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>( S_k )</th>
<th>( \hat{S}_k )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.33</td>
<td>1</td>
</tr>
<tr>
<td>3.08</td>
<td>3</td>
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<tr>
<td>4.55</td>
<td>5</td>
</tr>
<tr>
<td>5.67</td>
<td>6</td>
</tr>
<tr>
<td>6.23</td>
<td>6</td>
</tr>
<tr>
<td>6.56</td>
<td>7</td>
</tr>
<tr>
<td>6.86</td>
<td>7</td>
</tr>
<tr>
<td>7.00</td>
<td>7</td>
</tr>
</tbody>
</table>
**FIGURE 3.19** Illustration of histogram equalization of a 3-bit (8 intensity levels) image. (a) Original histogram. (b) Transformation function. (c) Equalized histogram.
Introducing the concept of intensity transformations and spatial filtering in digital image processing.

- **Real Experiment:**

  - Images and corresponding graphs illustrating the effects of different intensity transformations and spatial filtering techniques on digital images.
Gray-Level Transfer Function
Digital Image Processing

Intensity Transformations and Spatial Filtering

- Histogram Matching and Modification:
  - Goal: Specify the shape of the histogram:

\[
p_r(r) \rightarrow p_z(z)
\]

\[
s = T(r) = (L-1) \int_0^r p_r(w) \, dw
\]

\[
\Rightarrow z = G^{-1} \left[ T(r) \right] = G^{-1} [s]
\]

\[
G(z) = (L-1) \int_0^z p_z(t) \, dt
\]

- Example: Pages: 133-136
• Example (Mars image and its histogram):
Digital Image Processing

Intensity Transformations and Spatial Filtering

- Histogram Matching:
  - Washout Gray
Intense Transformations and Spatial Filtering

- Histogram Matching:

Transform using Initial CDF

Transform using Initial Modified CDF
• Local Histogram Enhancement

**FIGURE 3.26** (a) Original image. (b) Result of global histogram equalization. (c) Result of local histogram equalization applied to (a), using a neighborhood of size $3 \times 3$. 
Histogram Statistics For Image Enhancement:

- Use of Global Statistical Measures

\[ \mu_n (r) = \sum_{i=0}^{L-1} (r_i - m)^n p(r_i) \approx \frac{1}{MN} \sum_{x=1}^{M} \sum_{y=1}^{N} [f(x, y) - m]^n \]

\[ m = \sum_{i=0}^{L-1} r_i p(r_i) \approx \frac{1}{MN} \sum_{x=1}^{M} \sum_{y=1}^{N} f(x, y) \]

- Gross adjustments in overall intensity \((m)\) and contrast \((\mu_2)\)
Digital Image Processing

Intensity Transformations and Spatial Filtering

- Histogram Statistics For Image Enhancement:
  - Local mean and local variance:
    \[
    m_{S_{xy}} (x, y) = \sum_{i=0}^{L-1} r_i p_{S_{xy}} (r_i) \approx \frac{1}{|S_{xy}|} \sum_{(s,t) \in S_{xy}} f(s,t)
    \]
    \[
    \sigma_{S_{xy}}^2 (x, y) = \sum_{i=0}^{L-1} \left( r_i - m_{S_{xy}} (x, y) \right)^2 p_{S_{xy}} (r_i) \approx \frac{1}{|S_{xy}|} \sum_{(s,t) \in S_{xy}} \left[ f(s,t) - m_{S_{xy}} (x, y) \right]^2
    \]
  
  \( S_{xy} \) : Neighborhood centered on \((x, y)\)
  
- Local information intensity and contrast (edges)
• A simple enhancement algorithm for SEM image:

\[
g(x, y) = \begin{cases} 
E \cdot f(x, y) & m_S(x, y) \leq k_0 m_G \text{ and } k_1 \sigma_G \leq \sigma_S(x, y) \leq k_2 \sigma_G \\
\sigma(x, y) & \text{O.W}
\end{cases}
\]

\[E = 4.0, \ k_0 = 0.4, \ k_1 = 0.02, \ k_2 = 0.4\]
Graphical Illustration:

- Local Mean
- Local Var
- $E$ or one
- Enhanced Image
  - Global histogram equalization
  - Use of statistical moments

Original                Local Histogram         Local Statistics
**Digital Image Processing**

