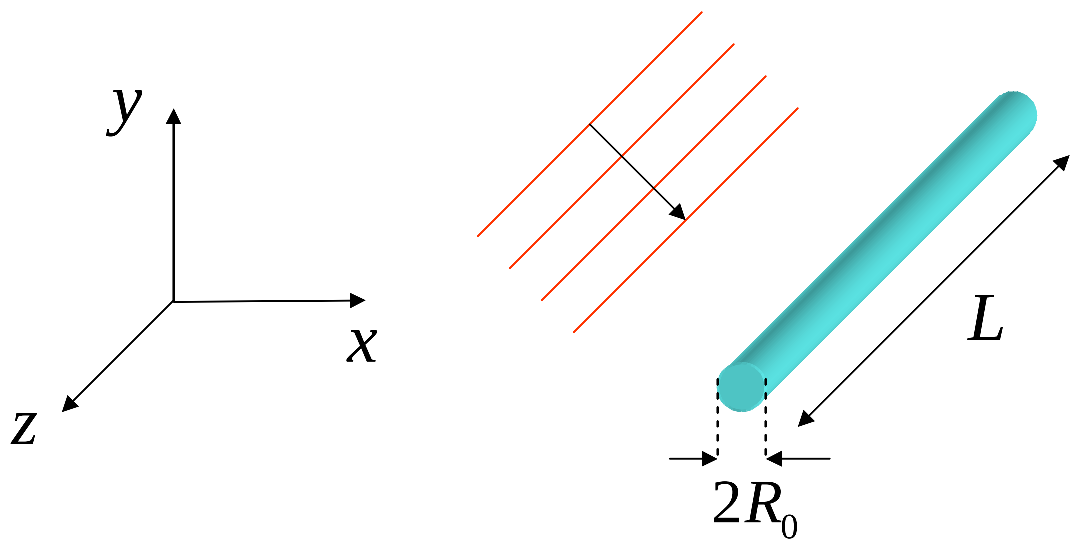
Equations for a finite wire

- We would now like to consider the case of a finite, but thin conductive cylindrical wire (along the z-axis)
- Note: the case of an infinite cylindrical wire was solved before by using the exact cylindrical vector function



Equations for a finite wire

- The surface (electric field) equation is

$$-\hat{\boldsymbol{n}}' \times \boldsymbol{E}_i(\boldsymbol{r}') = -j\omega\mu_0 \hat{\boldsymbol{n}}' \times \oint_S \bar{\bar{\boldsymbol{G}}}_0(\boldsymbol{r}', \boldsymbol{r}) \cdot [\hat{\boldsymbol{n}} \times \boldsymbol{H}(\boldsymbol{r})] dS$$

$$\boldsymbol{J}_s(\boldsymbol{r}) = \hat{\boldsymbol{n}} \times \boldsymbol{H}(\boldsymbol{r})$$

Surface current

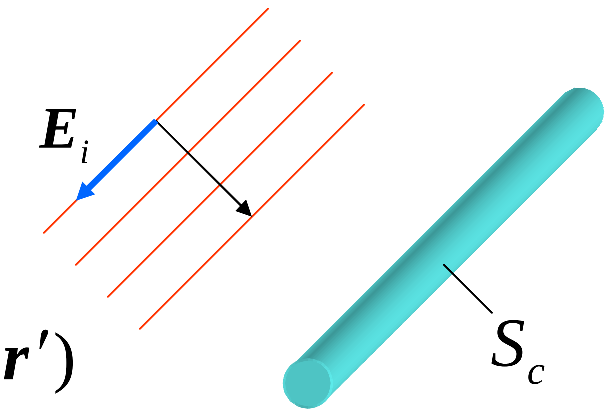
- If the wire is thin compared to its length and to wavelength:
  - We neglect the contribution of cross sectional surfaces at the beginning and end of the wire (only include the cylindrical surface)
  - Since the wire is assumed to be thin, electric fields normal to the wire cannot produce high currents. We only consider electric fields and surface currents *along* the wire.

