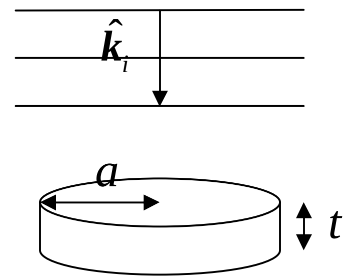
EM Scattering

Final Examination, 30.10.1388

Name: Student nr.:

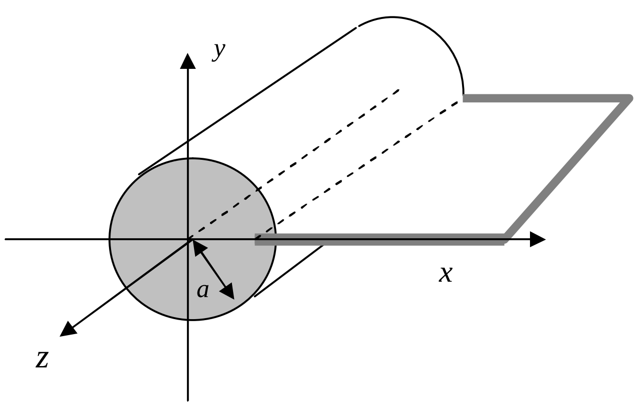
Problem 1:

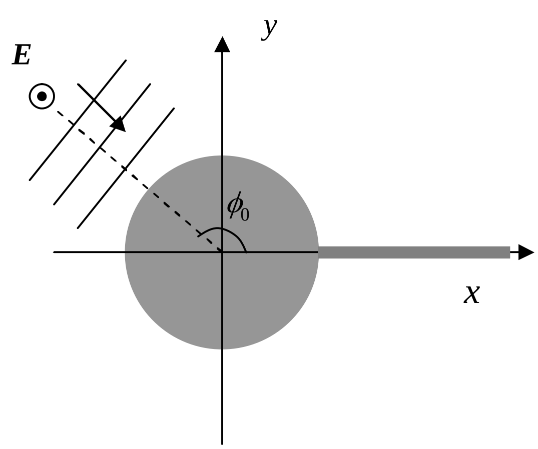
Using the Born approximation, find the forward and backward scattering amplitudes of a dielectric disk (thickness t, radius a, relative dielectric constant ε_r) when the incident wave propagates in a direction perpendicular to the disk. The background medium is vacuum.



Problem 2:

Consider the structure shown below which consists of a half-infinite perfectly conducting plane terminated by an infinitely long, perfectly conducting cylinder of radius a whose axis coincides with the z-axis. An incident TM wave with the electric field vector in the z-direction and a wave vector parallel to the x-y plane propagates along a line which makes an angle ϕ_0 with the x-axis (see figure) and is scattered by structure. Find the total electric field in this 2D scattering problem. The background medium is vacuum. Hint: First introduce an infinitely long line of carrying a constant electric current (line source) at a finite distance from the structure, and then let the line source move to infinity along the propagation line.

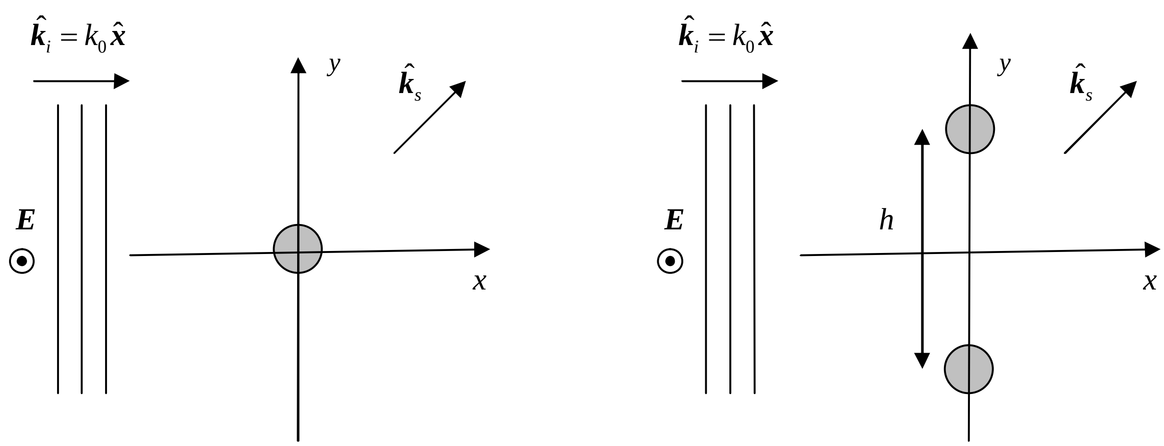




Problem 3:

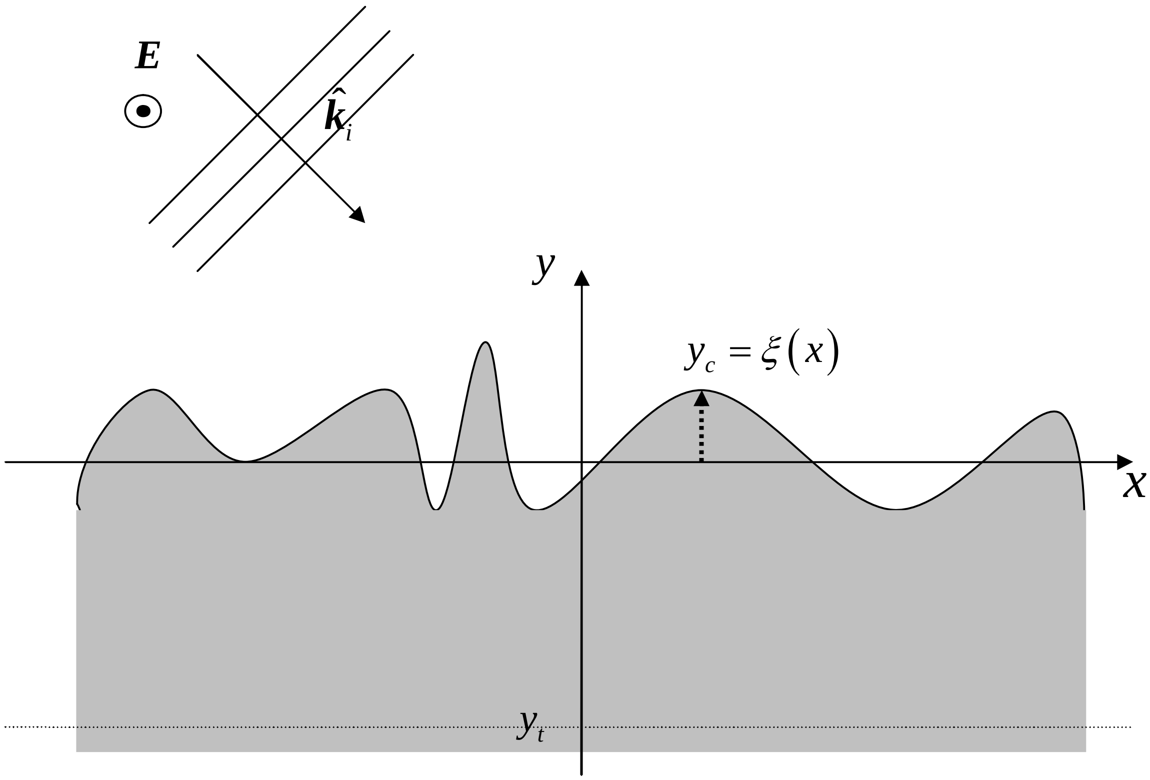
An incident plane wave is propagating in vacuum along the +x-axis and with a polarization along the z-axis (in the electric sense). The wave is scattered by a small dielectric sphere with the radius a and relative dielectric constant ε_r. The center of the sphere coincides with the origin of the coordinate system.

1. Find the far-zone scattered field along an arbitrary direction $\hat{\boldsymbol{k}}_s$ by using the Rayleigh approximation (small-particle, electrostatic approximation where the electric field is assumed constant inside the object and depolarization factors are used)
2. Next, consider two such spheres placed on the z-axis at $z = +h/2$ and $z = -h/2$. Assume that the distance h between the center of the two spheres is large enough (compared to wavelength) so that each sphere is in the far zone field of the other. Use the Rayleigh approximation again to find the (constant) electric field inside each sphere and solve the two-sphere scattering problem.



Problem 4:

Consider an infinitely large conductor whose surface is uniform along the z-direction, but is irregular along the x-direction. The surface of the conductor is thus defined by the function $y_c = \xi(x),\ -\infty < x < \infty$ as shown in the figure below. An incident TM wave with the electric field vector in the z-direction, and a wave vector parallel to the x-y plane propagates along $\hat{\boldsymbol{k}}_i$ and is scattered by structure.



1. Write down an integral equation for the surface current on the surface of the conductor in terms of the variable x.

2. The standard way of deriving surface integral equations (as in the previous question) is based on using the extinction theorem and letting the observation point approach the conductor surface from within the conductor. However, it is possible to derive an alternative integral equation by choosing the observation point on a flat surface well inside the conductor ($y = y_t$). Find this integral equation.

3. Assume that y_t is chosen such that $|\xi(x)| \ll |y_t|$. Can you sketch how perturbation theory can be used to find an approximate solution for the surface current?