Geometrical optics

- Consider the wave equation for the electric field propagating in a source-free, homogeneous medium

$$\nabla^2 \boldsymbol{E} + k^2 \boldsymbol{E} = 0$$

$$k^2 = \omega^2 \epsilon \mu$$

$$\nabla \cdot \boldsymbol{E} = 0$$

- Luneberg-Kline high frequency approximation:

$$\boldsymbol{E}(\boldsymbol{r};\omega) = \exp\left[-jk\psi(\boldsymbol{r})\right] \sum_{m=0}^{\infty} \frac{\boldsymbol{E}_m(\boldsymbol{r})}{(j\omega)^m}$$

$$\psi(\boldsymbol{r}) \text{ : real function of position}$$

- We substitute this equation into the wave equation, and separate different orders in $1/j\omega$
- 0^{th} order:

$$\nabla\psi\cdot\nabla\psi = \left|\nabla\psi\right|^2 = 1$$

Eikonal equation

- 1^{st} order terms:

$$\left(\nabla\psi\cdot\nabla\right)\boldsymbol{E}_0 + \frac{\nabla^2\psi}{2}\boldsymbol{E}_0 = 0$$

$$\nabla\psi\cdot\boldsymbol{E}_0 = 0$$

Geometrical optics

- Higher order terms

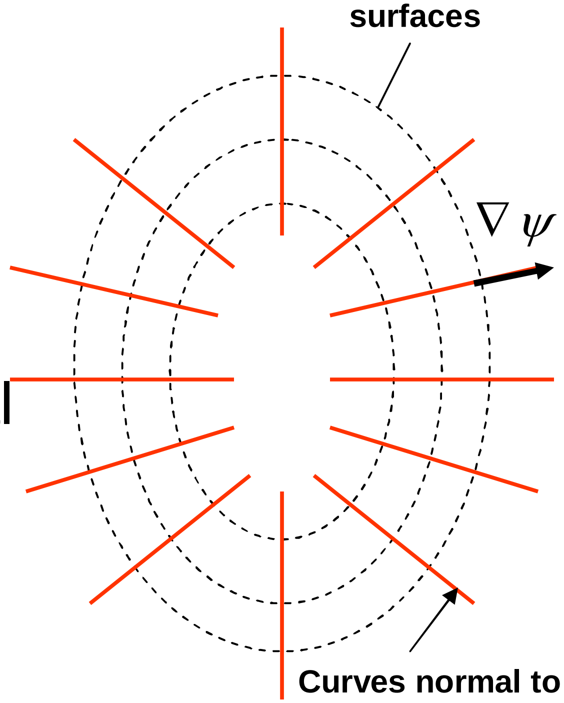
$$\left(\nabla\psi\cdot\nabla\right)\boldsymbol{E}_m+\frac{\nabla^2\psi}{2}\boldsymbol{E}_m=\frac{c}{2}\nabla^2\boldsymbol{E}_{m-1}$$

$$\nabla\psi\cdot\boldsymbol{E}_m=c\nabla\cdot\boldsymbol{E}_{m-1}$$

$$c=\frac{1}{\sqrt{\epsilon\mu}}$$

Geometrical optics

- Let us focus on the eikonal equation. It tells us that the gradient of ψ, which is a vector normal to surfaces of constant phase, always has a constant length (unity)
- Consider now the surfaces of constant ψ (or phase) and the curves perpendicular to these surfaces.
- At any point on these curves, the tangential vector to the curve is normal to a constant ψ surface



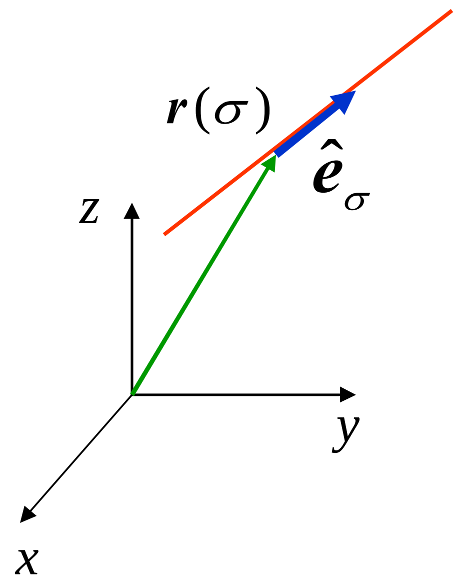
Geometrical optics

- Imagine that we parametrize these curves in the 3D space:

$$\boldsymbol{r} = \boldsymbol{r}(\sigma) = \left[x(\sigma), y(\sigma), z(\sigma)\right]$$

- Unit vector tangential to the curve

$$\hat{\boldsymbol{e}}_\sigma = \frac{\dfrac{d\boldsymbol{r}(\sigma)}{d\sigma}}{\left|\dfrac{d\boldsymbol{r}(\sigma)}{d\sigma}\right|} = \frac{\dfrac{d\boldsymbol{r}(\sigma)}{d\sigma}}{\sqrt{\dfrac{d\boldsymbol{r}(\sigma)}{d\sigma}\cdot\dfrac{d\boldsymbol{r}(\sigma)}{d\sigma}}}$$



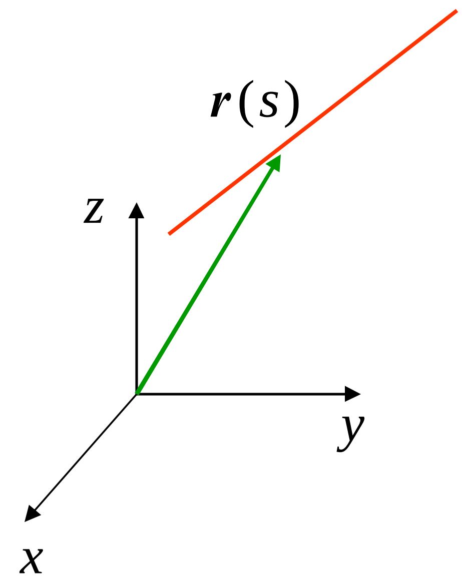
- Length along the curve: $l_{12} = \int_{\sigma_1}^{\sigma_2} \left|\dfrac{d\boldsymbol{r}(\sigma)}{d\sigma}\right| d\sigma$

Geometrical optics

- Imagine that we parametrize these curves by changing to a different parameter s such that

$$s = s(\sigma) \qquad \frac{ds}{d\sigma} = \left| \frac{d\boldsymbol{r}(\sigma)}{d\sigma} \right|$$

- Actually, s is the length measured along the line (with respect to a reference)



$$dl = \left| \frac{d\boldsymbol{r}}{d\sigma} \right| d\sigma = \left| \frac{d\boldsymbol{r}}{d\sigma} \right| \frac{d\sigma}{ds} ds = ds$$

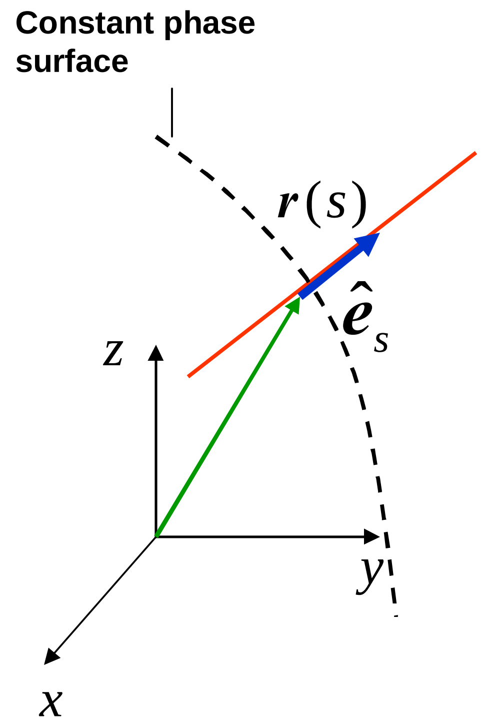
$$l_{12} = \int_{s_1}^{s_2} ds = s_2 - s_1$$

Geometrical optics

- With this new parameter:

$$\boldsymbol{r} = \boldsymbol{r}(s) = \left[x(s), y(s), z(s) \right]$$

$$\left| \frac{d\boldsymbol{r}}{ds} \right| = \left| \frac{d\boldsymbol{r}}{d\sigma} \right| \frac{d\sigma}{ds} = 1 \qquad \hat{\boldsymbol{e}}_s = \frac{d\boldsymbol{r}(s)}{ds}$$



- At each point in space we have a unit vector $\hat{\boldsymbol{e}}_s$ which is along such a curve and normal to a surface of constant ψ

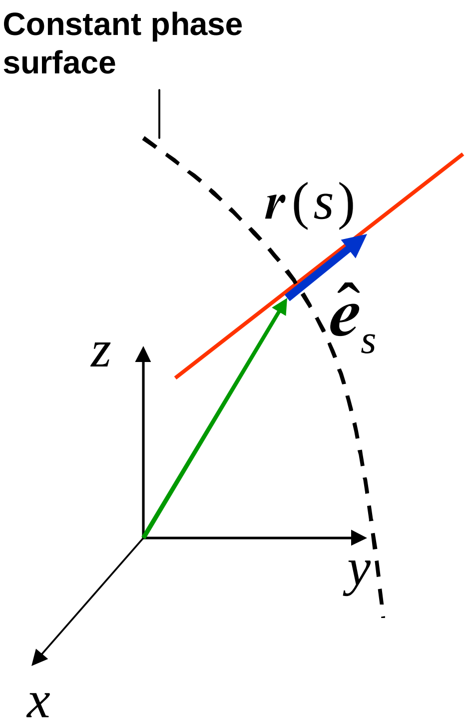
$$\rightarrow \hat{\boldsymbol{e}}_s = \nabla \psi$$

Because the gradient had a unit length!