**Intensity Transformations and Spatial Filtering**

- **Fundamentals of Spatial Filtering:**

\[
g(x, y) = \sum_{s=-a}^{a} \sum_{t=-b}^{b} w(s, t) f(x + s, y + t)
\]
Intensity Transformations and Spatial Filtering

• Spatial Correlation (⋆) and Convolution (★)

\[
w(x, y)★ f(x, y) = \sum_{s=-a}^{a} \sum_{t=-b}^{b} w(s, t)f(x-s, y-t)
\]

\[
w(x, y)⋆ f(x, y) = \sum_{s=-a}^{a} \sum_{t=-b}^{b} w(s, t)f(x+s, y+t)
\]

• Reflection/Rotation in convolution:

\[
\begin{bmatrix}
1 & 2 & 3 \\
4 & 5 & 6 \\
7 & 8 & 9 \\
\end{bmatrix}
\Rightarrow
\begin{bmatrix}
9 & 8 & 7 \\
6 & 5 & 4 \\
3 & 2 & 1 \\
\end{bmatrix}
\]
Digital Image Processing

Intensity Transformations and Spatial Filtering

- **Smoothing Spatial Filters:**
  - Linear Filters (averaging, lowpass)
    - General Formulation
      
      \[ g(x, y) = \sum_{s=-a}^{+a} \sum_{t=-b}^{+b} w(s, t) f(x + s, y + t) \]

      \[ \frac{1}{9} \times \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} \quad \frac{1}{16} \times \begin{bmatrix} 1 & 2 & 1 \\ 2 & 4 & 2 \\ 1 & 2 & 1 \end{bmatrix} \]

      \[ G_\sigma(x, y) = e^{-\frac{x^2 + y^2}{2\sigma^2}} \]
Intensity Transformations and Spatial Filtering

- Square averaging Filter:
  - Denoising/Smoothing
  - Blurring (Equal Value Kernel)
• Blurring Usage:
  
  – Delete unwanted (small) subjects.

Original  Smoothed  Thresholded
Order Statistics Filters:

- Impulsive noise:
  - Mono Level: Salt, Pepper noises
  - Bi Level: Salt-Pepper noises

- Filter
  - Median
  - Max
  - Min
Digital Image Processing

Intensity Transformations and Spatial Filtering

• Example:

Salt-Pepper Noise
3×3 Averaging
3×3 Median
Introducing the Spatial Filtering Techniques

- **Sharpening Spatial Filter**
  - Highlights Intensity Transitions
  - First and Second order Derivatives

\[
\frac{\partial f}{\partial x} \approx \begin{cases} 
  f(x+1, y) - f(x, y) \\
  f(x, y) - f(x-1, y) \\
  0.5 \left( f(x+1, y) - f(x-1, y) \right) 
\end{cases}
\]

\[
\frac{\partial^2 f}{\partial x^2} \approx f(x+1, y) - 2f(x, y) + f(x-1, y)
\]
• First Order Derivative:
  – Zero in flat region
  – Non-zero at start of step/ramp region
  – Non-zero along ramp
• Second Order Derivative:
  - Zero in flat region
  - Non-zero at start/end of step/ramp region
  - Zero along ramp
Digital Image Processing

Intensity Transformations and Spatial Filtering

Comparison:

Scan line
1st derivative
2nd derivative

Intensity transition
Constant intensity
Ramp
Step

Zero crossing

First derivative
Second derivative
1\textsuperscript{st} and 2\textsuperscript{nd} Order Derivative Comparison:

- First Derivative:
  - Thicker Edge;
  - Strong Response for step changes;

- Second Derivative:
  - Strong response for fine details and isolated points;
  - Double response at step changes.
Intensity Transformations and Spatial Filtering

• Laplacian as an isotropic Enhancer:

$$\nabla^2 f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2}$$

• Discrete Implementation:

$$\nabla^2 f = \left[ f(x+1, y) + f(x, y+1) + f(x-1, y) + f(x, y-1) - 4f(x, y) \right]$$

$$\begin{bmatrix}
0 & 1 & 0 \\
1 & -4 & 1 \\
0 & 1 & 0
\end{bmatrix} \quad 90° \text{ isotropic}$$

$$\begin{bmatrix}
1 & 1 & 1 \\
1 & -8 & 1 \\
1 & 1 & 1
\end{bmatrix} \quad 45° \text{ isotropic}$$
• Laplacian Masks

