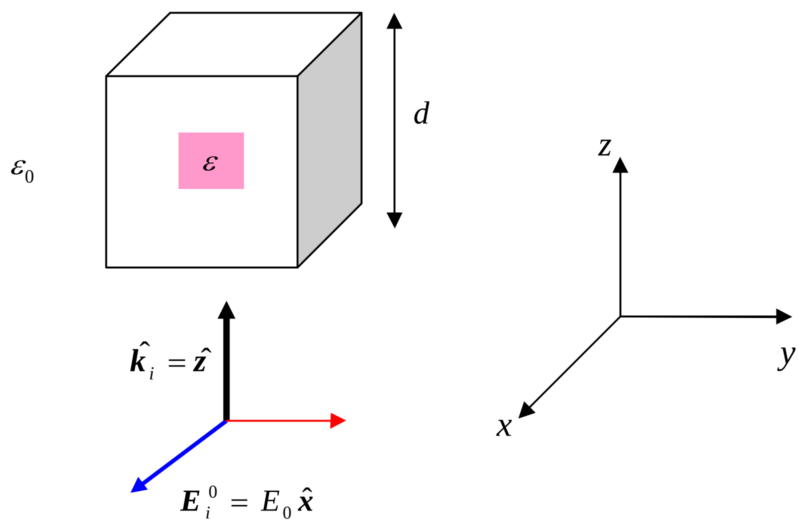
EM Scattering

Homework assignment 1

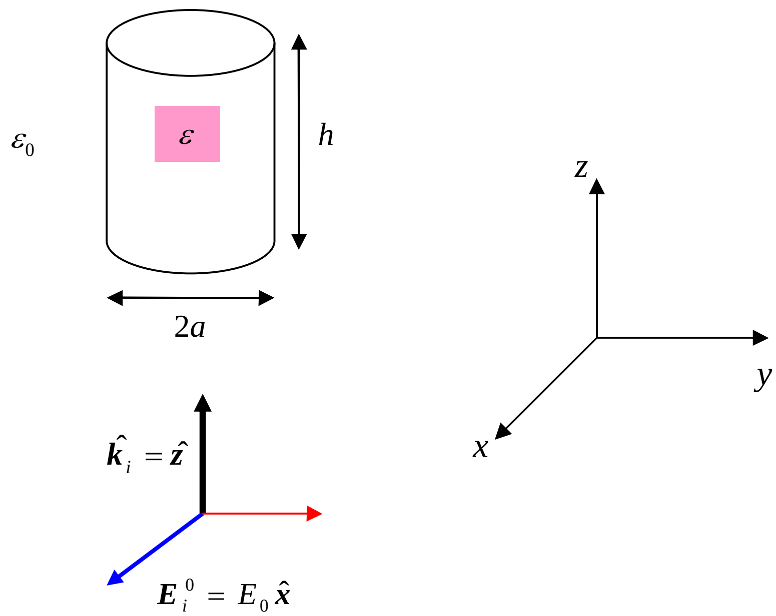
Problem 1:

An incident wave travels in free space (dielectric constant ε_0, permeability μ_0, wavenumber $k = \omega\sqrt{\varepsilon_0\mu_0}$) along the z-direction and impinges upon a cube of dielectric material with a constant permittivity of ε. The faces of the dielectric cube are parallel to the x-y, y-z, z-x planes respectively. The length of each side of the cube is d. The polarization of the incident electric field is along the x-axis. Use the Born approximation to calculate the scattered electric and magnetic fields in the far-field zone along the $\hat{\boldsymbol{k}}_s = \pm\hat{\boldsymbol{z}}$, $\hat{\boldsymbol{k}}_s = \pm\hat{\boldsymbol{x}}$, and $\hat{\boldsymbol{k}}_s = \pm\hat{\boldsymbol{y}}$ directions.



Problem 2:

Repeat this calculation for a dielectric cylinder with the radius a, height h, and dielectric constant ε. Now find the scattered field in any direction $\hat{\boldsymbol{k}}_s$.



Problem 3:

Consider a plane wave traveling in free space with $\boldsymbol{E}^i(\boldsymbol{r}) = \boldsymbol{E}_0^i \exp(-j\boldsymbol{k}_i \cdot \boldsymbol{r})$ where $\boldsymbol{k}_i$ is the incident wave vector with $|\boldsymbol{k}_i| = k = \omega\sqrt{\varepsilon_0 \mu_0}$. The wave is incident upon a certain dielectric object of arbitrary shape with a dielectric constant $\varepsilon(\boldsymbol{r})$ generating equivalent currents inside the object. Let $\boldsymbol{E}_s, \boldsymbol{H}_s$ denote the scattered field, i.e., the field induced by these equivalent currents. In the far field zone the scattered electric field is given by:

$$\boldsymbol{E}_s^f(\boldsymbol{r}) = \frac{\exp(-jkr)}{r} \boldsymbol{Q}_\perp\left(\hat{\boldsymbol{k}}_s, \hat{\boldsymbol{k}}_i\right)$$

Where, $\hat{\boldsymbol{k}}_s$ denotes the direction of scattering and $\boldsymbol{Q}_\perp\left(\hat{\boldsymbol{k}}_s, \hat{\boldsymbol{k}}_i\right)$ is the vector scattering amplitude (see the powerpoint slides for a description). Using the reciprocity theorem of electromagnetic theory show that

$$\boldsymbol{E}_0^i \cdot \boldsymbol{Q}_\perp\left(-\hat{\boldsymbol{k}}_i, \hat{\boldsymbol{k}}_i\right) = \frac{k\eta}{4\pi j}\left\{ \boldsymbol{E}_0^i \cdot \oint_S \left[\hat{\boldsymbol{n}} \times \boldsymbol{E}_s(\boldsymbol{r})\right] \exp(-j\boldsymbol{k}_i \cdot \boldsymbol{r})\, dS - \boldsymbol{H}_0^i \cdot \oint_S \left[\boldsymbol{H}_s(\boldsymbol{r}) \times \hat{\boldsymbol{n}}\right] \exp(-j\boldsymbol{k}_i \cdot \boldsymbol{r})\, dS \right\}$$

Where $\boldsymbol{H}_0^i = (1/\eta)\hat{\boldsymbol{k}}_i \times \boldsymbol{E}_0^i$, $\eta = \sqrt{\mu_0 / \varepsilon_0}$, and S is any arbitrary closed surface surrounding the object. (See figure below)