**Image Enhancement:**
- No Explicit definition

**Methods**
- Spatial Domain:
  - Linear
  - Nonlinear
- Frequency Domain:
  - Linear
  - Nonlinear
• Spatial Domain Process

\[ g(x, y) = T(f(x, y)) \]

**FIGURE 3.1**
A \(3 \times 3\) neighborhood about a point \((x, y)\) in an image in the spatial domain. The neighborhood is moved from pixel to pixel in the image to generate an output image.
Digital Image Processing

Intense Transformations and Spatial Filtering

- For $1 \times 1$ neighborhood: $s = T(r)$
  - Contrast Enhancement/Stretching/Point Process
- For $w \times w$ neighborhood:
  - Filtering/Mask/Kernel/Window/Template Processing

\[ s = T(r) \]

**FIGURE 3.2**
Intensity transformation functions.
(a) Contrast-stretching function.
(b) Thresholding function.
Intensity Transformations and Spatial Filtering

- Intensity Transformation Functions:

![Graph showing various intensity transformation functions](image)

**FIGURE 3.3** Some basic intensity transformation functions. All curves were scaled to fit in the range shown.
• Image Negatives:

\[ s = L - 1 - r \]
Log Transform:

\[ s = c \log(1 + r) \]

**FIGURE 3.5**
(a) Fourier spectrum.
(b) Result of applying the log transformation in Eq. (3.2-2) with \( c = 1 \).
• **Power-Law Transform:**

\[ s = cr^\gamma \]

**FIGURE 3.6** Plots of the equation \( s = cr^\gamma \) for various values of \( \gamma \) (\( c = 1 \) in all cases). All curves were scaled to fit in the range shown.
• **Gamma Correction:**

\[ r^{1.8 \ldots 2.5} \]

\[ r_{(1.8 \ldots 2.5)^{-1}} \]

**Figure 3.7**
(a) Intensity ramp image. (b) Image as viewed on a simulated monitor with a gamma of 2.5. (c) Gamma-corrected image. (d) Corrected image as viewed on the same monitor. Compare (d) and (a).
• Gamma Correction
  – Too Dark Image
• Gamma Correction
  – Too Bright Image

Original

\( \gamma = 3 \)

\( \gamma = 4 \)

\( \gamma = 5 \)
Digital Image Processing

Intensity Transformations and Spatial Filtering

- Contrast Stretching

![Graph showing contrast stretching transformation](image)

Original

C. S.

THR.
• Gray Level Slicing

Digital Image Processing

Intensity Transformations and Spatial Filtering
### Digital Image Processing

**Intensity Transformations and Spatial Filtering**

- **Example**
  - Using Fig 3.11 (a)
  - Using Fig 3.11 (b)

**FIGURE 3.12** (a) Aortic angiogram. (b) Result of using a slicing transformation of the type illustrated in Fig. 3.11(a), with the range of intensities of interest selected in the upper end of the gray scale. (c) Result of using the transformation in Fig. 3.11(b), with the selected area set to black, so that grays in the area of the blood vessels and kidneys were preserved. (Original image courtesy of Dr. Thomas R. Gest, University of Michigan Medical School.)

E. Fatemizadeh, Sharif University of Technology, 2012
Bit-Plane Slicing:
- Highlighting effect of a single bit!
Digital Image Processing

Intensity Transformations and Spatial Filtering

• Example:
  – LSB -> MSB (Left to Right and Top to Bottom)
• Image Reconstruction from bit-planes

Bits (7,8)  

Bits (6,7,8)  

Bits (5,6,7,8)
• Histogram Processing:
  – Enhancement based on statistical Properties:
    • Local
    • Global
  – Histogram Definition:

\[
h(r_k) = n_k, \quad r_k \in [0, L-1], \quad n_k \in [0, M \times N]
\]

\[
p(r_k) = \frac{n_k}{n} = \frac{1}{M \times N} n_k
\]
- Histogram Visual Meaning:
  - Dark
  - Light
  - Low Contrast
  - High Contrast
• Histogram Equalization:
  – Continuous Case.
  – Seek for a suitable transform (Except for negative):

\[ T(r) \]

\[
\begin{align*}
L - 1 & \quad \text{Single value, } s_k \\
T(r) & \\
L - 1 & \quad \text{Single value, } s_q \\
0 & \quad \text{Multiple values}
\end{align*}
\]
• Effect of Point Process on Histogram:

\[
\begin{cases}
  s = T(r) \\
  r = T^{-1}(s)
\end{cases}
\Rightarrow P_s(s) = P_r(r) \left| \frac{dr}{ds} \right|
\]

