Electromagnetic scattering

Graduate Course

Electrical Engineering (Communications)

1st Semester, 1388-1389

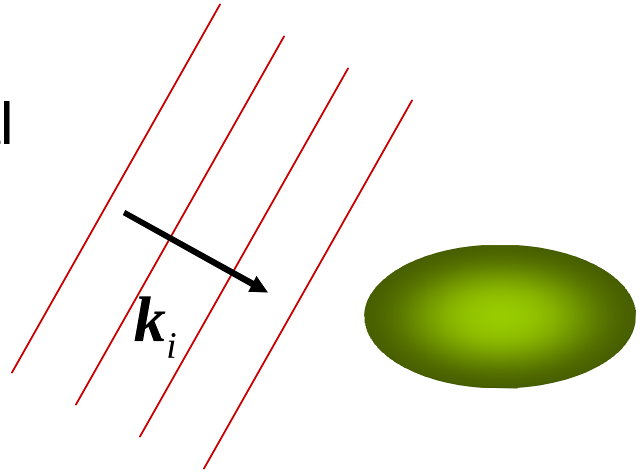
Sharif University of Technology

Contents of lecture 9

- **High frequency approximations:**
  - Physical optics
  - Geometrical optics
    - Maxwell equations at high frequencies
    - Eikonal equation and rays
    - Reflection of rays from conductive surfaces

Introduction

- We have, so far, approached the problem of scattering by
  - Exactly solution (cylinders, spheres, wedges,…)
  - Integral equation method (surface and volume formulations)
- If objects cannot be treated by exact methods, we have to resort to numerical solution of integral equations
- This is OK if the object is not much larger than the wavelength, otherwise the number of variables will be huge!



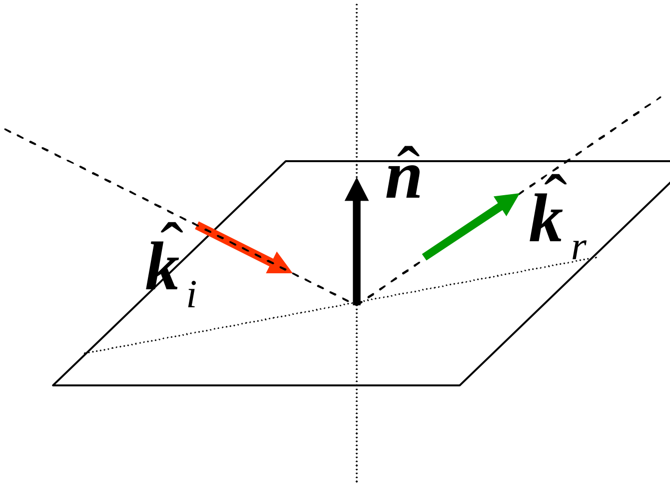
Introduction

- For such large objects, the of integral equations is impossible, given the limitations of today's computers
- The usual way to handle these cases is to use approximations (asymptotic solutions)

Physical optics

- Consider the simple problem of wave reflection by a flat perfect conductor. As in case of scattering by layered objects, one can distinguish a TE^z and a TM^z case.
- Rotate the coordinate system such that z is along the surface normal $\hat{\boldsymbol{n}}$ and y is along $\hat{\boldsymbol{n}} \times \hat{\boldsymbol{k}}_i$

$$\hat{\boldsymbol{k}}_r = \hat{\boldsymbol{k}}_i - 2\left(\hat{\boldsymbol{k}}_i \cdot \hat{\boldsymbol{n}}\right)\hat{\boldsymbol{n}}$$



Physical optics

- In this system:

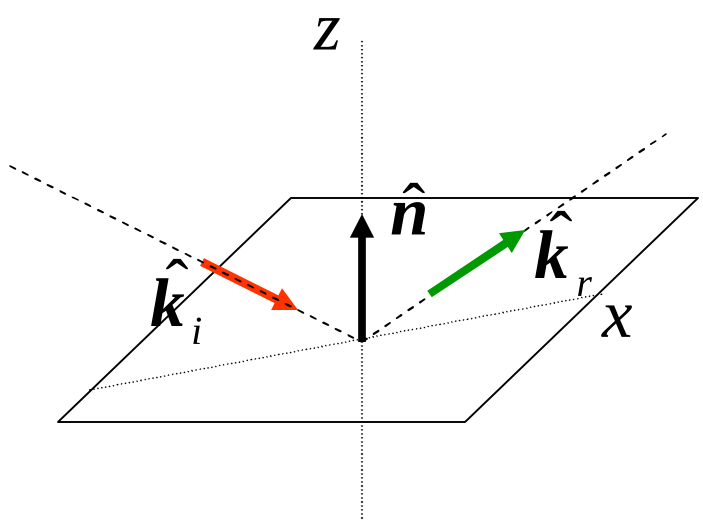
TE^z

$$\boldsymbol{E}_i(\boldsymbol{r}) = \hat{\boldsymbol{y}} E_0 \exp\left(-j\boldsymbol{k}_i \cdot \boldsymbol{r}\right)$$ Incident wave

$$\boldsymbol{E}_r(\boldsymbol{r}) = -\hat{\boldsymbol{y}} E_0 \exp\left(-j\boldsymbol{k}_r \cdot \boldsymbol{r}\right)$$ Reflected wave

$$\boldsymbol{k}_i = \left(k_{i,x}, 0, k_{i,z}\right)$$

$$\boldsymbol{k}_r = \left(k_{i,x}, 0, -k_{i,z}\right)$$



Physical optics

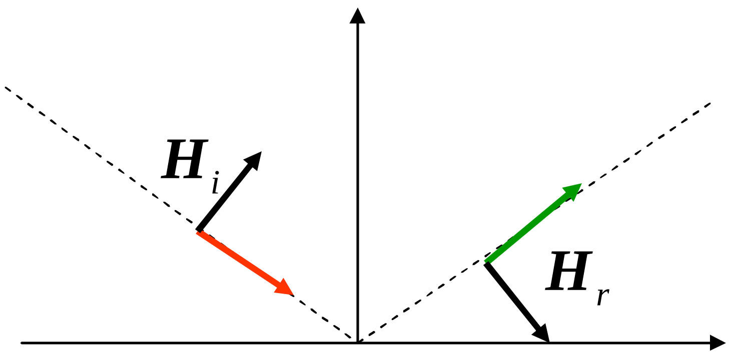
- Magnetic field

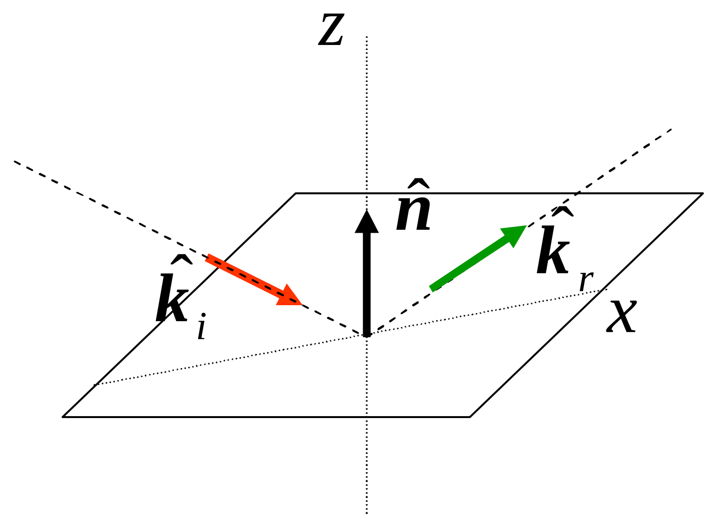
$$\boldsymbol{H}_i(\boldsymbol{r}) = \frac{1}{\eta}\hat{\boldsymbol{k}}_i \times \hat{\boldsymbol{y}} E_0 \exp\left(-j\boldsymbol{k}_i \cdot \boldsymbol{r}\right)$$

Incident wave

$$\boldsymbol{H}_r(\boldsymbol{r}) = -\frac{1}{\eta}\hat{\boldsymbol{k}}_r \times \hat{\boldsymbol{y}} E_0 \exp\left(-j\boldsymbol{k}_r \cdot \boldsymbol{r}\right)$$