Practically use:

\[
\begin{array}{ccc}
0 & 1 & 0 \\
1 & -4 & 1 \\
0 & 1 & 0 \\
\end{array}
\quad
\begin{array}{ccc}
1 & 1 & 1 \\
1 & -8 & 1 \\
1 & 1 & 1 \\
\end{array}
\quad
\begin{array}{ccc}
0 & -1 & 0 \\
-1 & -1 & -1 \\
-1 & 8 & -1 \\
\end{array}
\quad
\begin{array}{ccc}
0 & -1 & 0 \\
-1 & -1 & -1 \\
-1 & -1 & -1 \\
\end{array}
\]
Intensity Transformations and Spatial Filtering

- **Background Recovering:**

\[
g(x, y) = \begin{cases} 
  f(x, y) - \nabla^2 f(x, y) & -\text{sign} \\
  f(x, y) + \nabla^2 f(x, y) & +\text{sign}
\end{cases}
\]

\[
\begin{bmatrix}
  0 & -1 & 0 \\
  -1 & +5 & -1 \\
  0 & -1 & 0
\end{bmatrix}
\text{ 90° isotropic}
\]

\[
\begin{bmatrix}
  -1 & -1 & -1 \\
  -1 & +9 & -1 \\
  -1 & -1 & -1
\end{bmatrix}
\text{ 45° isotropic}
\]
Intensity Transformations and Spatial Filtering

Example:

Non-Scaled and scaled Laplacian

Sharpened using 90° and 45° degree isotropic Laplacian
Digital Image Processing

Intensity Transformations and Spatial Filtering

- **Unsharp Masking and High-Boost Filtering:**
  \[
  g_{\text{mask}}(x, y) = f(x, y) - \bar{f}(x, y), \quad \bar{f}(x, y) : \text{Blurred image}
  \]
  \[
  g(x, y) = f(x, y) + k \cdot g_{\text{mask}}(x, y)
  \]

- **\( k \geq 0 \)**
  - \( k=1 \): Unsharp Masking
  - \( k>1 \): High Boost

- **Another mask:**
  - Laplacian and any highpass filter
Digital Image Processing

Intensity Transformations and Spatial Filtering

• One Dimensional Illustration

- Original signal
- Blurred signal
- Unsharp mask
- Sharpened signal
Digital Image Processing

Intensity Transformations and Spatial Filtering

- Two Dimensional Example

Blurred, $5 \times 5$, $\sigma = 3$

Unsharp mask

Unsharp masking, $k = 1$

Highboost, $k = 4.5$
Intensity Transformations and Spatial Filtering

- First Derivative - Gradient:

\[ \nabla f = \left[ \begin{array}{cc} G_x & G_y \end{array} \right]^T = \left[ \frac{\partial f}{\partial x} \quad \frac{\partial f}{\partial y} \right]^T \]

\[ |\nabla f| = \left[ \left( \frac{\partial f}{\partial x} \right)^2 + \left( \frac{\partial f}{\partial y} \right)^2 \right]^{1/2} \approx \left| \frac{\partial f}{\partial x} \right| + \left| \frac{\partial f}{\partial y} \right| \]
Digital Image Processing

Intensity Transformations and Spatial Filtering

- Discrete Implementation

Roberts Cross Gradient

\[ G_x = (z_9 - z_5) \quad G_y = (z_8 - z_6) \]

Sobel Gradient

\[ G_x = (z_7 + 2z_8 + z_9) - (z_1 + 2z_2 + z_3) \]
\[ G_y = (z_3 + 2z_6 + z_9) - (z_1 + 2z_4 + z_7) \]
Digital Image Processing

Intensity Transformations and Spatial Filtering

- Example (Sobel Mask):
Intensity Transformations and Spatial Filtering

- **Image Subtraction:**

  \[ g(x, y) = f^+(x, y) - f^-(x, y) \]

  - Physically static object and background
  - Intensity changes in object but not in background
Example:
- Original
- Discard Some bits
- Difference
- Histogram Equalized
• **Image Averaging:**
  
  – Consider an additive noise condition:
  
  \[ g(x,y) = f(x,y) + \eta(x,y) \]
  