• Special Case (CDF):

\[
s = T(r) = (L-1) \int_0^r p_r(w)dw \Rightarrow \frac{ds}{dr} = \frac{dT(r)}{dr} = (L-1)p_r(r)
\]

\[
\Rightarrow p_s(s) = p_r(r) \left| \frac{dr}{ds} \right| = p_r(r) \left| \frac{1}{p_r(r)} \right| = \frac{1}{L-1}, \; 0 \leq s \leq L-1
\]

\[
\therefore \text{Uniform Distribution on } [0, L-1]
\]
**Figure 3.18** (a) An arbitrary PDF. (b) Result of applying the transformation in Eq. (3.3-4) to all intensity levels, r. The resulting intensities, s, have a uniform PDF, independently of the form of the PDF of the r’s.
Intensity Transformations and Spatial Filtering

• Discrete Case

\[ p_r(r_k) = \frac{n_k}{MN} = \frac{n_k}{n}, \quad k = 0, 1, 2, \ldots, L-1 \]

\[ S_k = T(r_k) = (L-1) \sum_{j=0}^{k} p_r(r_j) = \frac{L-1}{MN} \sum_{j=0}^{k} n_k, \quad k = 0, 1, \ldots, L-1 \]

\[ \hat{S}_k^1 = \lceil S_k + 0.5 \rceil = \text{round}(S_k) \]

\[ \hat{S}_k^2 = \left\lfloor \frac{S_k - S_k^{\text{min}}}{L-1-S_k^{\text{min}}} (L-1) + 0.5 \right\rfloor \]

• Perfect equalization is NOT possible
### Numerical Example:

\[
S_k = 7 \sum_{j=0}^{k} p_r(r_j)
\]

<table>
<thead>
<tr>
<th>(r_k)</th>
<th>(n_k)</th>
<th>(p_r(r_k) = n_k/MN)</th>
</tr>
</thead>
<tbody>
<tr>
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<td>790</td>
<td>0.19</td>
</tr>
<tr>
<td>1</td>
<td>1023</td>
<td>0.25</td>
</tr>
<tr>
<td>2</td>
<td>850</td>
<td>0.21</td>
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<td>656</td>
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<td>0.08</td>
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<td>245</td>
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<td>0.03</td>
</tr>
<tr>
<td>7</td>
<td>81</td>
<td>0.02</td>
</tr>
</tbody>
</table>

<table>
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<th>(S_k)</th>
<th>(\hat{S}_k)</th>
</tr>
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<tbody>
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<td>6.56</td>
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<tr>
<td>6.86</td>
<td>7</td>
</tr>
<tr>
<td>7.00</td>
<td>7</td>
</tr>
</tbody>
</table>
Numerical Examples (Cont.)

**FIGURE 3.19** Illustration of histogram equalization of a 3-bit (8 intensity levels) image. (a) Original histogram. (b) Transformation function. (c) Equalized histogram.
• Real Experiment:
Intensity Transformations and Spatial Filtering

- Gray-Level Transfer Function

![Graph of Gray-Level Transfer Function]
• Histogram Matching and Modification:
  – Goal: Specify the shape of the histogram:

\[
p_{r}(r) \xrightarrow{?} p_{z}(z)
\]

\[
s = T(r) = (L-1) \int_{0}^{r} p_{r}(w) dw
\]

\[
G(z) = (L-1) \int_{0}^{z} p_{z}(t) dt
\]

\[
\Rightarrow z = G^{-1}[T(r)] = G^{-1}[s]
\]

– Example: Pages: 133-136
Intensity Transformations and Spatial Filtering

- Example (Mars image and its histogram):
Histogram Matching:
- Washout Gray
• Histogram Matching:
Local Histogram Enhancement