Reflected wave

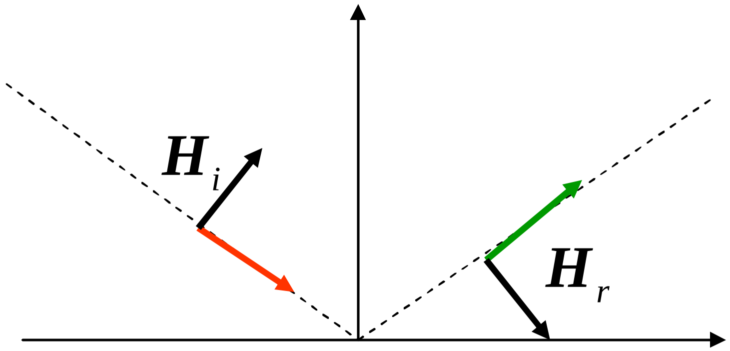




Physical optics

- On the surface of the conductor:

$$\hat{\boldsymbol{n}} \times \boldsymbol{H} = \hat{\boldsymbol{n}} \times \left(\boldsymbol{H}_i + \boldsymbol{H}_r \right)$$
$$= 2\hat{\boldsymbol{n}} \times \boldsymbol{H}_i$$



- The same can be shown for the TM case

$$\boldsymbol{H}_i(\boldsymbol{r}) = \hat{\boldsymbol{y}} H_0 \exp\left(-j\boldsymbol{k}_i \cdot \boldsymbol{r} \right)$$

$$\boldsymbol{H}_r(\boldsymbol{r}) = \hat{\boldsymbol{y}} H_0 \exp\left(-j\boldsymbol{k}_r \cdot \boldsymbol{r} \right)$$

$$\Rightarrow \quad \hat{\boldsymbol{n}} \times \boldsymbol{H} = \hat{\boldsymbol{n}} \times \left(\boldsymbol{H}_i + \boldsymbol{H}_r \right) = 2\hat{\boldsymbol{n}} \times \boldsymbol{H}_i$$

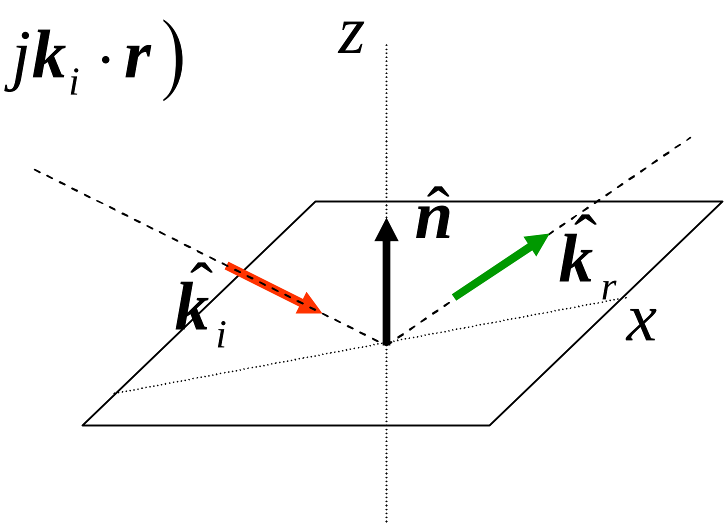
Physical optics

- In both cases the surface current density is given by

$$\boldsymbol{J}_s = \hat{\boldsymbol{n}} \times \boldsymbol{H} = 2\hat{\boldsymbol{n}} \times \boldsymbol{H}_i$$

$$\boldsymbol{H}_i(\boldsymbol{r}) = \frac{1}{\eta}\hat{\boldsymbol{k}}_i \times \hat{\boldsymbol{y}} E_0 \exp\left(-j\boldsymbol{k}_i \cdot \boldsymbol{r}\right)$$

$$= \frac{1}{\eta}\hat{\boldsymbol{k}}_i \times \left(\hat{\boldsymbol{n}} \times \hat{\boldsymbol{k}}_i\right) E_0 \exp\left(-j\boldsymbol{k}_i \cdot \boldsymbol{r}\right)$$



Physical optics

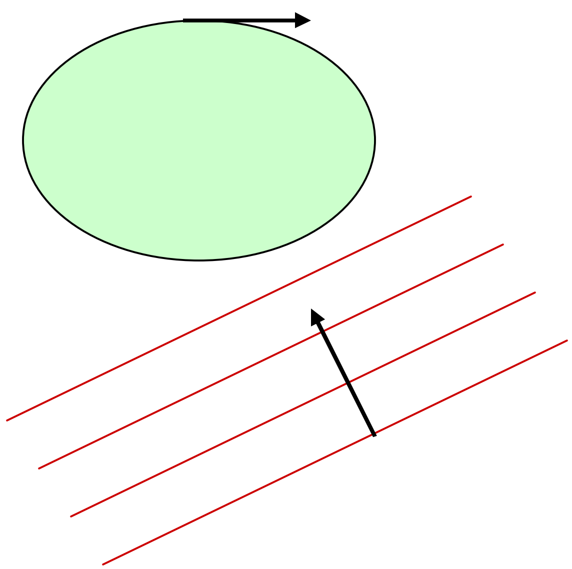
- Now consider a perfect conductor with an arbitrary shape
- The incident wave induced surface currents on such objects, which generate the scattering field