Geometrical optics

- Look at the change of the unit tangential vector along the curve

$$\frac{\partial \hat{\boldsymbol{e}}_s}{\partial s} = \left(\hat{\boldsymbol{e}}_s \cdot \nabla\right)\hat{\boldsymbol{e}}_s$$



- Next use the relation

$$\nabla\left(\boldsymbol{a}\cdot\boldsymbol{b}\right) = \left(\boldsymbol{a}\cdot\nabla\right)\boldsymbol{b} + \left(\boldsymbol{b}\cdot\nabla\right)\boldsymbol{a} + \boldsymbol{a}\times\left(\nabla\times\boldsymbol{b}\right) + \boldsymbol{b}\times\left(\nabla\times\boldsymbol{a}\right)$$

$$\rightarrow \nabla\left(\boldsymbol{a}\cdot\boldsymbol{a}\right) = 2\left(\boldsymbol{a}\cdot\nabla\right)\boldsymbol{a} + 2\boldsymbol{a}\times\left(\nabla\times\boldsymbol{a}\right)$$

Geometrical optics

- It follows that

$$\frac{\partial \hat{\boldsymbol{e}}_s}{\partial s} = 0$$

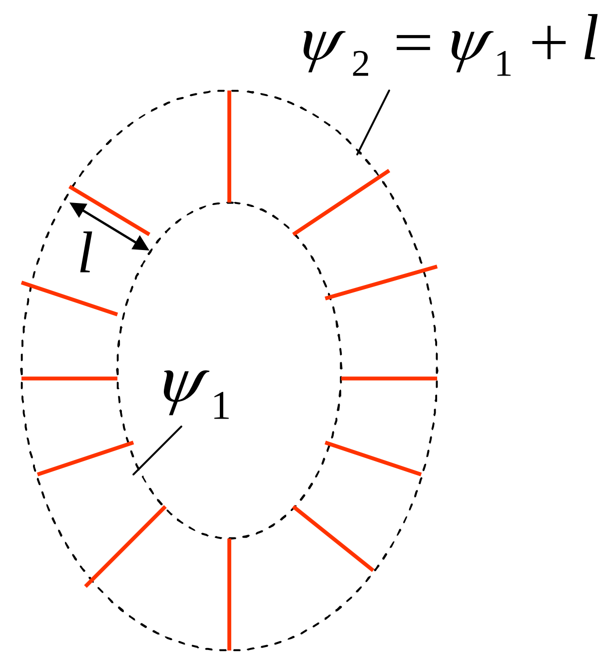
- This means that the curves perpendicular to the surfaces of constant phase are straight lines: they have no curvature
- These lines are called rays

Geometrical optics

- Starting from a certain wave front corresponding to $\psi=\psi_1$, look at the change in ψ along each ray:

$$\psi_2 = \psi_1 + \int_{s_1}^{s_2} \hat{\boldsymbol{e}}_s \cdot \nabla \psi \, ds = \psi_1 + \int_{s_1}^{s_2} \hat{\boldsymbol{e}}_s \cdot \hat{\boldsymbol{e}}_s ds = \psi_1 + \left(s_2 - s_1 \right)$$

- Thus, another surface of constant ψ can be constructed but starting from $\psi=\psi_1$ and drawing normal rays each with the same length $l = s_2 - s_1$
- Note: each ray is normal to both surfaces



Geometrical optics

- Now let us look at the 1st order equations which we write as

$$\frac{\partial \boldsymbol{E}_0}{\partial s} + \frac{\nabla^2 \psi}{2} \boldsymbol{E}_0 = 0$$

$$\hat{\boldsymbol{e}}_s \cdot \boldsymbol{E}_0 = 0$$

- From 1st Maxwell equation it follows for the lowest order magnetic field that

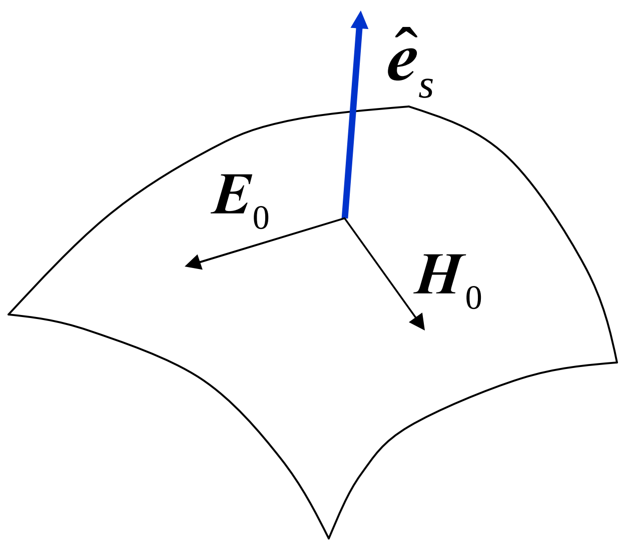
$$\hat{\boldsymbol{e}}_s \times \boldsymbol{E}_0 = \eta \boldsymbol{H}_0$$

- Locally, this is like a TEM plane wave propagating along ***s*** (along the ray). We neglect higher order terms.

Geometrical optics

- Note that the wave fronts (surfaces of constant ψ) are, in general, curved surfaces. The electric and magnetic fields are tangent to such a surface, and normal to the propagation direction which is along the ray.

- The amplitude of the electric field vector changes in space. But its polarization does not change along a ray (follows from the ray equation for the electric field)



Geometrical optics

- Example: plane waves along z:

$$\psi = z,\ \hat{\boldsymbol{e}}_s = \hat{\boldsymbol{z}}, \nabla^2 \psi = 0 \ \rightarrow \frac{\partial \boldsymbol{E}_0}{\partial z} = 0 \rightarrow \boldsymbol{E}_0 \ : \text{constant}$$

- Example: cylindrical waves

$$\psi = \rho, \hat{\boldsymbol{e}}_s = \hat{\boldsymbol{\rho}}, \nabla^2 \psi = \frac{1}{\rho} \ \rightarrow \frac{\partial \boldsymbol{E}_0}{\partial \rho} + \frac{1}{2\rho} \boldsymbol{E}_0 = 0 \rightarrow \boldsymbol{E}_0(\rho) = \frac{\boldsymbol{E}_0(\rho_0)}{\sqrt{\rho / \rho_0}}$$

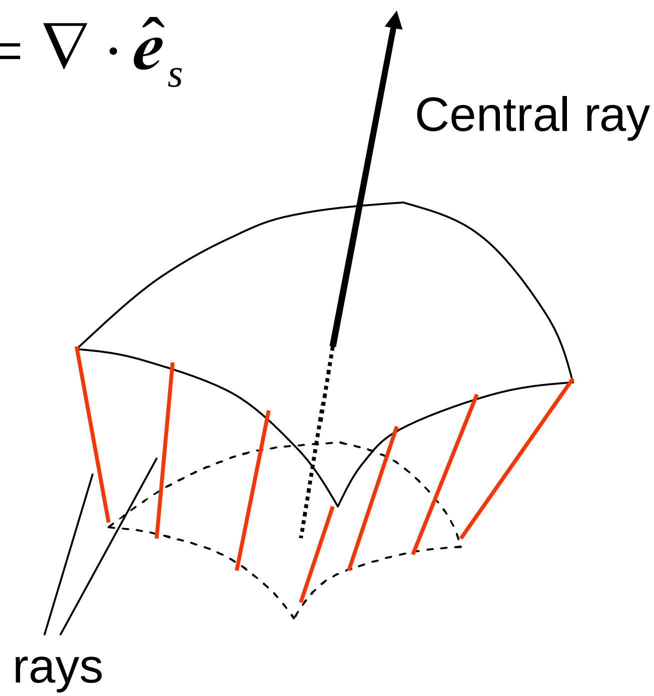
- Spherical waves:

$$\psi = r,\ \hat{\boldsymbol{e}}_s = \hat{\boldsymbol{r}}, \nabla^2 \psi = \frac{2}{r} \ \rightarrow \frac{\partial \boldsymbol{E}_0}{\partial r} + \frac{1}{r} \boldsymbol{E}_0 = 0 \rightarrow \boldsymbol{E}_0(r) = \frac{\boldsymbol{E}_0(r_0)}{r / r_0}$$

Geometrical optics

- In general, however, a surface may be none of these
- To see how one can still solve the ray equation note that

$$\frac{\partial \boldsymbol{E}_0}{\partial s} + \frac{\nabla^2 \psi}{2} \boldsymbol{E}_0 = 0 \qquad \nabla^2 \psi = \nabla \cdot \hat{\boldsymbol{e}}_s$$



- Consider small portions of two constant ψ surfaces around a central ray, and "cut" by walls of rays around the central ray.
- Length of all ray segments:

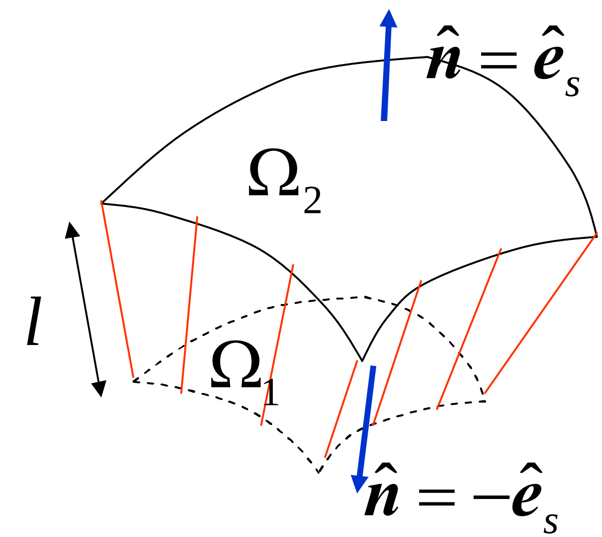
$$\psi_2 - \psi_1 = s_2 - s_1 = l$$

Geometrical optics

- In this volume in space, let us use the divergence theorem

$$\int_v \nabla \cdot \hat{\boldsymbol{e}}_s dV = \oint_S \hat{\boldsymbol{e}}_s \cdot \hat{\boldsymbol{n}} dS$$

- Contribution from side walls is zero, only the top and bottom surfaces contribute



$$\int_v \nabla \cdot \hat{\boldsymbol{e}}_s dV = \Omega_2 - \Omega_1$$

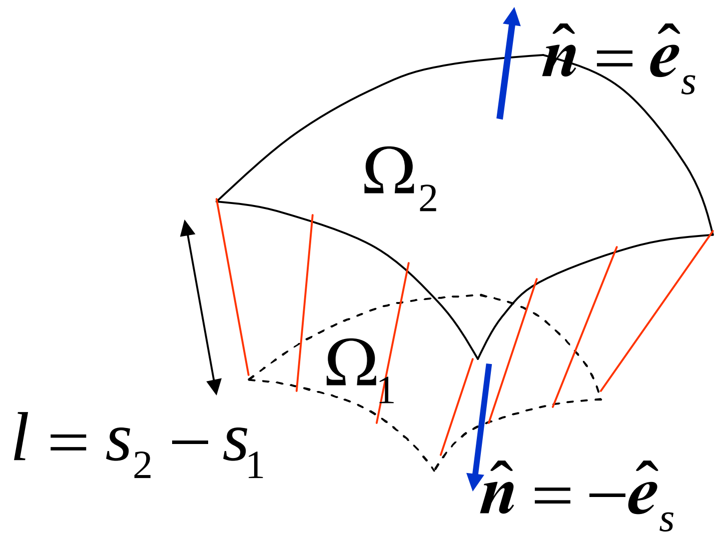
- If the volume is small:

$$\int_v \nabla \cdot \hat{\boldsymbol{e}}_s dV \sim \nabla \cdot \hat{\boldsymbol{e}}_s \int_v dV \sim \nabla \cdot \hat{\boldsymbol{e}}_s \frac{l}{2}\left(\Omega_2 + \Omega_1\right)$$