Equations for a finite wire

- With these approximations the equation becomes

$$-\hat{\boldsymbol{n}}'\times\hat{\boldsymbol{z}}\,E_{i,z}(\boldsymbol{r}') = -j\omega\mu_0\hat{\boldsymbol{n}}'\times\int_{S_c}\overline{\overline{\boldsymbol{G}}}_0(\boldsymbol{r}',\boldsymbol{r})\cdot\left[\hat{\boldsymbol{z}}\,J_{s,z}(\boldsymbol{r})\right]dS$$

- We next use the definition of the dyadic Green's function

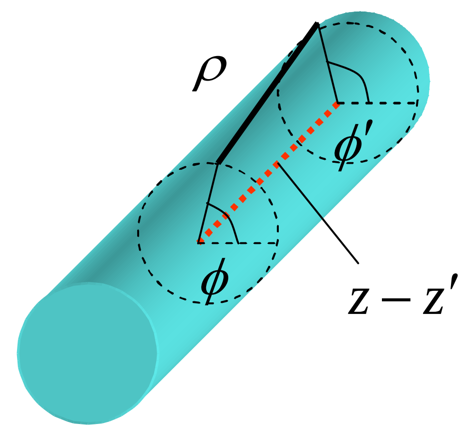


$$\overline{\overline{\boldsymbol{G}}}_0(\boldsymbol{r},\boldsymbol{r}') = G_0(\boldsymbol{r},\boldsymbol{r}')\overline{\overline{\boldsymbol{I}}} - \frac{1}{k_0^2}\nabla\nabla' G_0(\boldsymbol{r},\boldsymbol{r}')$$

$$E_{i,z}(\boldsymbol{r}') = j\omega\mu_0\int_{S_c}\left[G_0(\boldsymbol{r}',\boldsymbol{r}) - \frac{1}{k_0^2}\frac{\partial^2 G_0(\boldsymbol{r}',\boldsymbol{r})}{\partial z'\partial z}\right]J_{s,z}(\boldsymbol{r})dS$$

Equations for a finite wire

- Note that $$G_0(\boldsymbol{r},\boldsymbol{r}') = \frac{\exp\left(-jk_0\left|\boldsymbol{r}-\boldsymbol{r}'\right|\right)}{4\pi\left|\boldsymbol{r}-\boldsymbol{r}'\right|}$$



$$G_0(\boldsymbol{r},\boldsymbol{r}') = \frac{\exp\left(-jk_0\rho\right)}{4\pi\rho}$$

$$\rho = \sqrt{\left(z'-z\right)^2 + 2R_0^2\left[1-\cos\left(\phi-\phi'\right)\right]}$$

- Let us average the left hand side over the circumference of a cross section of the wire at z'

Equations for a finite wire

- Result:

$$\left\langle E_{i,z}(z')\right\rangle = \frac{j\omega\mu_0}{2\pi}\int_0^{2\pi}\left\{\int_{S_c}\left[G_0(\mathbf{r}',\mathbf{r}) - \frac{1}{k_0^2}\frac{\partial^2 G_0(\mathbf{r}',\mathbf{r})}{\partial z'\partial z}\right]J_{s,z}(\mathbf{r})dS\right\}d\phi'$$

$$\left\langle E_{i,z}(z')\right\rangle \equiv \frac{1}{2\pi}\int_0^{2\pi} E_{i,z}\left(R_0\cos\phi', R_0\sin\phi', z'\right)d\phi'$$

- Because of the dependence of the Green's function on $\phi' - \phi$, the integrated Green's function becomes independent of ϕ. Note that

$$G_0(\mathbf{r}',\mathbf{r}) = \frac{\exp\left(-jk_0\rho\right)}{4\pi\rho} \qquad \rho = \sqrt{\left(z'-z\right)^2 + 2R_0^2\left[1-\cos\left(\phi-\phi'\right)\right]}$$

Equations for a finite wire

$$\frac{1}{2\pi}\int_0^{2\pi} G_0(\mathbf{r}',\mathbf{r})d\phi' = \frac{1}{2\pi}\int_0^{2\pi} \frac{\exp\left(-jk_0\sqrt{(z'-z)^2+2R_0^2\left[1-\cos\phi'\right]}\right)}{4\pi\sqrt{(z'-z)^2+2R_0^2\left[1-\cos\phi'\right]}}d\phi'$$

$$\equiv g_0(z'-z)$$

- Now the equation becomes:

$$\left\langle E_{i,z}(z')\right\rangle = j\omega\mu_0\int_0^L \left[g_0(z'-z)-\frac{1}{k_0^2}\frac{\partial^2 g_0(z'-z)}{\partial z'\partial z}\right]I(z)dz$$

$$I(z) = R_0\int_0^{2\pi} J_{s,z}(z,\phi)d\phi$$

Total current at z

Equations for a finite wire

- The integral equation can be represented in the form

$$-j\omega\epsilon_0 \left\langle E_{i,z}(z') \right\rangle = \left(k_0^2 + \frac{d^2}{dz'^2} \right) \int_0^L g_0(z'-z) I(z) dz$$

- This is the Pocklington integral equation for a finite length wire