$$\boldsymbol{F}(\hat{\boldsymbol{r}}) = \oint_S \exp\left(jk\hat{\boldsymbol{r}}\cdot\boldsymbol{r}'\right)\boldsymbol{J}_s(\boldsymbol{r}')dS'$$

$$\boldsymbol{E}^f(\boldsymbol{r}) = jk\eta\frac{\exp(-jkr)}{4\pi r}\hat{\boldsymbol{r}}\times\left[\hat{\boldsymbol{r}}\times\boldsymbol{F}(\hat{\boldsymbol{r}})\right]$$

$$\boldsymbol{H}^f(\boldsymbol{r}) = -\frac{jk\exp(-jkr)}{4\pi r}\hat{\boldsymbol{r}}\times\boldsymbol{F}(\hat{\boldsymbol{r}})$$

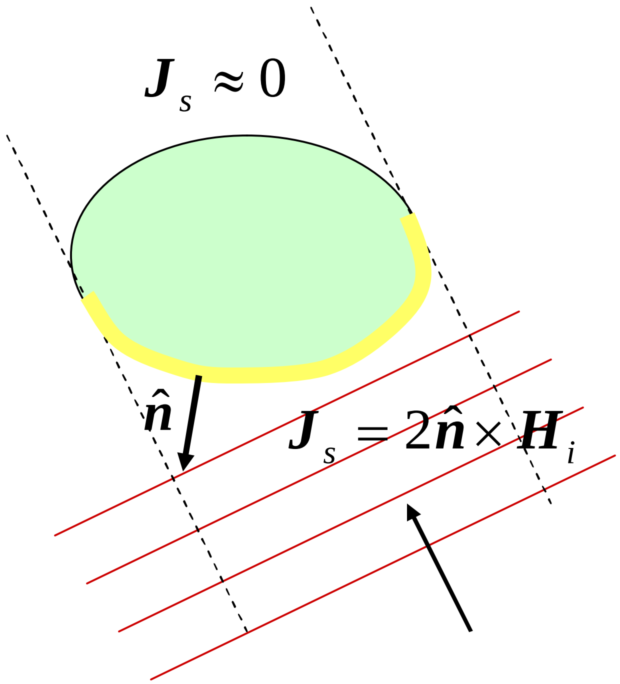
$$\boldsymbol{J}_s = \hat{\boldsymbol{n}}\times\boldsymbol{H}$$



Physical optics

- Physics optics approximation: if the curvature of the surface of the perfect conductor is much larger than wavelength then surface locally approximated by a flat tangential surface
- Current on the surface approximated by

$$\boldsymbol{J}_s = \begin{cases} 2\hat{\boldsymbol{n}} \times \boldsymbol{H}_i & \text{in the illuminated region} \\ 0 & \text{in the shadow region} \end{cases}$$



$$\boldsymbol{F}(\hat{\boldsymbol{r}}) = \int_{S_{ill}} \exp\left(jk\hat{\boldsymbol{r}} \cdot \boldsymbol{r}'\right) \boldsymbol{J}_s(\boldsymbol{r}') dS'$$

Physical optics

$$\boldsymbol{H}_i = \boldsymbol{H}_{i,0}\exp\left(-j\boldsymbol{k}_i\cdot\boldsymbol{r}\right),\quad \boldsymbol{H}_{i,0} = \frac{1}{\eta}\hat{\boldsymbol{k}}_i\times\boldsymbol{E}_{i,0}$$

$$\boldsymbol{F}\left(\hat{\boldsymbol{r}}\right) = 2\left\{\int_{S_{ill}}\exp\left[j\left(\boldsymbol{k}_s-\boldsymbol{k}_i\right)\cdot\boldsymbol{r}'\right]\hat{\boldsymbol{n}}(\boldsymbol{r}')dS'\right\}\times\boldsymbol{H}_{i,0}$$

- Physics optics can also be generalized to include diffraction effects. For this we need to compute additional surface currents at the edges of rapidly varying surfaces