Geometrical optics

- Consequently:

$$\nabla \cdot \hat{\boldsymbol{e}}_s = \lim_{l \to 0}\left(\frac{2}{l}\frac{\Omega_2 - \Omega_1}{\Omega_2 + \Omega_1}\right)$$



- Recall that

$$\Omega_1 = \Omega(s_1), \Omega_2 = \Omega(s_2), l = s_2 - s_1$$

$$\nabla \cdot \hat{\boldsymbol{e}}_s = \frac{1}{\Omega}\frac{d\Omega}{ds}$$

Ω: Small element of the surface around the central ray

Geometrical optics

- The ray equation now becomes:

$$\frac{\partial \boldsymbol{E}_0}{\partial s} + \frac{1}{2\Omega}\frac{d\Omega}{ds}\boldsymbol{E}_0 = 0 \rightarrow \boldsymbol{E}_0(s) = \boldsymbol{E}_0(s_1)\sqrt{\frac{\Omega(s_1)}{\Omega(s)}}$$

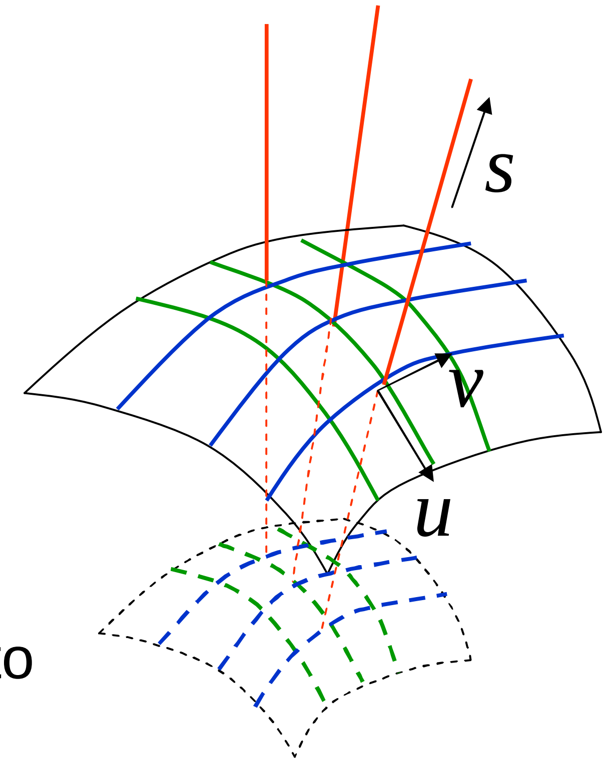
- To know the behavior of the field along each ray (with respect to a reference point), we have to know how a small area around the ray, on the constant phase surface, changes with s

Geometrical optics

- We define a system of curvilinear coordinates in space, where s is defined along the rays, an u and v are defined by curves on the constant phase surfaces

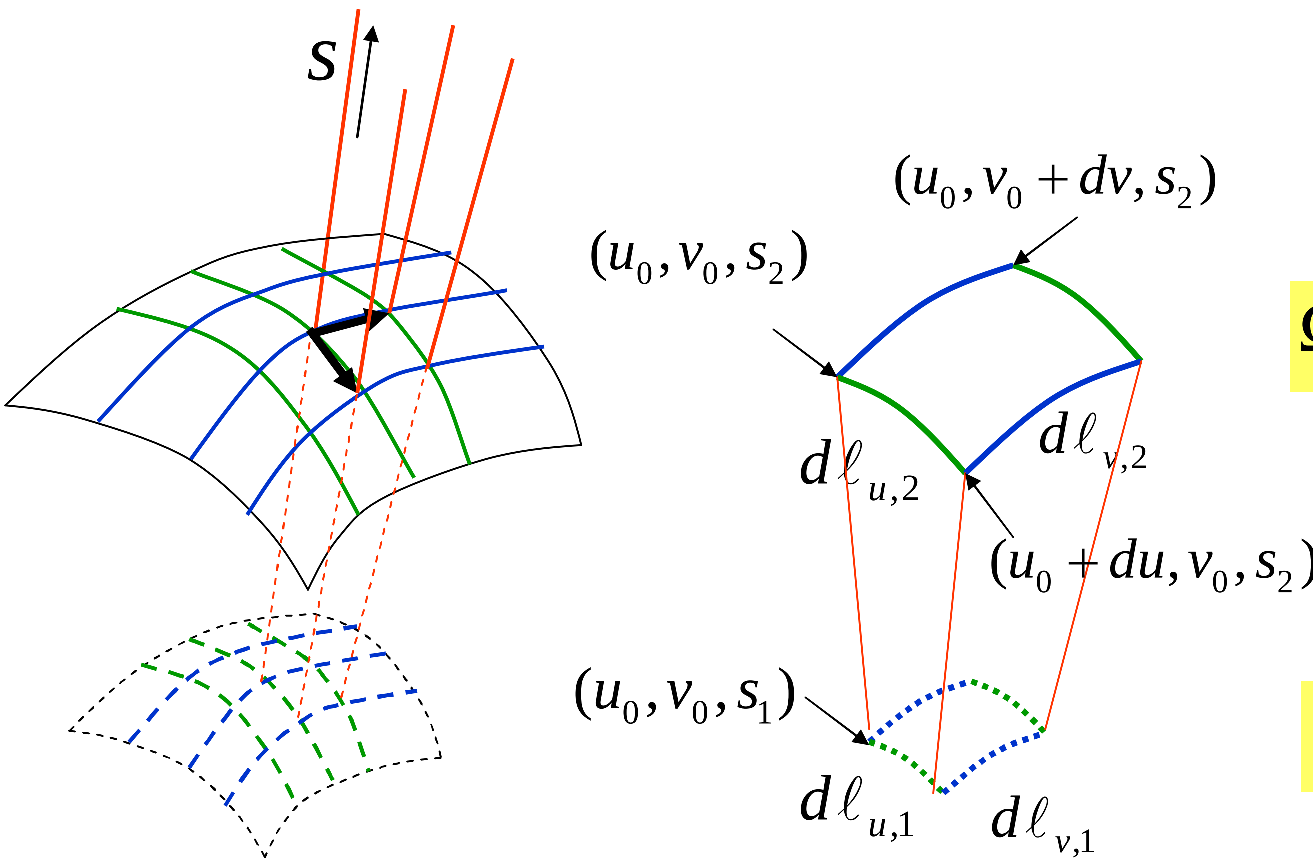
- Two points on a ray have different s values, but the same u and v values

- Let us restrict ourselves to orthogonal coordinate systems where lines along s (constant u,v), along u (constant s,v) and along v (constant u,s) are perpendicular to each other at each point



Geometrical optics

- What is the area of a small segment determined by du,dv?



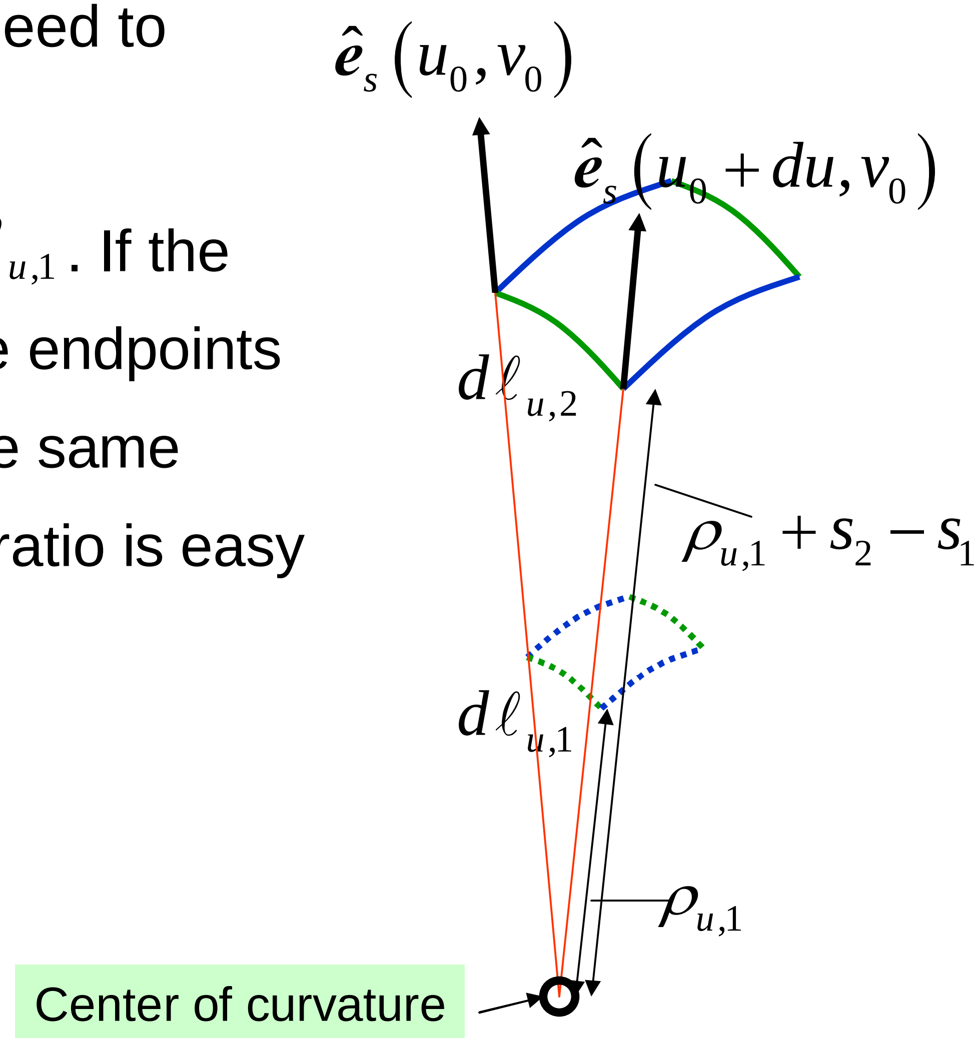
$$\Omega_2 = d\ell_{u,2} d\ell_{v,2}$$

$$\Omega_1 = d\ell_{u,1} d\ell_{v,1}$$

Geometrical optics

- To find the electric field we need to know $\Omega(s_2)/\Omega(s_1)$
- Consider the ratio $d\ell_{u,2}/d\ell_{u,1}$. If the two rays passing through the endpoints of the line segments lie in the same plane, the calculation of the ratio is easy

