EM Scattering

Homework assignment 1

Problem 1:

A plane wave which is propagating in vacuum along the z-axis (wave number k_0) has an electric field which has a left-handed circular polarization: $\boldsymbol{E}_i = E_0\left(\hat{\boldsymbol{x}} + j\hat{\boldsymbol{y}}\right)\exp\left(-jk_0 z\right)$. The wave is scattered by a dielectric cylinder of length L and radius of a. The axis of the cylinder coincides with the z-axis. The permeability of the cylinder equals that of vacuum, but its permittivity is given by the tensor

$$\overline{\overline{\boldsymbol{\varepsilon}}} = \varepsilon_0 \begin{bmatrix} \varepsilon & j\varepsilon_a & 0 \\ -j\varepsilon_a & \varepsilon & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

(i) Using the Born approximation, find the field scattered in the forward and backward directions.

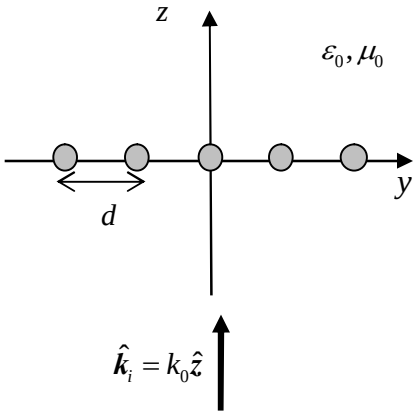
(ii) Repeat the calculation for a right-hand polarized incident field, e.g.,

$$\boldsymbol{E}_i = E_0\left(\hat{\boldsymbol{x}} - j\hat{\boldsymbol{y}}\right)\exp\left(-jk_0 z\right)$$

Problem 2:

Consider an infinite, one-dimensional array of small dielectric spheres in vacuum whose centers lie on the y-axis. The radius and relative dielectric constant of each sphere is a and ε_r, respectively. The distance between the centers of adjacent spheres is d. An incident plane wave whose electric field is polarized along the x-axis and travels in the z-direction hits the array. The distance between the spheres is such that they are in each other's far field.

(i) Use the small particle approximation and calculate the electric field inside each sphere. You must take into account the interaction between the spheres. Hint: calculate the far field generated by each sphere with respect to the center of that sphere.



(ii) Find the field scattered by each sphere in the far zone

Problem 3:

A plane wave traveling in vacuum the *x*-direction and linearly polarized along the *y*-direction is incident upon an infinitely long dielectric cylinder whose axis coincides with the z-axis. The radius and (relative) dielectric constant of the cylinder are given by a and ε_r, respectively. The amplitude of the incident wave is E_0.

i. Find the far-zone scattered field in Born approximation

ii. Is it possible to define a scattering cross section? If not which quantity must be calculated? Compute this quantity using the results of (i).