EM Scattering

Homework assignment 1

Problem 1:

For a dielectric object with a volume V and space-dependent dielectric constant $\epsilon_p(\boldsymbol{r}')$, the scattered field in the far field zone is given by

$$\boldsymbol{E}_s(\boldsymbol{r}) = -\frac{k^2 \exp(-jkr)}{4\pi\epsilon_0 r}\hat{\boldsymbol{k}}_s \times \left[\hat{\boldsymbol{k}}_s \times \int_V \exp\left(j\boldsymbol{k}_s \cdot \boldsymbol{r}'\right)\left[\epsilon_p(\boldsymbol{r}') - \epsilon_0\right]\boldsymbol{E}(\boldsymbol{r}')dV'\right]$$

Here $\boldsymbol{k}_s = k\hat{\boldsymbol{k}}_s$ is the wave vector of the scattered wave along the direction of observation $\hat{\boldsymbol{k}}_s$, $\boldsymbol{E}(\boldsymbol{r}')$ is the total electric field inside the object, and $k^2 = \omega^2\epsilon_0\mu_0$ with μ_0, ϵ_0 denoting the permeability, respectively, the dielectric constant of the background medium. This result was obtained by computing the far field generated by polarization currents inside the object. It applies to materials which have no magnetic properties.

i. How should this equation be modified to include materials with magnetic properties, i.e., which are described by a permeability $\mu(\boldsymbol{r})$ different from the background permeability?

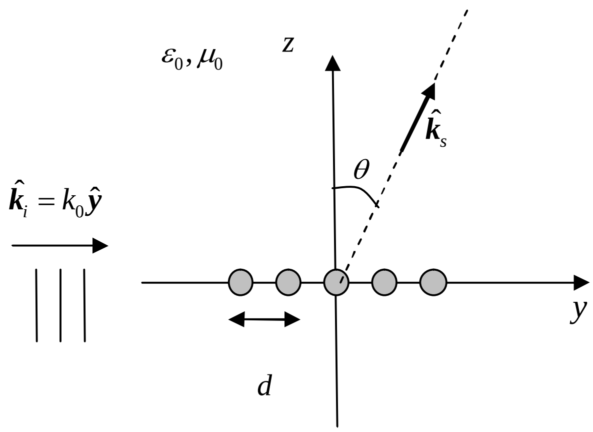
ii. Derive the scattered electric field in the Born approximation for this general case by assuming that $\mu(\boldsymbol{r}) - \mu_0$ is a small quantity. Assume that the incident wave is given by

$$\boldsymbol{E}_i(\boldsymbol{r}) = \boldsymbol{E}_i^0 \exp\left(-j\boldsymbol{k}_i \cdot \boldsymbol{r}\right)$$

Problem 2 :

Consider an array of N small dielectric spheres in vacuum whose centers lie on the y axis. The radius and relative dielectric constant of each sphere is a and ε_r, respectively. The distance between the centers of adjacent spheres is d. An incident plane wave traveling in the y-direction hits the array. The electric field of the incident wave is along the x-axis.

Use the Born approximation and calculate the far-zone scattered electric field on the $y-z$ plane as function of the scattering angle θ. Neglect the interaction between the spheres. Approximate the integrals by assuming the spheres to be much smaller than the wavelength.



Problem 3 :

A plane wave traveling in vacuum the *x*-direction and linearly polarized along the *z*-direction is incident upon an infinitely long dielectric cylinder whose axis coincides with the z-axis. The radius and (relative) dielectric constant of the cylinder are given by a and ε_r, respectively. The amplitude of the incident wave is E_0.

i. Find the far-zone scattered field in Born approximation

ii. Is it possible to define a scattering cross section? If not which quantity must be calculated? Compute this quantity using the results of (i).