$$\frac{d\ell_{u,2}}{d\ell_{u,1}} = \frac{\rho_{u,1}+s_2-s_1}{\rho_{u,1}}$$



Geometrical optics

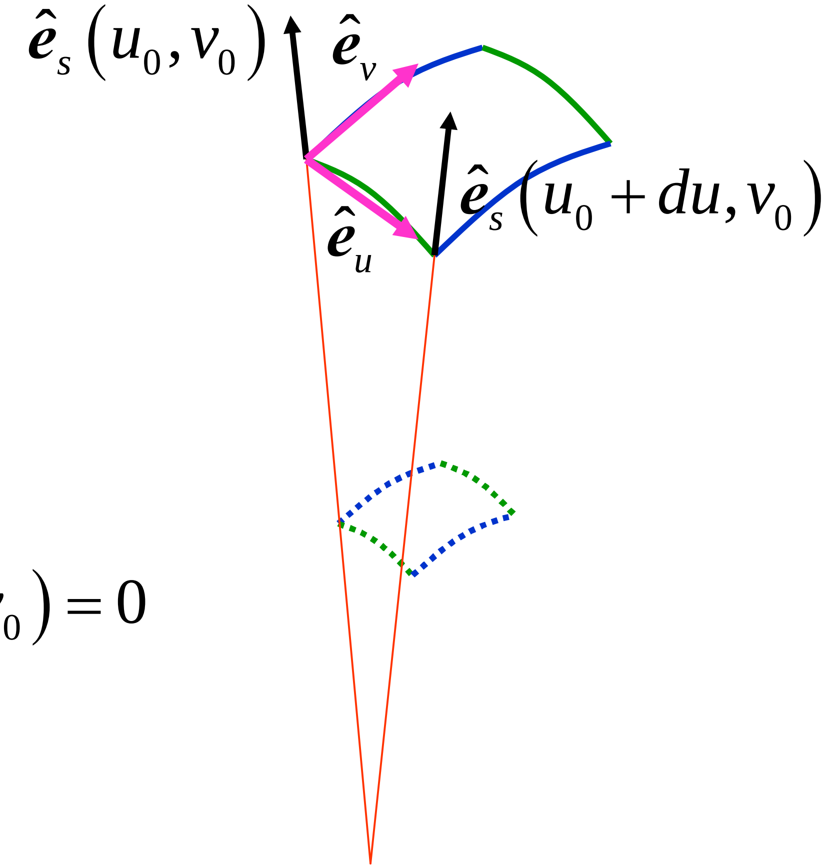
- But for an arbitrary coordinate system there is no guarantee that the two ray cross each other. They may not lie on the same plane.

- The only way to ensure this is that

$$\left[\hat{\boldsymbol{e}}_s(u_0+du,v_0)\times\hat{\boldsymbol{e}}_s(u_0,v_0)\right]\cdot\hat{\boldsymbol{e}}_u(u_0,v_0)=0$$

$$\Rightarrow\quad \left[\frac{\partial\hat{\boldsymbol{e}}_s}{\partial u}\times\hat{\boldsymbol{e}}_s\right]\cdot\hat{\boldsymbol{e}}_u=0\ \text{ at } u_0,v_0$$

$$\Rightarrow\quad \frac{\partial\hat{\boldsymbol{e}}_s}{\partial u}\cdot\hat{\boldsymbol{e}}_v=0\ \text{ at } u_0,v_0$$



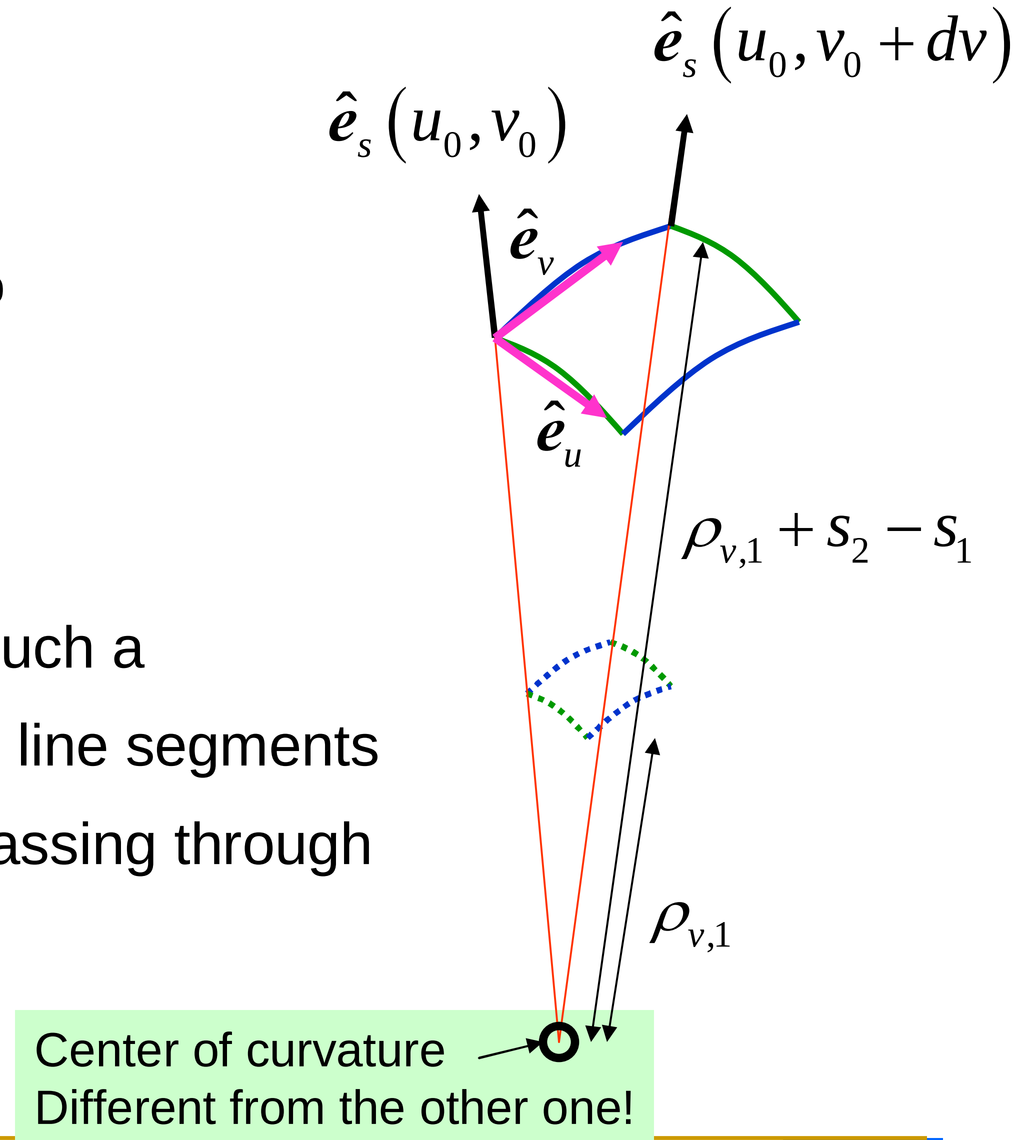
Geometrical optics

- Similarly, we demand that

$$\left[\frac{\partial \hat{\boldsymbol{e}}_s}{\partial v} \times \hat{\boldsymbol{e}}_s\right] \cdot \hat{\boldsymbol{e}}_v = 0 \quad \text{at } u_0, v_0$$

$$\frac{\partial \hat{\boldsymbol{e}}_s}{\partial v} \cdot \hat{\boldsymbol{e}}_u = 0 \quad \text{at } u_0, v_0$$

- It is always possible to find such a coordinate system. Then the line segments along u and v lie in planes passing through the ray.

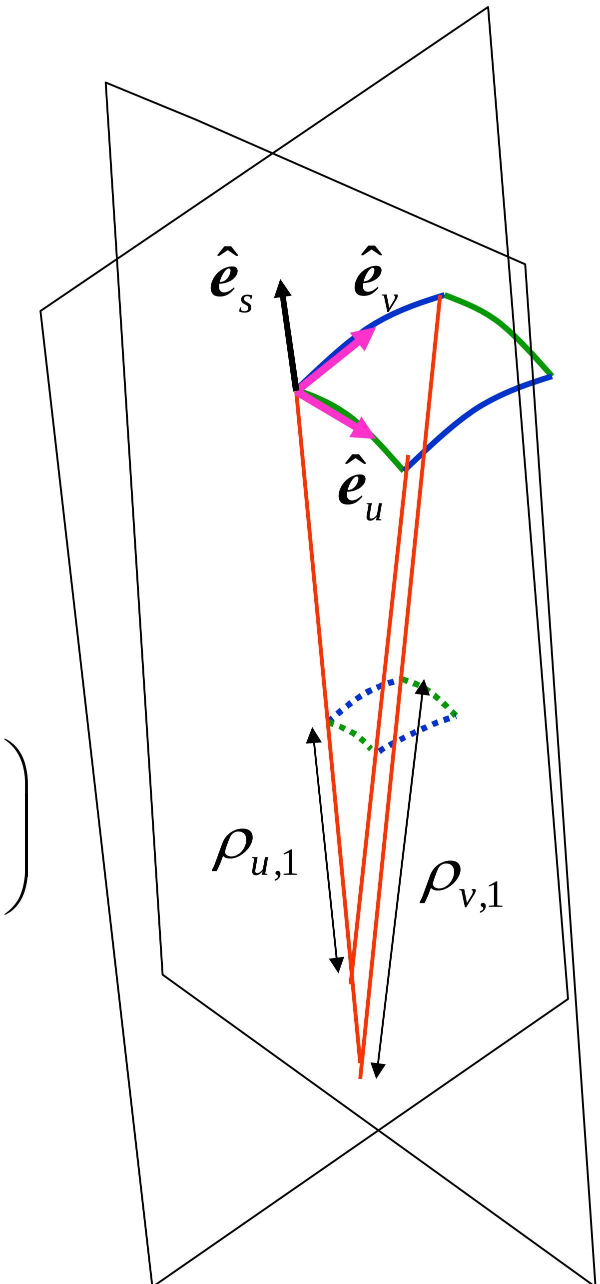


Geometrical optics

- Now, in this coordinate system we need to know the distances to the point of coincidence of the rays for the reference surface (s_1) and from there we get

$$\frac{\Omega_2}{\Omega_1} = \frac{d\ell_{u,2}\, d\ell_{v,2}}{d\ell_{u,1}\, d\ell_{v,1}} = \left(\frac{\rho_{u,1} + s_2 - s_1}{\rho_{u,1}}\right)\left(\frac{\rho_{v,1} + s_2 - s_1}{\rho_{v,1}}\right)$$

