EM Scattering

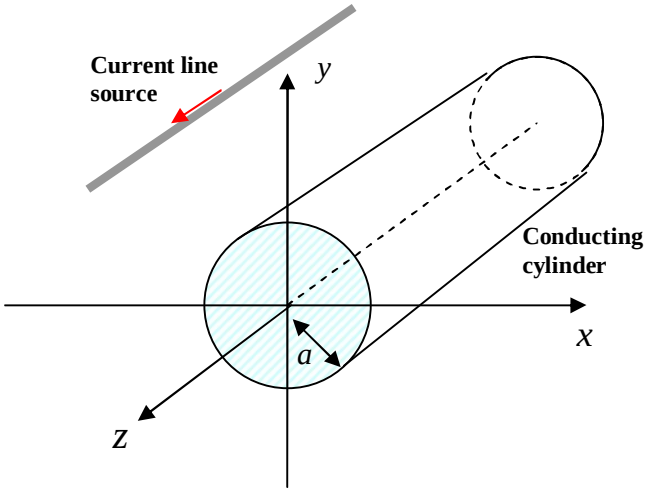
Homework assignment 3

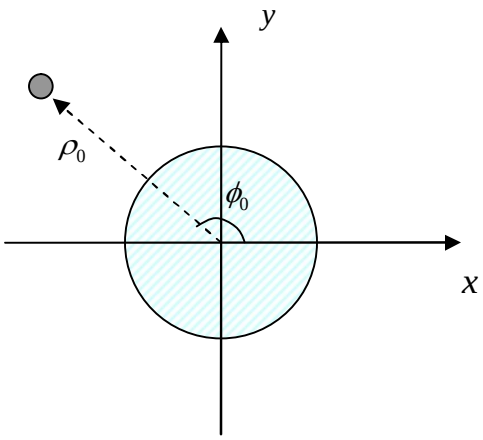
Problem 1:

A plane wave is normally incident on a perfectly conducting, infinitely long cylinder of radius a. For both the TE_z and TM_z case find the total current (integral over ϕ) on the cylinder. The dielectric constant and permeability of the surrounding medium are ε_0, μ_0.

Problem 2:

We would like to solve the TM scattering problem for an infinite, perfectly conducting cylinder (radius a) using a technique based on introducing an infinitely long line of electric current (line source). The line current is parallel to the cylinder and has the cylindrical coordinates ρ_0, ϕ_0. To simplify the situation, we assume the current along the line source to be constant (I) so that the variations of the field along the cylinder (along the z-axis) are neglected. The dielectric constant and permeability of the outside medium is ϵ, respectively, μ.





1. Write down the equation for the longitudinal component of the electric field E_z in cylindrical coordinates.
2. What is the solution of this equation for the electric current line source in the absence of the conducting cylinder?
3. What is the solution in the presence of the conducting cylinder?
4. How can we find the solution to the problem of the scattering of an incident plane wave with a wave vector parallel to the x-y plane by using the solution of problem 3?
5. Compare the solution to the one presented during the lectures.
6. How can we solve the TE scattering case by using this technique?

Problem 3:

Consider an infinitely long, dielectric circular cylinder whose axis coincides with the *z*-axis. A plane wave with the wave vector $\boldsymbol{k}_i = (k_{i,x}, k_{i,y}, 0)$ is normally incident on the cylinder where $|\boldsymbol{k}_i| = k_0 = \omega\sqrt{\varepsilon_0\mu_0}$ with ε_0, μ_0 respectively denoting the permittivity and permeability of the background medium. The electric field of the incident wave is polarized along the z-direction. The dielectric constant and radius of the cylinder are ε and a, respectively.

1. Write down the equations governing the electric field inside and outside the cylinder and give their general solution
2. Compute the scattered field by using the expansion of a (scalar) plane wave and matching the solutions at the boundary of the cylinder.