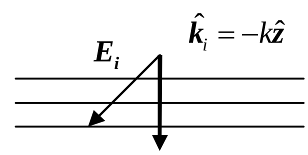
EM Scattering

Final Examination, 28.10.1389

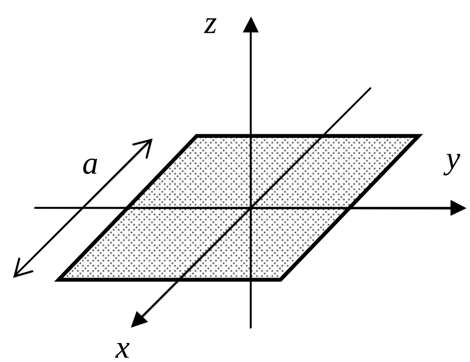
Name: Student nr.:

Problem 1:



A plane wave which is propagating in vacuum along the -z-axis is normally incident on a perfectly conducting, square-shaped, thin metallic plate. The plate has a side length of a and lies on the x-y plane. The electric field of the incident wave is linearly polarized along the x-axis and has an amplitude of E_0 (see figure).

Find the scattered, far-zone electric field along an arbitrary direction $\hat{\boldsymbol{k}}_s = (\sin\theta_s \cos\phi_s, \sin\theta_s \sin\phi_s, \cos\theta_s)$ by using the approximation of physical optics.



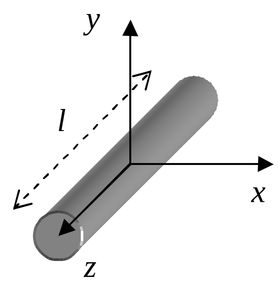
Problem 2:

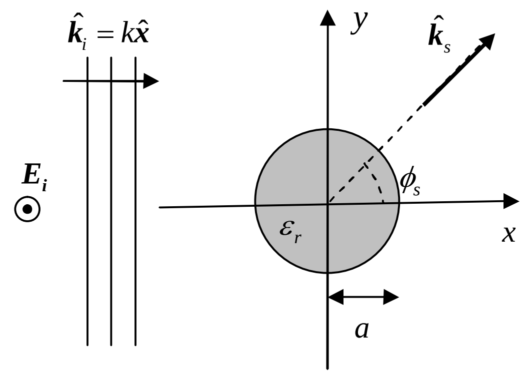
An incident plane wave which is propagating in vacuum along the +x-axis has an electric field linearly polarized along the z-axis with an amplitude of E_0. The wave is scattered by a small dielectric cylinder along the z-axis with the relative dielectric constant ε_r, length l, and radius a. Both l and a are small compared to the wavelength, i.e., $kl << 1,\ ka << 1$ where k is the wave number in vacuum. Besides, we have $l >> a$. We would like to calculate the scattered, far-zone electric field along an arbitrary direction vector $\hat{\boldsymbol{k}}_s$ on the x-y plane with $\hat{\boldsymbol{k}}_s = (\cos\phi_s, \sin\phi_s, 0)$.

1. Find the scattered electric field using the Born approximation.

2. Find the scattered field using the small particle approximation using depolarization factors.

(In both cases, approximate the integrals by using the fact that the object is small compared to the wavelength.)

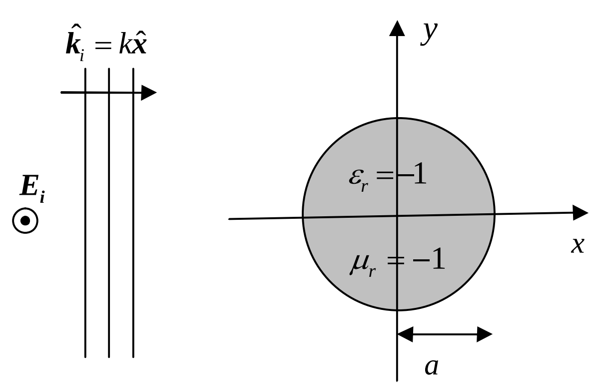




Problem 3:

Consider an infinitely long circular cylinder whose axis coincides with the z-axis. A plane wave with the wave vector $\boldsymbol{k}_i = k\hat{\boldsymbol{x}}$ is normally incident on the cylinder where $k = \omega\sqrt{\varepsilon_0 \mu_0}$ with ε_0, μ_0 respectively denoting the permittivity and permeability of the background medium. The electric field of the incident wave is polarized along the z-direction and has an amplitude of E_0. The relative dielectric constant and permeability of the cylinder are given by $\varepsilon_r = -1$ and $\mu_r = -1$. The radius of the cylinder is a.

Compute the scattered field using the cylindrical coordinates and the expansion of a (scalar) plane wave in Bessel functions: $\exp(-jkx) = \sum_{m=-\infty}^{\infty} (-j)^m J_m(k\rho)\exp(-jm\phi)$. You do not have to calculate the far-zone scattered field, just the exact result.



Problem 4:

Consider an infinitely long (along the z-axis), 90° perfectly conducting corner. The plates forming the corner are also infinite in extent. An infinitely-long, dielectric, quarter-cylinder of radius a is placed on the structure as shown in the figure below. The relative dielectric constant of the quarter-cylinder is ε_r.

1. Consider an electric line-source placed parallel to the z-axis and carrying a constant current I_0 with the radial frequency ω. The position of this source on the x-y plane is denoted by (ρ_0, ϕ_0) in cylindrical coordinates. Write down the field equations in each region and solve them to find the electric field.

2. Using the above result, solve the scattering problem (find the total electric field) for an incident plane wave whose electric field is polarized along the z-direction and is propagating towards the structure along a line through the origin which makes an angle ϕ_0 with the x-axis.

If necessary use $J_\nu(z)Y_\nu'(z) - J_\nu'(z)Y_\nu(z) = 2/(\pi z)$ or
$J_\nu(z)H_\nu^{(2)\prime}(z) - J_\nu'(z)H_\nu^{(2)}(z) = 2/(j\pi z)$.