- These distances are the principal radii of curvature of the surface at the point of coincidence with the ray



Geometrical optics

- The principal radius of curvature of the surfaces can also be expressed as

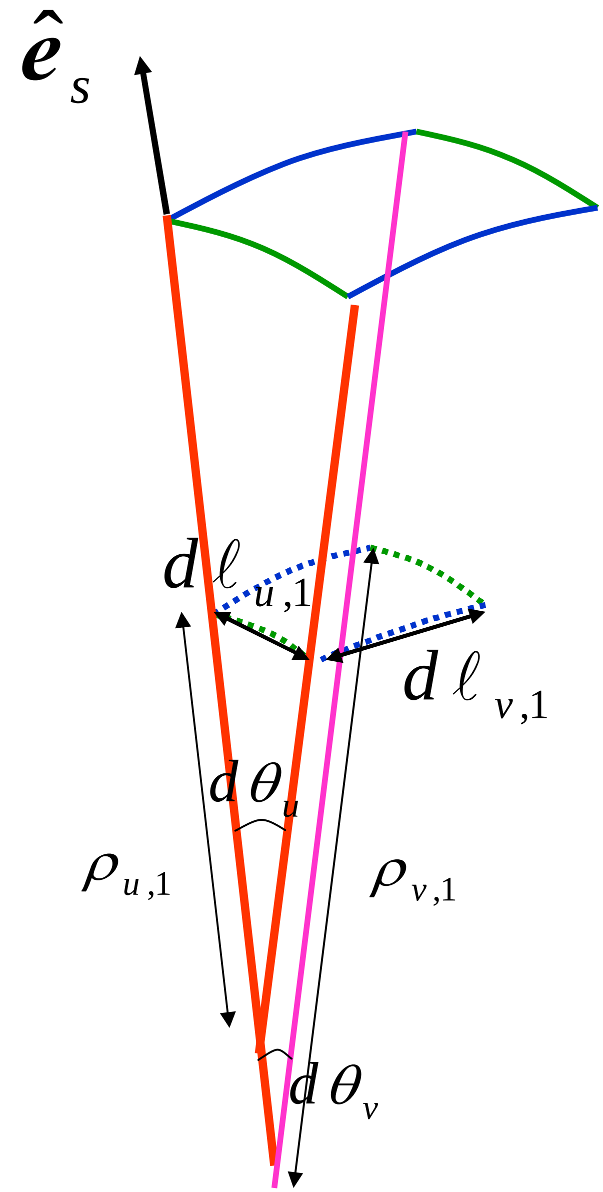
$$\rho_{u,1} = \frac{d\ell_{u,1}}{d\theta_u} = \frac{d\ell_{u,1}}{du}\left(\frac{d\theta_u}{du}\right)^{-1} = \frac{d\ell_{u,1}}{du}\left|\frac{\partial \hat{\boldsymbol{e}}_s}{\partial u}\right|^{-1}$$

$$\rho_{v,1} = \frac{d\ell_{v,1}}{d\theta_v} = \frac{d\ell_{v,1}}{dv}\left(\frac{d\theta_v}{dv}\right)^{-1} = \frac{d\ell_{v,1}}{dv}\left|\frac{\partial \hat{\boldsymbol{e}}_s}{\partial v}\right|^{-1}$$

- Note that:

$$\frac{d\ell_{u,1}}{du} = h_u, \quad \frac{d\ell_{v,1}}{dv} = h_v$$

Metric coefficients!



Geometrical optics

- Alternatively, if instead of u and v we use the length parameter along these curves, then

$$\rho_u = \frac{d\ell_u}{du}\left|\frac{\partial \hat{\boldsymbol{e}}_s}{\partial u}\right|^{-1} = \left|\frac{\partial \hat{\boldsymbol{e}}_s}{\partial \ell_u}\right|^{-1} \qquad \rho_v = \frac{d\ell_v}{dv}\left|\frac{\partial \hat{\boldsymbol{e}}_s}{\partial v}\right|^{-1} = \left|\frac{\partial \hat{\boldsymbol{e}}_s}{\partial \ell_v}\right|^{-1}$$

Geometrical optics

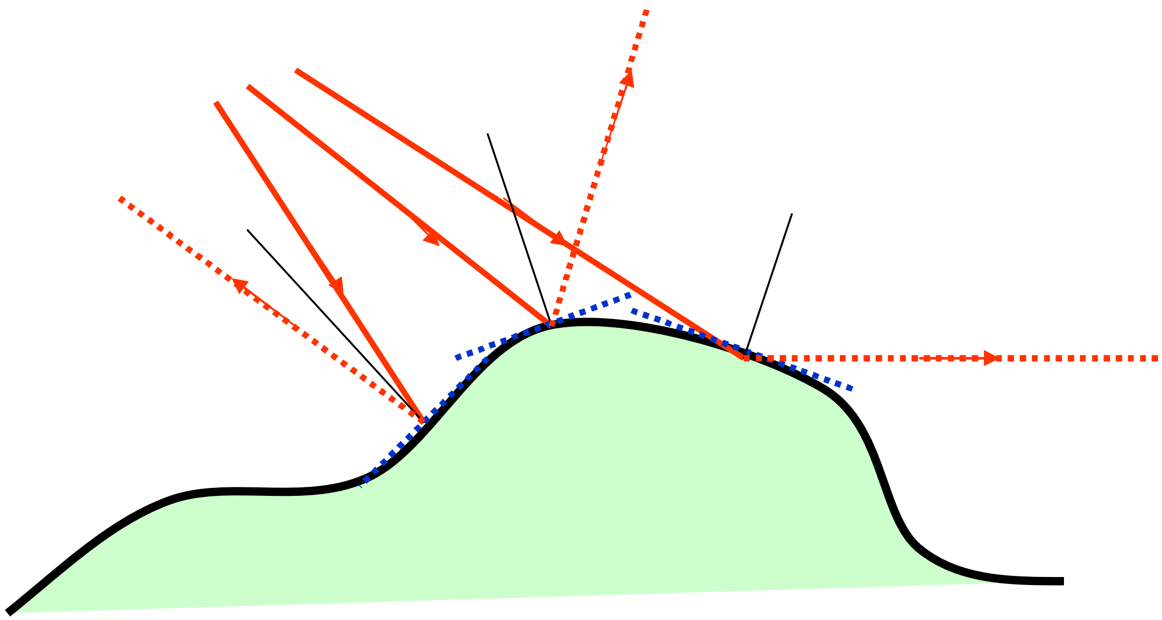
- Now we know how a wave propagates in space in the ray approximation. But happens when it hits an object? (We restrict ourselves to perfectly conducting surfaces)
- The reflected wave can also be represented in terms of fields along rays:

$$\boldsymbol{E}_s(\boldsymbol{r};\omega) \approx \boldsymbol{E}_{0,s}(\boldsymbol{r}) \exp\left[-jk\psi_s(\boldsymbol{r})\right]$$

$$\frac{\partial \boldsymbol{E}_{0,s}}{\partial s} + \frac{\nabla^2 \psi_s}{2}\boldsymbol{E}_{0,s} = 0 \qquad \hat{\boldsymbol{e}}_s \cdot \boldsymbol{E}_{0,s} = 0$$

Geometrical optics

- But how can we find the ray directions? We can use the short wave length approximation: reflection from a conductive surface happens as if the wave is locally reflected from a plane surface tangent to the true surface. Then the Snell's reflection law is employed.

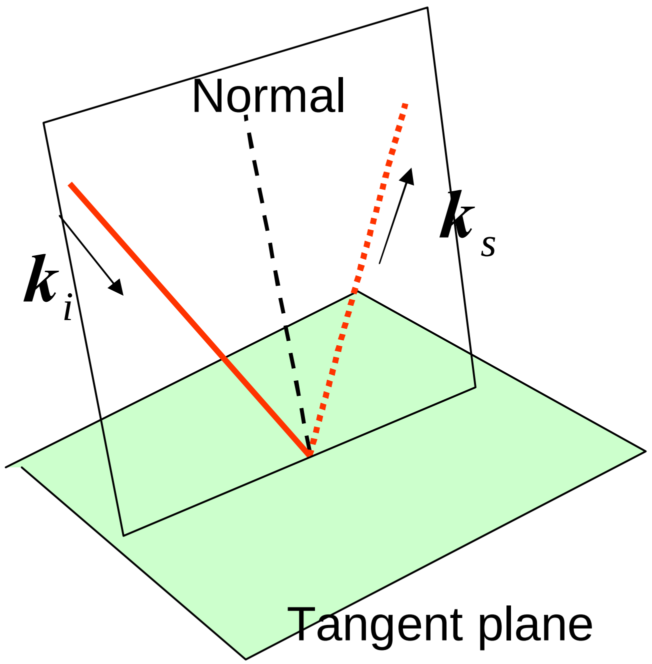


Geometrical optics

- Now, along each reflected ray again the polarization does not change and the amplitude changes according to the radii of the curvature of the reflected wave fronts
- If we know the polarization and radii of curvature directly after reflection, we know on every point along the ray
- Then, to write down the electric field at any point, we should first see which ray (or rays) pass through that point, calculate the electric field (polarization and amplitude) along the rays, and add them up

Geometrical optics

- How can we calculate the polarization along the reflected ray?
- Again use the plane approximation: view the incident ray as a plane wave (locally), and a plane tangent to the surface (at the point of incidence) as the reflecting plane
- We can now define the polarizations in terms of the normal to the local tangent plane and the plane of reflection, as in case of a plane wave



Geometrical optics

- Use local system of coordinates defined by unit vectors normal and tangent to local tangent plane