**FIGURE 3.26** (a) Original image. (b) Result of global histogram equalization. (c) Result of local histogram equalization applied to (a), using a neighborhood of size $3 \times 3$. 
Digital Image Processing

Intensity Transformations and Spatial Filtering

• Histogram Statistics For Image Enhancement:
  – Use of Global Statistical Measures

\[
\mu_n(r) = \sum_{i=0}^{L-1} (r_i - m)^n \quad p(r_i) \approx \frac{1}{MN} \sum_{x=1}^{M} \sum_{y=1}^{N} [f(x, y) - m]^n
\]

\[
m = \sum_{i=0}^{L-1} r_i p(r_i) \approx \frac{1}{MN} \sum_{x=1}^{M} \sum_{y=1}^{N} f(x, y)
\]

– Gross adjustments in overall intensity \(m\) and contrast \(\mu_2\)
Digital Image Processing

Intercity Transformations and Spatial Filtering

- Histogram Statistics For Image Enhancement:
  - Local mean and local variance:

\[
m_{S_{xy}}(x, y) = \sum_{i=0}^{L-1} r_i p_{S_{xy}}(r_i) \approx \frac{1}{|S_{xy}|} \sum_{(s,t) \in S_{xy}} f(s,t)
\]

\[
\sigma^2_{S_{xy}}(x, y) = \sum_{i=0}^{L-1} \left( r_i - m_{S_{xy}}(x, y) \right)^2 p_{S_{xy}}(r_i) \approx \frac{1}{|S_{xy}|} \sum_{(s,t) \in S_{xy}} \left[ f(s,t) - m_{S_{xy}}(x, y) \right]^2
\]

\(S_{xy}\) : Neighborhood centered on \((x, y)\)

- Local information intensity and contrast (edges)
• A simple enhancement algorithm for SEM image:

\[
g(x, y) = \begin{cases} 
E \cdot f(x, y) & \text{if } m_S(x, y) \leq k_0 m_G \text{ and } k_1 \sigma_G \leq \sigma_S(x, y) \leq k_2 \sigma_G \\
 f(x, y) & \text{otherwise (O.W)}
\end{cases}
\]

\[E = 4.0, \ k_0 = 0.4, \ k_1 = 0.02, \ k_2 = 0.4\]
Digital Image Processing

Intensity Transformations and Spatial Filtering

• Graphical Illustration:

Local Mean  Local Var  E or one

[Images of graphical illustrations showing local mean, local variance, and E or one]
Enhanced Image

- Global histogram equalization
- Use of statistical moments

Original                  Local Histogram                  Local Statistics

Digital Image Processing

Intensity Transformations and Spatial Filtering
Digital Image Processing

Intensity Transformations and Spatial Filtering

- Fundamentals of Spatial Filtering:

\[ g(x, y) = \sum_{s=-a}^{a} \sum_{t=-b}^{b} w(s, t) f(x + s, y + t) \]
Spatial Correlation (⋆) and Convolution (★)

\[ w(x, y) \star f(x, y) = \sum_{s=-a}^{a} \sum_{t=-b}^{b} w(s, t) f(x-s, y-t) \]

\[ w(x, y) \bigodot f(x, y) = \sum_{s=-a}^{a} \sum_{t=-b}^{b} w(s, t) f(x+s, y+t) \]

Reflection/Rotation in convolution:

\[
\begin{bmatrix}
1 & 2 & 3 \\
4 & 5 & 6 \\
7 & 8 & 9
\end{bmatrix}
\Rightarrow
\begin{bmatrix}
9 & 8 & 7 \\
6 & 5 & 4 \\
3 & 2 & 1
\end{bmatrix}
\]
Smoothing Spatial Filters:

- Linear Filters (averaging, lowpass)

\[
G_\sigma (x, y) = e^{-\frac{x^2 + y^2}{2\sigma^2}}
\]

- General Formulation

\[
g(x, y) = \frac{\sum_{s=-a}^{+a} \sum_{t=-b}^{+b} w(s, t) f(x + s, y + t)}{\sum_{s=-a}^{+a} \sum_{t=-b}^{+b} w(s, t)}
\]
Intensity Transformations and Spatial Filtering

- Square averaging Filter:
  - Denoising/Smoothing
  - Blurring (Equal Value Kernel)
• Blurring Usage:
  – Delete unwanted (small) subjects.

