

# EM Scattering

## Homework assignment 2

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### Problem 1:

A plane wave traveling in the  $x$  direction in vacuum is scattered by a perfectly conducting, infinite cylinder with the radius of 1cm whose axis coincides with the  $z$ -axis. The frequency of the incident wave is 20GHz. The amplitude of the incident electric field is  $E_0$ . For **both** cases where the electric field is polarized along the  $y$ - and  $z$ -axes:

- Calculate the electric field in the far zone and plot its amplitude as function of angle
- Calculate the current density on the surface of the cylinder and plot it as function of angle

(You need routines for Hankel and Bessel functions. Matlab would be OK.)

### Problem 2:

Consider a region in vacuum confined by two infinite, perfectly conducting plates at  $z = 0$  and  $z = b$ . We would like to use cylindrical coordinates  $(\rho, \phi, z)$  to analyze a scattering problem in this configuration.

(i) Find the general solution of the vector-wave equation for the electric field in terms of the  $\mathbf{M}$  and  $\mathbf{N}$  functions in cylindrical coordinates. Use  $\hat{\mathbf{z}}$  for the constant vector field needed to define these functions. Explicitly express  $\mathbf{M}$  and  $\mathbf{N}$  in terms of  $\rho, \phi, z$ .

Consider now a perfectly conducting cylinder of radius  $a$  and height  $b$  put in between the two plates.

The axis of the cylinder coincides with the  $z$ -axis. An incident wave whose fields are given by

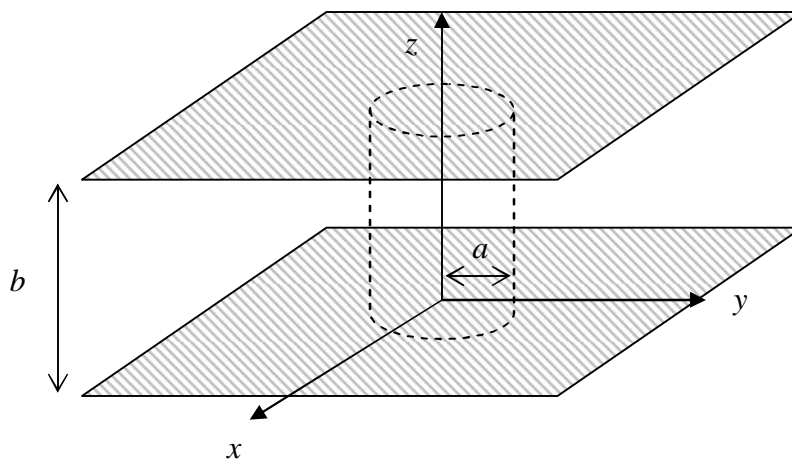
$$\mathbf{E}_i = \hat{\mathbf{y}}E_0 \sin\left(\frac{\pi z}{b}\right) \exp(-j\beta x), \quad \beta = \sqrt{k_0^2 - \left(\frac{\pi}{b}\right)^2},$$
$$\mathbf{H}_i = \frac{E_0}{j\omega\mu_0} \left[ \hat{\mathbf{x}} \frac{\pi}{b} \cos\left(\frac{\pi z}{b}\right) + \hat{\mathbf{z}} j\beta \sin\left(\frac{\pi z}{b}\right) \right] \exp(-j\beta x),$$

travels along  $x$  with the wave number  $\beta$  and hits the cylinder ( $k_0 > \pi/b$ ).

(ii) Write down the scattered electric field as a series with unknown coefficients and find these coefficients by imposing appropriate boundary conditions on the surface of the cylinder.

**Hint:** if needed you may use

$$\exp(-j\beta x) = \sum_{m=-\infty}^{\infty} (-j)^m J_m(\beta\rho) \exp(-jm\phi)$$



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