  – **Conditions:**
    
    • *Noise, \( \eta(x,y) \):*
      
      – Uncorrelated
      
      – i.i.d
      
      – Zero Mean
    
    • *Subject, \( f(x,y) \):*
      
      – Physical Stationary
      
      – Repeatable Experiments
• **Image Averaging:**

  – For i.i.d RV’s:

  \[
  \bar{\eta} = \frac{1}{N} \sum_{i=1}^{N} \eta_i \Rightarrow E\{\bar{\eta}\} = E\{\eta\}, \quad \text{Var}\{\bar{\eta}\} = \frac{\text{Var}\{\eta\}}{N}
  \]

  \[
  \begin{align*}
  g_i(x, y) &= f(x, y) + \eta_i(x, y) \\
  \bar{g}(x, y) &= \frac{1}{N} \sum_{i=1}^{N} g_i(x, y)
  \end{align*}
  \Rightarrow
  \begin{align*}
  E\{\bar{g}(x, y)\} &= f(x, y) \\
  \sigma^2_{\bar{g}(x,y)} &= \frac{1}{N} \sigma^2_{\eta(x,y)}
  \end{align*}
  \]
Astronomical Application:
- Repeatable Experiments!

Original

Noisy

N=8

N=16

N=64

N=128
• Difference Histogram:

![Difference Histogram](image)

- N=8
- N=16
- N=64
- N=128

Digital Image Processing

Intensity Transformations and Spatial Filtering
• **Combination:**

**Bone Scan**

**Laplacian**

**Original + Laplacian**

**Soble of Original**
Intensity Transformations and Spatial Filtering

- **Combination:**

  - Smoothed Sobel
  - (Orig. + L.)*S.Sobel
  - Orig. + (Orig.+L.)*S.Sobel
  - Apply Power-Law
Digital Image Processing

Intensity Transformations and Spatial Filtering

• Fuzzy Image Enhancement:
  – Brief Introduction to Fuzzy set
  – Fuzzy Intensity Transformation
  – Fuzzy Spatial Filtering
Introduction

- Logic of real world
- "He is young man" is true or false?
- Fuzzy and Crisp sets:
  - Membership function:
Fuzzy Set:

- Definition:

\[ A = \left\{ z, \mu_A(z) \mid z \in Z \right\}, \quad Z : \text{Set of Elements} \]

Empty Set: \( \mu_A(z) = 0 \)

Equality: \( \mu_A(z) = \mu_B(z) \)

Subset: \( \mu_A(z) \leq \mu_B(z) \)

Complement: \( \mu_A(z) = 1 - \mu_A(z) \)

Union (A \( \cup \) B, A OR B): \( \mu_C(z) = \max\{\mu_A(z), \mu_B(z)\} \)

Intersection (A \( \cap \) B, A AND B): \( \mu_C(z) = \min\{\mu_A(z), \mu_B(z)\} \)
Digital Image Processing

Intensity Transformations and Spatial Filtering

- Graphical Illustration

![Graphical Illustration](image)

---

E. Fatemizadeh, Sharif University of Technology, 2012
Digital Image Processing

Intensity Transformations and Spatial Filtering

• Some Membership Functions:
• Using Fuzzy Sets in inferring:
  – Example: Using color to categorize fruits

\[ R_1: \text{If color is } \textit{green} \text{ then the fruit is } \textit{verdant} \]

  \text{OR}

\[ R_2: \text{If color is } \textit{yellow} \text{ then the fruit is } \textit{half-mature} \]

  \text{OR}

\[ R_3: \text{If color is } \textit{red} \text{ then the fruit is } \textit{mature} \]
We should to do:

- Fuzzify antecedent and consequent of each rule
- Evaluate membership function of each rules \((R_1, R_2, R_3)\),
- Rules aggregation,
- Final crisp value (Defuzzification).
• Fuzzification of antecedents and consequents
• *if-then* rule fuzzification:
  
  - Possibility of *consequents* is less than *antecedents*
  
  • Example: “if x is A then y is D”

  ![Image](image-url)  