Original                    Smoothed                    Thresholded
Order Statistics Filters:

- Impulsive noise:
  - Mono Level: Salt, Pepper noises
  - Bi Level: Salt-Pepper noises

- Filter
  - Median
  - Max
  - Min
Example:

Salt-Pepper Noise  3×3 Averaging  3×3 Median
Intensity Transformations and Spatial Filtering

- Sharpening Spatial Filter
  - Highlights Intensity Transitions
  - First and Second order Derivatives

\[
\frac{\partial f}{\partial x} \approx \begin{cases} 
  f(x+1, y) - f(x, y) \\
  f(x, y) - f(x-1, y) \\
  0.5(f(x+1, y) - f(x-1, y)) 
\end{cases}
\]

\[
\frac{\partial^2 f}{\partial x^2} \approx f(x+1, y) - 2f(x, y) + f(x-1, y)
\]
Intensity Transformations and Spatial Filtering

- **First Order Derivative:**
  - Zero in flat region
  - Non-zero at start of step/ramp region
  - Non-zero along ramp
Second Order Derivative:
- Zero in flat region
- Non-zero at start/end of step/ramp region
- Zero along ramp
• Comparison:

![Graph showing intensity transformations and spatial filtering](image)

- **Intensity transition**: Constant intensity, Ramp, Step.
- **Scan line**: Show the intensity values and derivatives for different intensity transitions.
- **Derivatives**:
  - 1st derivative: 0 0 -1 -1 -1 -1 0 0 0 0 5 0 0 0 0
  - 2nd derivative: 0 0 -1 0 0 0 0 1 0 0 0 0 5 -5 0 0 0

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1st and 2nd Order Derivative Comparison:

- First Derivative:
  - Thicker Edge;
  - Strong Response for step changes;

- Second Derivative:
  - Strong response for fine details and isolated points;
  - Double response at step changes.
Digital Image Processing

Intensity Transformations and Spatial Filtering

• Laplacian as an isotropic Enhancer:

\[ \nabla^2 f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} \]

• Discrete Implementation:

\[ \nabla^2 f = \left[ f(x+1, y) + f(x, y+1) + f(x-1, y) + f(x, y-1) - 4f(x, y) \right] \]

\[
\begin{bmatrix}
0 & 1 & 0 \\
1 & -4 & 1 \\
0 & 1 & 0
\end{bmatrix}
\] 90° isotropic

\[
\begin{bmatrix}
1 & 1 & 1 \\
1 & -8 & 1 \\
1 & 1 & 1
\end{bmatrix}
\] 45° isotropic
Laplacian Masks

Practically use:

\[
\begin{array}{|ccc|}
\hline
0 & 1 & 0 \\
\hline
1 & -4 & 1 \\
\hline
0 & 1 & 0 \\
\hline
\end{array}
\]

\[
\begin{array}{|ccc|}
\hline
1 & 1 & 1 \\
\hline
1 & -8 & 1 \\
\hline
1 & 1 & 1 \\
\hline
\end{array}
\]

\[
\begin{array}{|ccc|}
\hline
0 & -1 & 0 \\
\hline
-1 & 4 & -1 \\
\hline
0 & -1 & 0 \\
\hline
\end{array}
\]

\[
\begin{array}{|ccc|}
\hline
-1 & -1 & -1 \\
\hline
-1 & 8 & -1 \\
\hline
-1 & -1 & -1 \\
\hline
\end{array}
\]
Intensity Transformations and Spatial Filtering

- Background Recovering:

\[ g(x, y) = \begin{cases} 
  f(x, y) - \nabla^2 f(x, y) & -\text{sign} \\
  f(x, y) + \nabla^2 f(x, y) & +\text{sign}
\end{cases} \]

\[
\begin{bmatrix}
  0 & -1 & 0 \\
  -1 & 5 & -1 \\
  0 & -1 & 0
\end{bmatrix} \quad 90^\circ \text{ isotropic} \quad \begin{bmatrix}
  -1 & -1 & -1 \\
  -1 & 9 & -1 \\
  -1 & -1 & -1
\end{bmatrix} \quad 45^\circ \text{ isotropic}
\]
Example:

Non-Scaled and scaled Laplacian

Sharpened using 90° and 45° degree isotropic Laplacian
Intensity Transformations and Spatial Filtering

- **Unsharp Masking and High-Boost Filtering:**
  \[ g_{mask}(x, y) = f(x, y) - \bar{f}(x, y), \quad \bar{f}(x, y) : \text{Blurred image} \]
  \[ g(x, y) = f(x, y) + k \times g_{mask}(x, y) \]

- **k ≥ 0**
  - k=1: Unsharp Masking
  - k>1: High Boost

- **Another mask:**
  - Laplacian and any highpass filter
Intensity Transformations and Spatial Filtering

- One Dimensional Illustration
• Two Dimensional Example

Blurred, 5×5, σ=3

Unsharp mask

Unsharp masking, k=1

Highboost, k=4.5
**First Derivative - Gradient:**

\[
\nabla f = \begin{bmatrix} G_x & G_y \end{bmatrix}^T = \begin{bmatrix} \frac{\partial f}{\partial x} & \frac{\partial f}{\partial y} \end{bmatrix}^T
\]

\[
|\nabla f| = \left[ \left( \frac{\partial f}{\partial x} \right)^2 + \left( \frac{\partial f}{\partial y} \right)^2 \right]^{1/2} \approx \left| \frac{\partial f}{\partial x} \right| + \left| \frac{\partial f}{\partial y} \right|
\]
Digital Image Processing

Intensity Transformations and Spatial Filtering

- Discrete Implementation

Roberts Cross Gradient

\[
G_x = (z_9 - z_5) \quad G_y = (z_8 - z_6)
\]

Sobel Gradient

\[
G_x = (z_7 + 2z_8 + z_9) - (z_1 + 2z_2 + z_3)
\]

\[
G_y = (z_3 + 2z_6 + z_9) - (z_1 + 2z_4 + z_7)
\]
Example (Sobel Mask):
Digital Image Processing

Intensity Transformations and Spatial Filtering

- **Image Subtraction:**
  \[ g(x, y) = f^+(x, y) - f^-(x, y) \]
  - Physically static object and background
  - Intensity changes in object but not in background
Example:
- Original
- Discard Some bits
- Difference
- Histogram Equalized
• **Image Averaging:**
  
  – Consider an additive noise condition:
  
  \[ g(x,y) = f(x,y) + \eta(x,y) \]
  
  – **Conditions:**
    
    • *Noise, \( \eta(x,y) \):*
      
      – Uncorrelated
      – i.i.d
      – Zero Mean
    
    • *Subject, \( f(x,y) \):*
      
      – Physical Stationary
      – Repeatable Experiments
Intensity Transformations and Spatial Filtering

- **Image Averaging:**
  - For i.i.d RV’s:

\[
\bar{\eta} = \frac{1}{N} \sum_{i=1}^{N} \eta_i \Rightarrow E\{\bar{\eta}\} = E\{\eta\}, \quad \text{Var}\{\bar{\eta}\} = \frac{\text{Var}\{\eta\}}{N}
\]

\[
\begin{align*}
\{ g_i(x, y) &= f(x, y) + \eta_i(x, y) \\
\bar{g}(x, y) &= \frac{1}{N} \sum_{i=1}^{N} g_i(x, y) \Rightarrow \{ E\{\bar{g}(x, y)\} &= f(x, y) \\
& \quad \sigma_{\bar{g}(x,y)}^2 = \frac{1}{N} \sigma_{\eta(x,y)}^2 \}
\end{align*}
\]
Digital Image Processing

Intensity Transformations and Spatial Filtering

- Astronomical Application:
  - Repeatable Experiments!