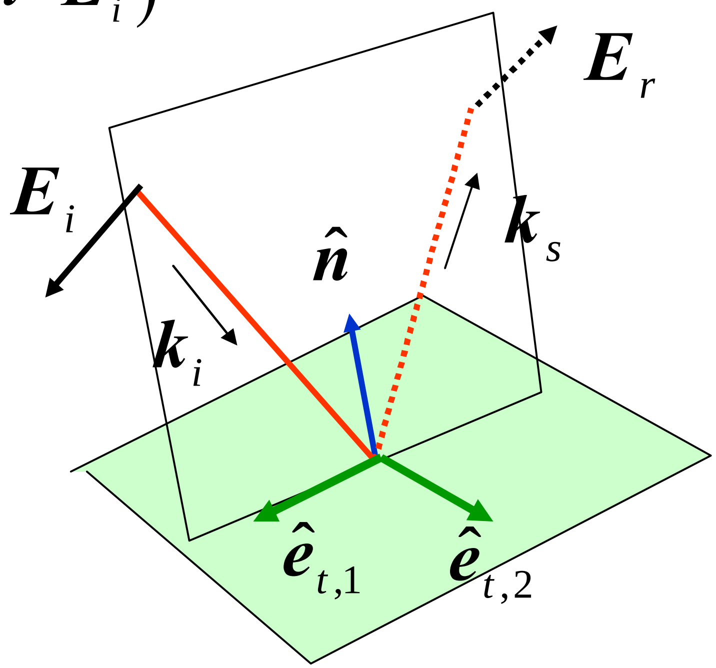
$$\hat{\boldsymbol{e}}_{t,2} = \frac{\hat{\boldsymbol{k}}_i \times \hat{\boldsymbol{n}}}{\left|\hat{\boldsymbol{k}}_i \times \hat{\boldsymbol{n}}\right|}$$

$$\hat{\boldsymbol{e}}_{t,1} = \hat{\boldsymbol{e}}_{t,2} \times \hat{\boldsymbol{n}}$$

$$\boldsymbol{E}_i = \hat{\boldsymbol{e}}_{t,1}\left(\hat{\boldsymbol{e}}_{t,1} \cdot \boldsymbol{E}_i\right) + \hat{\boldsymbol{e}}_{t,2}\left(\hat{\boldsymbol{e}}_{t,2} \cdot \boldsymbol{E}_i\right) + \hat{\boldsymbol{n}}\left(\hat{\boldsymbol{n}} \cdot \boldsymbol{E}_i\right)$$

- Reflected field:

$$\boldsymbol{E}_r = -\hat{\boldsymbol{e}}_{t,1}\left(\hat{\boldsymbol{e}}_{t,1} \cdot \boldsymbol{E}_i\right) - \hat{\boldsymbol{e}}_{t,2}\left(\hat{\boldsymbol{e}}_{t,2} \cdot \boldsymbol{E}_i\right) + \hat{\boldsymbol{n}}\left(\hat{\boldsymbol{n}} \cdot \boldsymbol{E}_i\right)$$



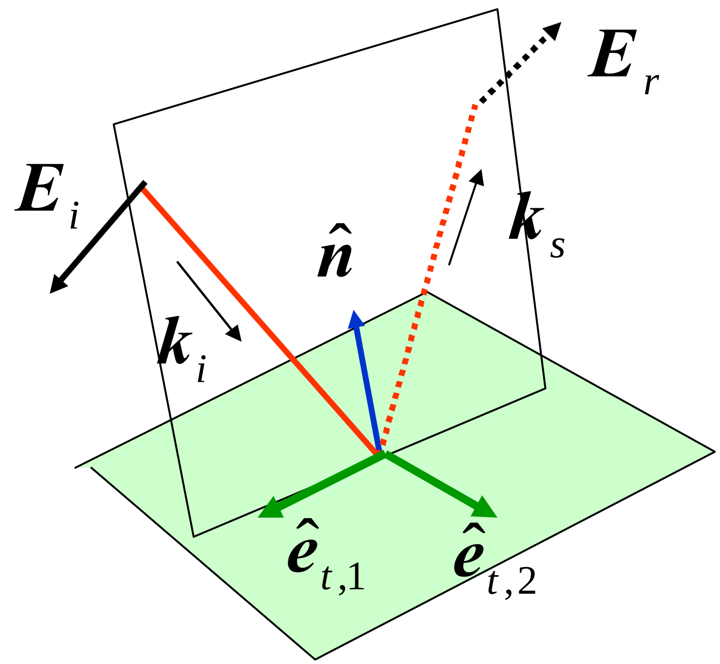
Tangent plane

Geometrical optics

- Can be written as $\boldsymbol{E}_r = \bar{\bar{\boldsymbol{R}}} \cdot \boldsymbol{E}_i \qquad \bar{\bar{\boldsymbol{R}}} = -\hat{\boldsymbol{e}}_{t,1}\hat{\boldsymbol{e}}_{t,1} - \hat{\boldsymbol{e}}_{t,2}\hat{\boldsymbol{e}}_{t,2} + \hat{\boldsymbol{n}}\hat{\boldsymbol{n}}$

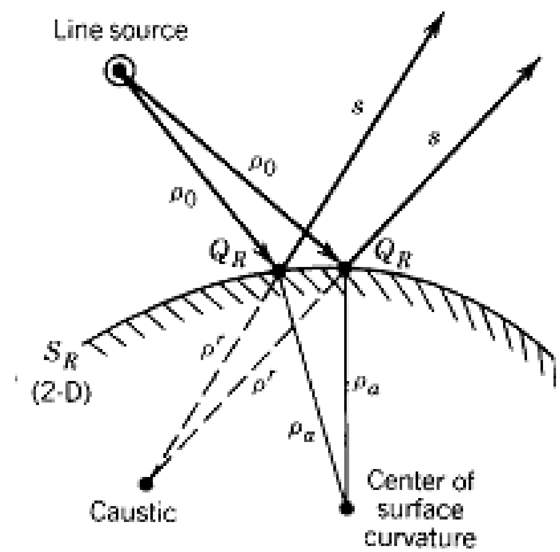
$$\hat{\boldsymbol{e}}_{t,1} = \hat{\boldsymbol{e}}_{t,2} \times \hat{\boldsymbol{n}} \qquad \hat{\boldsymbol{e}}_{t,2} = \frac{\hat{\boldsymbol{k}}_i \times \hat{\boldsymbol{n}}}{\left|\hat{\boldsymbol{k}}_i \times \hat{\boldsymbol{n}}\right|}$$

- The polarization of the reflected wave is thus expressed in terms of the dyadic reflection matrix **R** which itself depends on direction of incidence and the (local) unit normal to the surface



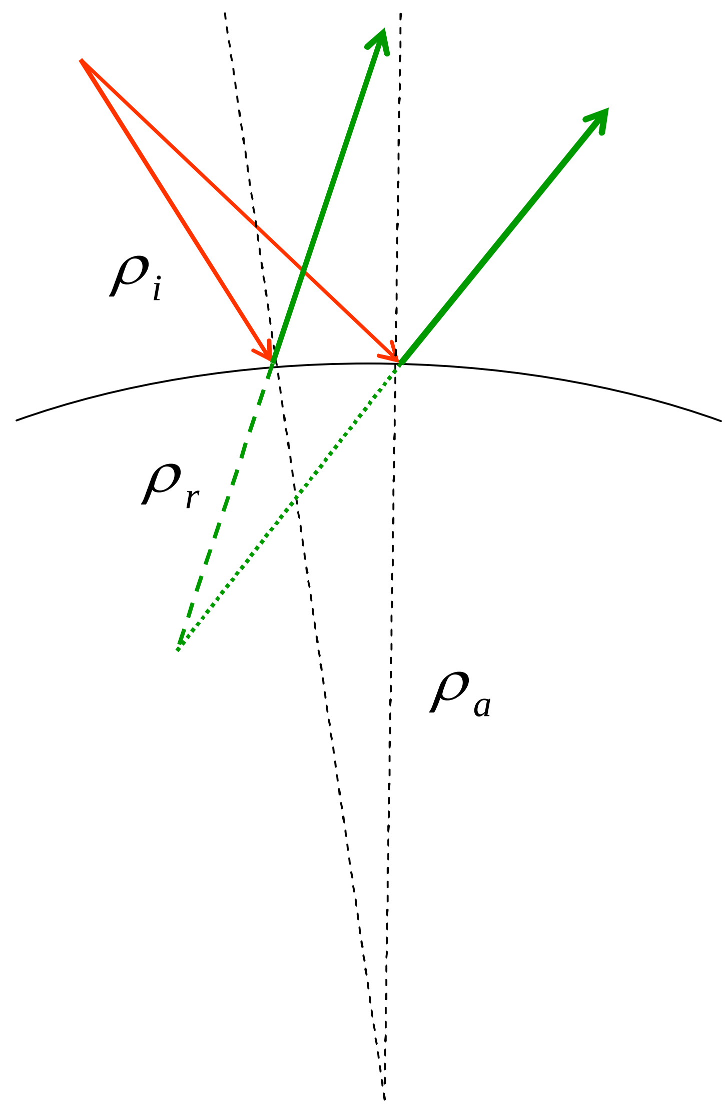
Geometrical optics

- So far the polarization, but how can we calculate the radii of curvature of the reflected surface of constant phase?
- This is a complicated problem in 3D. Let us restrict ourselves to 2D. (For 3D case see Balanis, Advanced Electromagnetic Engineering, chapter 13)
- Now, in 2D, consider a wave impinging on a curved, perfectly conducting surface
- Each ray is reflected according to Snell's law



Geometrical optics

- For the 2D problem consider a narrow tube of rays hitting the conducting surface. Locally, assume the radius of curvature of the incident rays to be ρ_i .
- The local radius of curvature of the surface is ρ_a . Thus, locally, we can view the surface as part of a circular cylinder with the radius ρ_a



Geometrical optics

- If follows that

$$\frac{1}{\rho_i} = \frac{\sin \Delta\varphi_i}{\Delta l \sin \vartheta_1} \approx \frac{\Delta\varphi_i}{\Delta l \sin \vartheta_1'}$$

$$\frac{1}{\rho_r} = \frac{\sin \Delta\varphi_r}{\Delta l \sin \vartheta_2} \approx \frac{\Delta\varphi_r}{\Delta l \sin \vartheta_2'}$$