  - Our Example:

  \[
  \mu_1(z_0, v) = \min\left[\mu_{\text{green}}(z_0), \mu_{\text{verd}}(v)\right]
  \]

  \[
  \mu_2(z_0, v) = \min\left[\mu_{\text{yellow}}(z_0), \mu_{\text{half}}(v)\right]
  \]

  \[
  \mu_3(z_0, v) = \min\left[\mu_{\text{red}}(z_0), \mu_{\text{mat}}(v)\right]
  \]
Rule aggregation

\[ R = R_1 \text{ OR } R_2 \text{ OR } \ldots \text{ OR } R_n \]

rule 1: IF \( x \) IS \( A \) THEN \( n \) IS \( D \):

\[ \mu(x) \]

rule 2: IF \( y \) IS \( B \) THEN \( n \) IS \( E \):

\[ \mu(y) \]

rule 3: IF \( z \) IS \( C \) THEN \( n \) IS \( F \):

\[ \mu(z) \]
• Rule aggregation in our Example

\[ Q = Q_1 \text{ OR } Q_2 \text{ OR } Q_3 \]

\[ \mu(z_0, v) = \max \left[ \mu_1(z_0, v), \mu_2(z_0, v), \mu_3(z_0, v) \right] \]
• Defuzzification:
  – Max Membership
    \[ x^* : \mu(x^*) \geq \mu(x), \forall x \]
  – Center of Gravity
    \[ x^* = \frac{\int x \mu(x) dx}{\int \mu(x) dx}, \quad x^* = \frac{\sum_{i=1}^{N} x_i \mu(x_i)}{\sum_{i=1}^{N} \mu(x_i)} \]
  – ....
Intensity Transformations and Spatial Filtering

- **Complex antecedent:**

  \[ R: \text{IF } (x \text{ is } A_1) \text{ AND } (x \text{ is } A_2) \text{ … AND } (x \text{ is } A_n) \text{ THEN } y \text{ is } B \]

  \[ R: \text{IF } x \text{ is } A \text{ THEN } y \text{ is } B \]

  \[ \mu_A(x) = \min\left[\mu_{A_1}(x), \mu_{A_2}(x), \ldots, \mu_{A_n}(x)\right] \]

  \[ R: \text{IF } (x \text{ is } A_1) \text{ OR } (x \text{ is } A_2) \text{ … OR } (x \text{ is } A_n) \text{ THEN } y \text{ is } B \]

  \[ R: \text{IF } x \text{ is } A \text{ THEN } y \text{ is } B \]

  \[ \mu_A(x) = \max\left[\mu_{A_1}(x), \mu_{A_2}(x), \ldots, \mu_{A_n}(x)\right] \]
• Fuzzy Intensity Transformation:

$R_1$: If a pixel is Dark then make it Darker

$R_2$: If a pixel is Gray then make it Gray

$R_3$: If a pixel is Bright then make it Brighter
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• Results

Original Histogram  Equalization  Fuzzy Enhancement
Introductions: Intensity Transformations and Spatial Filtering

**Analysis:**

![Histograms](image)

**FIGURE 3.55** (a) and (b) Histograms of Figs. 3.54(a) and (b). (c) Input membership functions superimposed on (a). (d) Histogram of Fig. 3.54(c).
Spatial Filtering Using Fuzzy Sets

- Goal: Boundary Extraction
- General Rule:

\[
\text{if "a pixel belongs to a uniform region", then "make it white, else make it black"}
\]

- Uniformity: Intensity difference central pixel and its neighbors
- For a 3x3 mask: \( d_i = z_i - z_5 \)
Filtering Rule:

IF $d_2$ is zero AND $d_6$ is zero THEN $z_5$ is white

IF $d_6$ is zero AND $d_8$ is zero THEN $z_5$ is white

IF $d_8$ is zero AND $d_4$ is zero THEN $z_5$ is white

IF $d_4$ is zero AND $d_2$ is zero THEN $z_5$ is white

ELSE $z_5$ is black
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- Membership Function

![Graph showing membership function](image)
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- Rules Graphical Interpretation

**FIGURE 3.58**
Fuzzy rules for boundary detection.
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Intensity Transformations and Spatial Filtering

- Example:

![Image](image.png)

**FIGURE 3.59** (a) CT scan of a human head. (b) Result of fuzzy spatial filtering using the membership functions in Fig. 3.57 and the rules in Fig. 3.58. (c) Result after intensity scaling. The thin black picture borders in (b) and (c) were added for clarity; they are not part of the data. (Original image courtesy of Dr. David R. Pickens, Vanderbilt University.)
• MATLAB Command:
  – Image Statistics:
    • means2, std2, corr2, imhist, regionprops
  – Image Intensity Adjustment:
    • imadjust, histeq, adapthisteq, imnoise
  – Linear Filter:
    • imfilter, fspecial, conv2, corr2,
  – Nonlinear filter:
    • medfilt2, ordfilt2,