EM Scattering

Homework assignment 2

Problem 1:

A plane wave traveling in the x direction in vacuum is scattered by a perfectly conducting, infinite cylinder with the radius of 1cm whose axis coincides with the z-axis. The frequency of the incident wave is 20GHz. The amplitude of the incident electric field is E_0. For **both** cases where the electric field is polarized along the y- and z-axes:

- Calculate the electric field in the far zone and plot its amplitude as function of angle
- Calculate the current density on the surface of the cylinder and plot it as function of angle

(You need routines for Hankel and Bessel functions. Matlab would be OK.)

Problem 2:

Consider a region in vacuum confined by two infinite, perfectly conducting plates at $z=0$ and $z=b$. We would like to use cylindrical coordinates (ρ,ϕ,z) to analyze a scattering problem in this configuration.

(i) Find the general solution of the vector-wave equation for the electric field in terms of the $\boldsymbol{M}$ and $\boldsymbol{N}$ functions in cylindrical coordinates. Use $\hat{\boldsymbol{z}}$ for the constant vector field needed to define these functions. Explicitly express $\boldsymbol{M}$ and $\boldsymbol{N}$ in terms of ρ,ϕ,z.

Consider now a perfectly conducting cylinder of radius a and height b put in between the two plates. The axis of the cylinder coincides with the z-axis. An incident wave whose fields are given by

$$\boldsymbol{E}_i = \hat{\boldsymbol{y}}E_0 \sin\left(\frac{\pi z}{b}\right)\exp(-j\beta x),\ \beta = \sqrt{k_0^2 - \left(\frac{\pi}{b}\right)^2},$$

$$\boldsymbol{H}_i = \frac{E_0}{j\omega\mu_0}\left[\hat{\boldsymbol{x}}\frac{\pi}{b}\cos\left(\frac{\pi z}{b}\right) + \hat{\boldsymbol{z}}j\beta\sin\left(\frac{\pi z}{b}\right)\right]\exp(-j\beta x),$$

travels along x with the wave number β and hits the cylinder ($k_0 > \pi / b$).

(ii) Write down the scattered electric field as a series with unknown coefficients and find these coefficients by imposing appropriate boundary conditions on the surface of the cylinder.

Hint: if needed you may use

$$\exp\left(-j\beta x\right) = \sum_{m=-\infty}^{\infty} (-j)^m J_m(\beta\rho) \exp\left(-jm\phi\right)$$