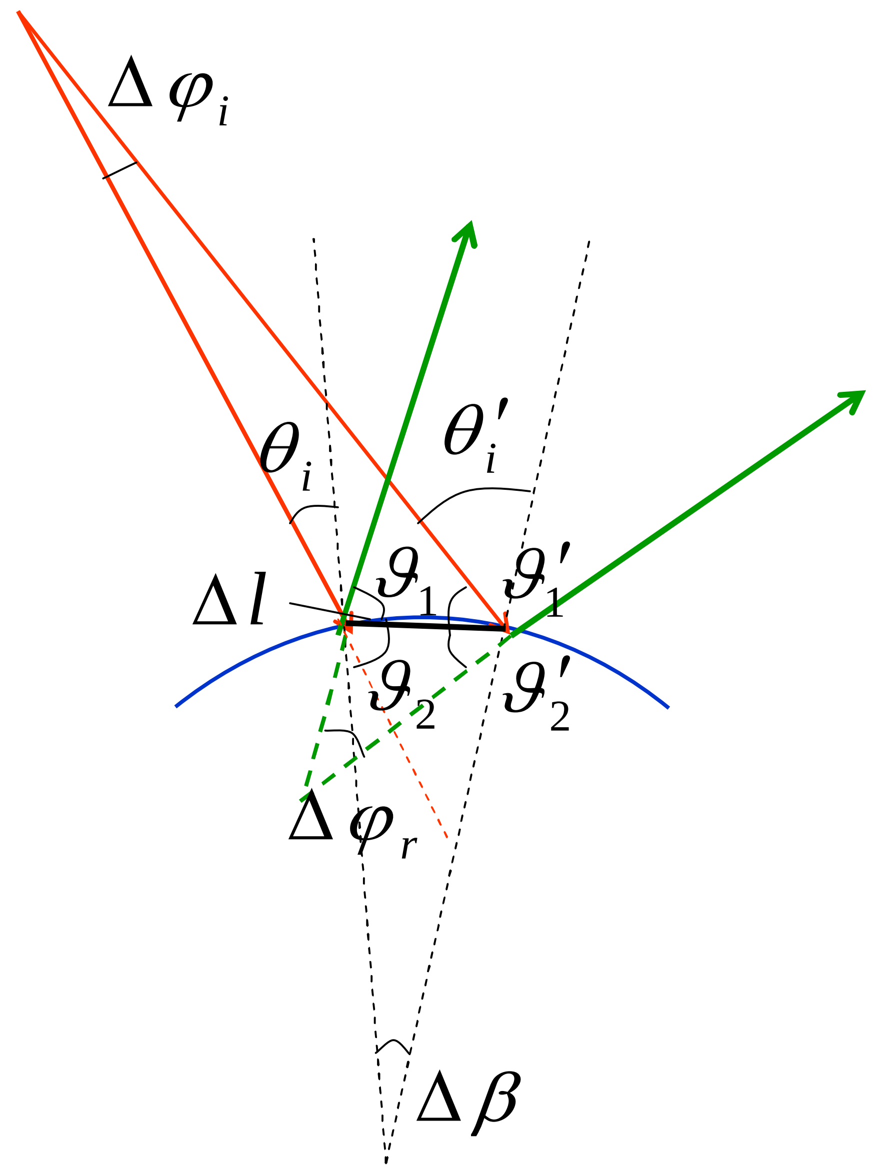
- If can be shown that

$$\vartheta_1 = \pi/2 - \theta_i + \Delta\beta/2$$

$$\vartheta_2 = \pi/2 - \theta_i - \Delta\beta/2$$

$$\vartheta_1' = \pi/2 - \theta_i' + \Delta\beta/2$$

$$\vartheta_2' = \pi/2 - \theta_i' - \Delta\beta/2$$



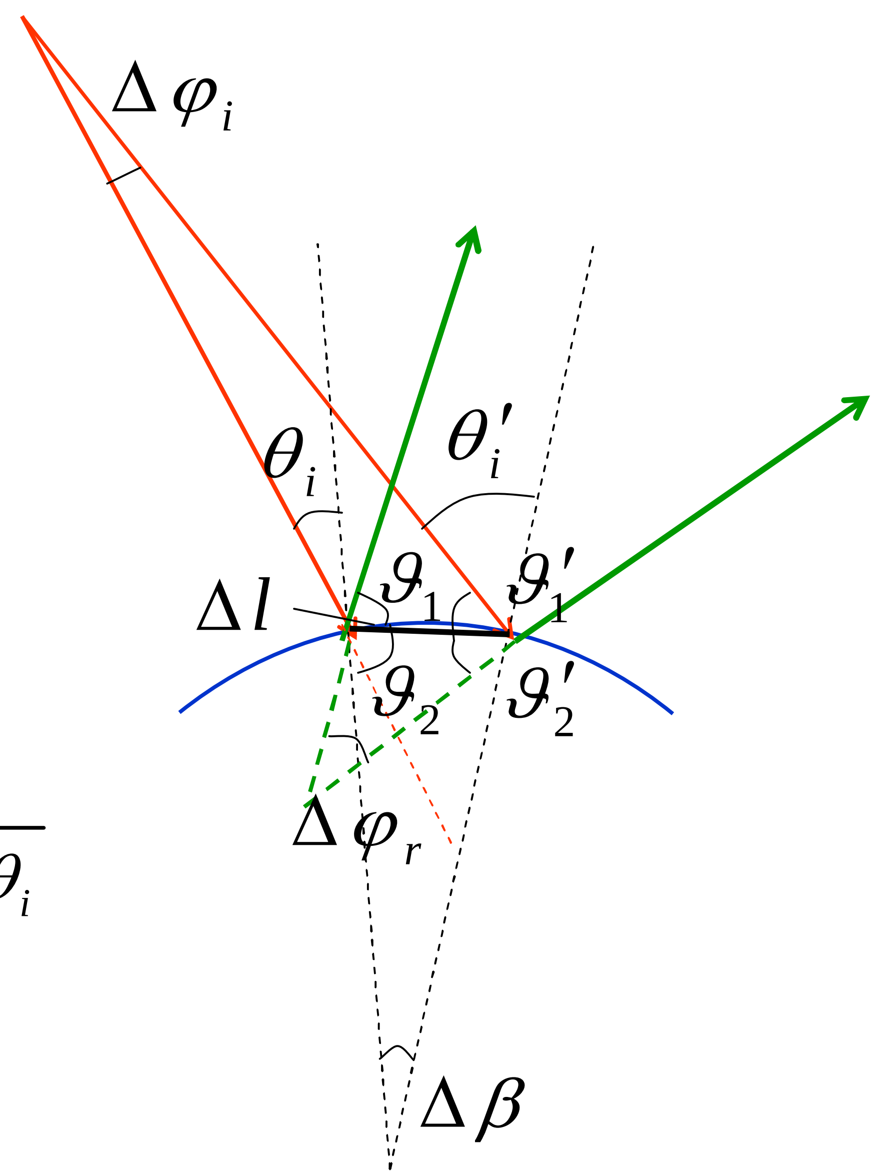
Geometrical optics

- We have

$$\Delta\varphi_r - \Delta\varphi_i = \vartheta_1 - \vartheta_2' - \vartheta_2 + \vartheta_1'$$
$$= 2\Delta\beta$$

- Hence:

$$\frac{1}{\rho_r} - \frac{1}{\rho_i} \approx \frac{2\Delta\beta}{\Delta l \cos\theta_i} = \frac{2}{\rho_a \cos\theta_i}$$



Geometrical optics

- Thus we have all the elements to compute the electric field along any reflected ray
- Considering any point in space, we should trace the reflected rays which pass through that point, compute the electric field using the properties of the surface and the field on the corresponding incident rays, and add up the results
- Note that multiple reflections should also be included (reflected ray may be again reflected by another part of the surface, etc)
- This constitutes the geometrical optics approximation

Geometrical optics

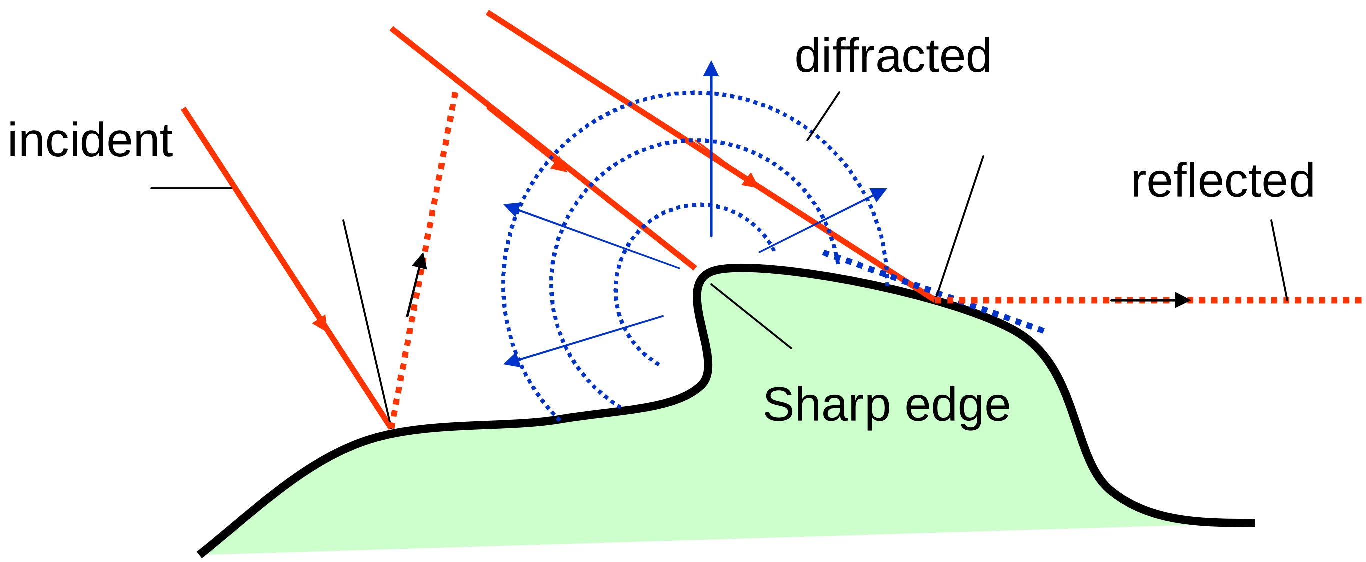
- Limitations:
  - Ray theory just an approximation (first order terms in Kline-Luneberg expansion)
  - In case of caustics (points or lines where infinite rays pass through a single point or line, e.g. in a dish reflector) the theory breaks down: adding up the fields leads to infinity
  - Scattering from surfaces was also approximated by expressions which are (strictly speaking) only valid for a perfectly flat surface. If the surface changes rapidly (compared to wavelength) this approximation is not valid.