![Image with original and noisy images with different N values](image_url)
Digital Image Processing

Intensity Transformations and Spatial Filtering

• Difference Histogram:

\[ N = 8 \]

\[ N = 16 \]

\[ N = 64 \]

\[ N = 128 \]
Intensity Transformations and Spatial Filtering

- Combination:

  - Bone Scan
  - Laplacian
  - Original + Laplacian
  - Sobel of Original
Intensity Transformations and Spatial Filtering

- Combination:
  - Smoothed Sobel
  - $(\text{Orig.} + \text{L.}) \ast \text{S.Sobel}$
  - $(\text{Orig.} + (\text{Orig.} + \text{L.}) \ast \text{S.Sobel})$
  - Apply Power-Law
Fuzzy Image Enhancement:
- Brief Introduction to Fuzzy set
- Fuzzy Intensity Transformation
- Fuzzy Spatial Filtering
• Introduction
  - Logic of real world
  - “He is young man” is true or false?
  - Fuzzy and Crisp sets:
    • Membership function:
Intensity Transformations and Spatial Filtering

- Fuzzy Set:
  - Definition:
    \[ A = \{ z, \mu_A(z) \mid z \in Z \}, \quad Z : \text{Set of Elements} \]
    
    Empty Set: \( \mu_A(z) = 0 \)
    
    Equality: \( \mu_A(z) = \mu_B(z) \)
    
    Subset: \( \mu_A(z) \leq \mu_B(z) \)
    
    Complement: \( \mu_{\overline{A}}(z) = 1 - \mu_A(z) \)
    
    Union (A \( \cup \) B, A OR B): \( \mu_C(z) = \max \{ \mu_A(z), \mu_B(z) \} \)
    
    Intersection (A \( \cap \) B, A AND B): \( \mu_C(z) = \min \{ \mu_A(z), \mu_B(z) \} \)
Digital Image Processing

Intensity Transformations and Spatial Filtering

- Graphical Illustration

\[ \mu_A(z) \]
\[ \mu_B(z) \]
\[ \mu_U(z) = \max[\mu_A(z), \mu_B(z)] \]
\[ \mu_I(z) = \min[\mu_A(z), \mu_B(z)] \]
\[ \mu_{\bar{A}}(z) = 1 - \mu_A(z) \]

Complement

Union

Intersection
Some Membership Functions:

- Triangular
- Trapezoidal
- Sigma
- S-shape
- Bell-shape
- Truncated Gaussian

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• Using Fuzzy Sets in inferring:
  – Example: Using color to categorize fruits

\[ R_1: \text{If color is } green \text{ then the fruit is } \text{verdant} \]
\[ \text{OR} \]
\[ R_2: \text{If color is } yellow \text{ then the fruit is } \text{half-mature} \]
\[ \text{OR} \]
\[ R_3: \text{If color is } red \text{ then the fruit is } \text{mature} \]
We should to do:

- Fuzzify *antecedent* and *consequent* of each rule
- Evaluate membership function of each rules ($R_1, R_2, R_3$),
- Rules aggregation,
- Final crisp value (Defuzzification).
• Fuzzification of antecedents and consequents
• *if-then* rule fuzzification:
  – Possibility of *consequents* is less than *antecedents*
  • Example: “if x is A then y is D”

- Our Example:

\[
\begin{align*}
\mu_1(z_0, v) &= \min \left[ \mu_{\text{green}}(z_0), \mu_{\text{verd}}(v) \right] \\
\mu_2(z_0, v) &= \min \left[ \mu_{\text{yellow}}(z_0), \mu_{\text{half}}(v) \right] \\
\mu_3(z_0, v) &= \min \left[ \mu_{\text{red}}(z_0), \mu_{\text{mat}}(v) \right]
\end{align*}
\]
• Rule aggregation

\[ R = R_1 \text{ OR } R_2 \text{ OR } ... \text{ OR } R_n \]

rule 1: IF \( x \) IS \( A \) THEN \( n \) IS \( D \):

rule 2: IF \( y \) IS \( B \) THEN \( n \) IS \( E \):

rule 3: IF \( z \) IS \( C \) THEN \( n \) IS \( F \):
• Rule aggregation in our Example

\[ Q = Q_1 \text{ OR } Q_2 \text{ OR } Q_3 \]

\[ \mu(z_0, v) = \max\left[ \mu_1(z_0, v), \mu_2(z_0, v), \mu_3(z_0, v) \right] \]
Defuzzification:

- Max Membership

\[ x^* : \mu(x^*) \geq \mu(x), \forall x \]

- Center of Gravity

\[ x^* = \frac{\int x \mu(x) \, dx}{\int \mu(x) \, dx}, \quad x^* = \frac{\sum_{i=1}^{N} x_i \mu(x_i)}{\sum_{i=1}^{N} \mu(x_i)} \]

- ....
• Complex antecedent:

$$R: \text{IF} \ (x \text{ is } A_1) \ \text{AND} \ (x \text{ is } A_2) \ \ldots \ \text{AND} \ (x \text{ is } A_n) \ \text{THEN} \ y \text{ is } B$$

$$R: \text{IF} \ x \text{ is } A \ \text{THEN} \ y \text{ is } B$$

$$\mu_A(x) = \min\left[ \mu_{A_1}(x), \mu_{A_2}(x) \ldots, \mu_{A_n}(x) \right]$$

$$R: \text{IF} \ (x \text{ is } A_1) \ \text{OR} \ (x \text{ is } A_2) \ \ldots \ \text{OR} \ (x \text{ is } A_n) \ \text{THEN} \ y \text{ is } B$$

$$R: \text{IF} \ x \text{ is } A \ \text{THEN} \ y \text{ is } B$$

$$\mu_A(x) = \max\left[ \mu_{A_1}(x), \mu_{A_2}(x) \ldots, \mu_{A_n}(x) \right]$$
Digital Image Processing

Intensity Transformations and Spatial Filtering

- Fuzzy Intensity Transformation:
  \( R_1 \): If a pixel is Dark then make it Darker
  \( R_2 \): If a pixel is Gray then make it Gray
  \( R_3 \): If a pixel is Bright then make it Brighter
Digital Image Processing

Intensity Transformations and Spatial Filtering

• Results

- Original Histogram
- Equalization
- Fuzzy Enhancement
Intensity Transformations and Spatial Filtering

- Analysis:

**FIGURE 3.55** (a) and (b) Histograms of Figs. 3.54(a) and (b). (c) Input membership functions superimposed on (a). (d) Histogram of Fig. 3.54(c).
Spatial Filtering Using Fuzzy Sets

- Goal: Boundary Extraction
- General Rule:

\[
\text{if } \text{"a pixel belongs to a uniform region"}, \text{ then } \text{"make it white, else make it black"}
\]

- Uniformity: Intensity difference central pixel and its neighbors
- For a 3×3 maks: \( d_i = z_i - z_5 \)
Digital Image Processing

Intensity Transformations and Spatial Filtering

- Filtering Rule:

\[
\text{IF } d_2 \text{ is zero AND } d_6 \text{ is zero THEN } z_5 \text{ is white}
\]

\[
\text{IF } d_6 \text{ is zero AND } d_8 \text{ is zero THEN } z_5 \text{ is white}
\]

\[
\text{IF } d_8 \text{ is zero AND } d_4 \text{ is zero THEN } z_5 \text{ is white}
\]

\[
\text{IF } d_4 \text{ is zero AND } d_2 \text{ is zero THEN } z_5 \text{ is white}
\]

ELSE \(z_5\) is black

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Digital Image Processing

Intensity Transformations and Spatial Filtering

- Membership Function

![Graphs showing membership functions](image-url)
Intensity Transformations and Spatial Filtering

- Rules Graphical Interpretation

**FIGURE 3.58**
Fuzzy rules for boundary detection.
Example:

**FIGURE 3.59** (a) CT scan of a human head. (b) Result of fuzzy spatial filtering using the membership functions in Fig. 3.57 and the rules in Fig. 3.58. (c) Result after intensity scaling. The thin black picture borders in (b) and (c) were added for clarity; they are not part of the data. (Original image courtesy of Dr. David R. Pickens, Vanderbilt University.)
Intensity Transformations and Spatial Filtering

• MATLAB Command:
  – Image Statistics:
    • means2, std2, corr2, imhist, regionprops
  – Image Intensity Adjustment:
    • imadjust, histeq, adapthisteq, imnoise
  – Linear Filter:
    • imfilter, fspecial, conv2, corr2,
  – Nonlinear filter:
    • medfilt2, ordfilt2,