EM Scattering
Homework assignment 2

Problem 1:
A plane wave traveling in the $x$ direction in vacuum is scattered by a perfectly conducting, infinite cylinder with the radius of 1cm whose axis coincides with the z-axis. The frequency of the incident wave is 20GHz. The amplitude of the incident electric field is $E_0$. For both cases where the electric field is polarized along the y- and z-axes:

- Calculate the electric field in the far zone and plot its amplitude as a function of angle
- Calculate the current density on the surface of the cylinder and plot it as a function of angle

(You need routines for Hankel and Bessel functions. Matlab would be OK.)

Problem 2:
Consider a region in vacuum confined by two infinite, perfectly conducting plates at $z = 0$ and $z = b$.

We would like to use cylindrical coordinates $(\rho, \phi, z)$ to analyze a scattering problem in this configuration.

(i) Find the general solution of the vector-wave equation for the electric field in terms of the $M$ and $N$ functions in cylindrical coordinates. Use $\hat{z}$ for the constant vector field needed to define these functions. Explicitly express $M$ and $N$ in terms of $\rho, \phi, z$.

Consider now a perfectly conducting cylinder of radius $a$ and height $b$ put in between the two plates. The axis of the cylinder coincides with the $z$-axis. An incident wave whose fields are given by

\[ E_i = \hat{y}E_0 \sin \left( \frac{\pi z}{b} \right) \exp(-j\beta x), \quad \beta = \sqrt{k_0^2 - \left( \frac{\pi}{b} \right)^2}, \]

\[ H_i = \frac{E_0}{j\omega\mu_0} \left[ \hat{x} \frac{\pi}{b} \cos \left( \frac{\pi z}{b} \right) + \hat{z} \beta \sin \left( \frac{\pi z}{b} \right) \right] \exp(-j\beta x), \]
travels along $x$ with the wave number $\beta$ and hits the cylinder ($k_0 > \pi / b$).

(ii) Write down the scattered electric field as a series with unknown coefficients and find these coefficients by imposing appropriate boundary conditions on the surface of the cylinder.

**Hint**: if needed you may use

$$\exp(-j\beta x) = \sum_{m=-\infty}^{\infty} (-j)^m J_m(\beta \rho) \exp(-jm\phi)$$