Extestion of Geometrical optics

- One of these limitations (scattering from bends or surfaces of small radius of curvature) can be lifted to a certain extent by extending the theory
- The known exact results from structures such as wedges, cylinders, and spheres can be used (in high frequency approximation) to calculate the diffracted field in addition to the usual incident and reflected fields

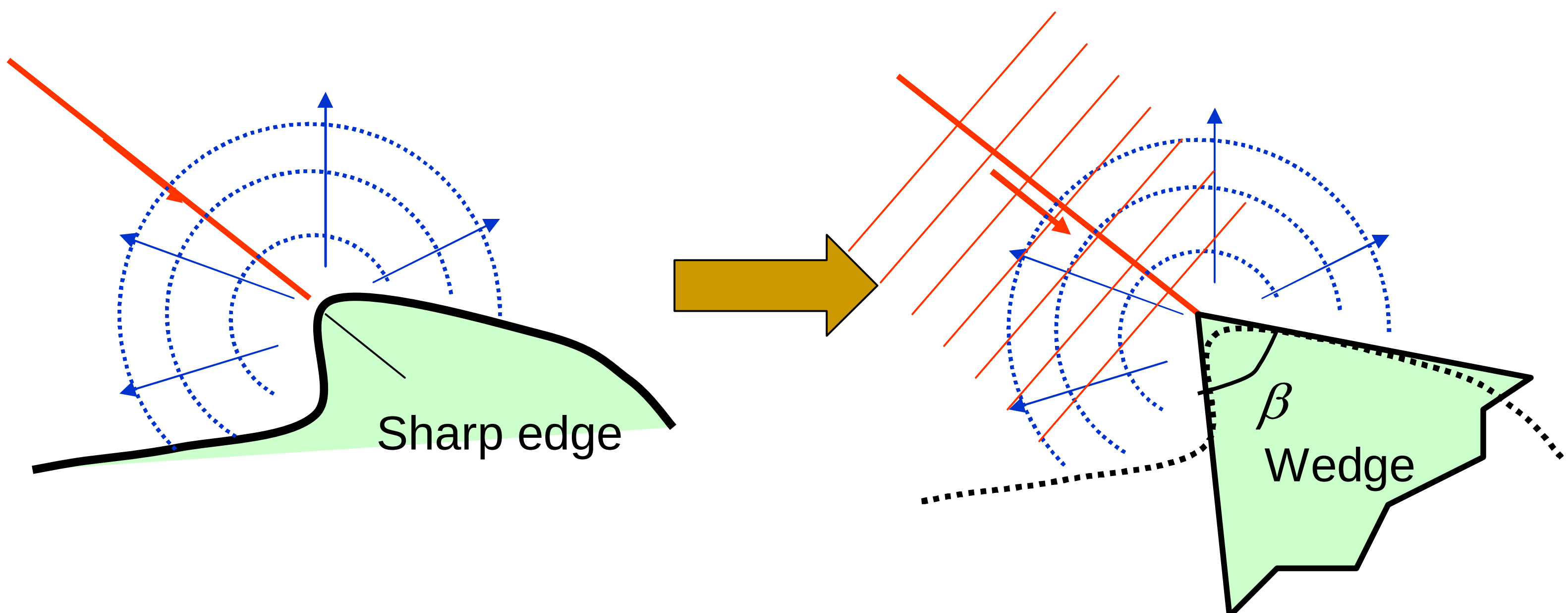
Extension of Geometrical optics

- Consider a surface edge (or rapid bend): it can be considered as the tip of a wedge. We know that in addition to incident and reflected fields we also have a diffracted field which behaves as if its source is on the tip of the wedge



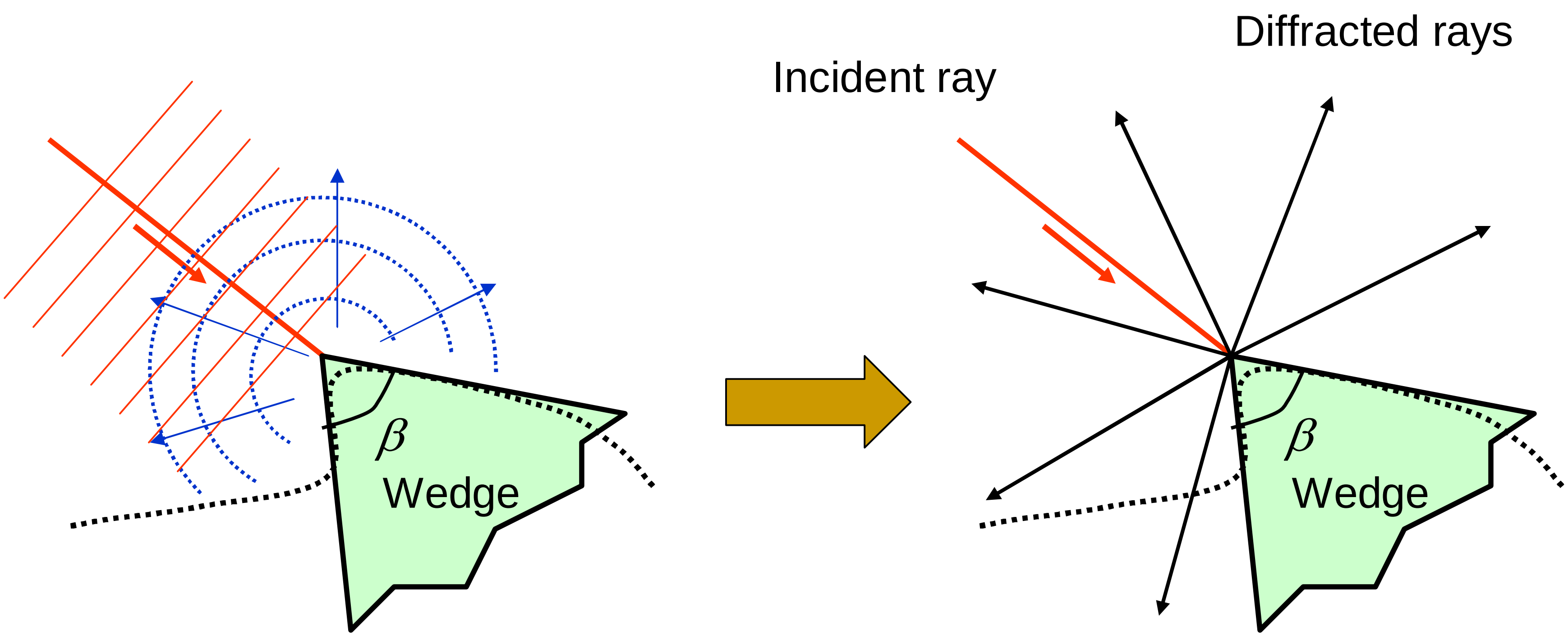
Extension of Geometrical optics

- So, as in case of scattering from a slowly varying surface, we consider the scattering to be a ***local*** phenomena
- We 1st find the ray which hits the wedge tip, and consider that as the incoming incident plane wave in the wedge problem



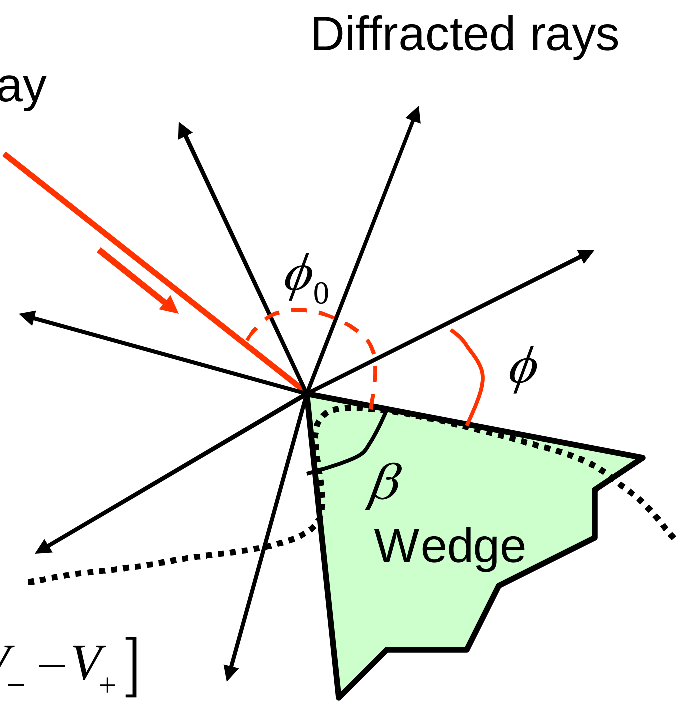
Extension of Geometrical optics

- Next, to use the language of rays, we view the wedge tip as the source of infinite many rays



Extension of Geometrical optics

- To find the field along each ray, we again use the result of a wedge for the diffracted field:



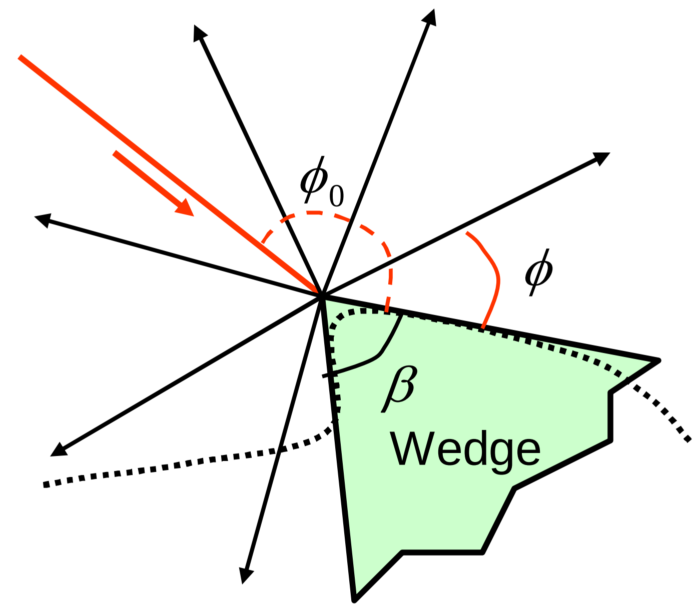
$$E_z^{TM,diff}(\rho,\phi) = \frac{1}{\psi_0}\exp\left(-\frac{j\pi}{4}\right)\sqrt{\frac{\pi}{2k\rho}}\exp(-jk\rho)\left[V_- - V_+\right]$$

$$H_z^{TE,diff}(\rho,\phi) = \frac{1}{\psi_0}\exp\left(-\frac{j\pi}{4}\right)\sqrt{\frac{\pi}{2k\rho}}\exp(-jk\rho)\left[V_- + V_+\right]$$

$$V_\pm = V_B(\phi \pm \phi_0), \quad V_B(\phi \pm \phi_0) = \frac{\sin\left(\pi^2/\psi_0\right)}{\cos\left(\pi^2/\psi_0\right) - \cos\left[\pi(\phi \pm \phi_0)/\psi_0\right]}$$

Extension of Geometrical optics

- These results are in line with the ray theory equations as well: for a cylindrical wave surfaces of constant phase for the diffracted rays are cylindrical and the fields drop as $1/\sqrt{\rho}$
- Only the amplitude depends on the angle of a refracted ray



Extension of Geometrical optics

- Remarks:
  - We considered the 2D case, in the 3D case we get a cone of refracted rays since we have to consider the z-component of the wave-vector of the incident and diffracted waves
  - In 3D the length of the surface edge does not have to be infinite: the theory is still applicable since the diffraction is considered to be local
  - To apply ray theory we replaced the incoming ray by a plane wave hitting the edge: this is strictly speaking not correct. One has to include the curvature of the incoming rays as well

Extension of Geometrical optics

- Remarks (continued):
  - After diffraction, the rays may again be reflected or even diffracted by other parts of the surface, this should be taken into account
  - For a more comprehensive account see for instance:

Balanis, *Advanced Electromagnetic Engineering*, chapter 13