Transmission line model for extraction of transmission characteristics in photonic crystal waveguides with stubs: optical filter design

Amin Khavasi,* Mehdi Miri, Mohsen Rezaei, Khashayar Mehrany, and Bizhan Rashidian

Electrical Engineering Department, Sharif University of Technology, P.O. Box: 11155-4363 Tehran, Iran

Received November 28, 2011; revised February 19, 2012; accepted February 20, 2012; posted February 23, 2012 (Doc. ID 158999); published April 6, 2012

A simple and efficient transmission line model is proposed here to study how the transmission characteristics of photonic crystal waveguides are tailored by introduction of stubs patterned in the photonic crystal lattice. It is shown that band-pass and band-stop optical filters can be easily designed and optimized when stubs of appropriate length are brought in. Since the lengths of the designed stubs are not necessarily integer multiples of the photonic crystal lattice constant, a geometric shift in a portion of the photonic crystal structure is shown to be essential. The proposed model is verified by using a rigorous numerical method. An excellent agreement is observed between the numerical results and the transmission characteristics as extracted by the proposed model. © 2012 Optical Society of America

OCIS codes: 130.7408, 130.5296, 050.5298.

The capability of photonic crystals in manipulating the flow of light has spurred much interest in controllable conduction of optical signal in a manner analogous to the conduction of electrical signals in electrical circuits. It is therefore no surprise that different types of photonic circuits have been reported in photonic crystal-based structures [1–5]. In particular, the idea of using stubs in tuning the transmission characteristics of a photonic crystal waveguide (PCW) is borrowed from microwave theory to realize optical filters. It was first reported by removing a row of rods in a square lattice photonic crystal to mimic a single stub [6]. The potential for using stubs to generate zeros in optical transmission of dielectric PCW; however, was not investigated until later by Ogusu et al., who brought double stubs into play and designed optical bandstop filters in [7]. They proposed a simple equivalent circuit for qualitative explanation of how the transmission characteristics are affected by using stubs but failed to employ the equivalent circuit in quantitative extraction of the transmission spectrum. Since the availability of a simple circuit model can render applying burdensome numerical methods unnecessary, thus facilitating optical filter design, a transmission line model capable of providing the transmission spectra of stub-tuned PCW is warmly welcome. This is presented here using a scalar impedance theory.

Consider a two-dimensional square lattice photonic crystal made of circular dielectric rods in air. A row of rods is removed to create a PCW supporting a single E-polarized mode whose electric field is parallel to the axis of the rods. The transmission spectrum of such PCW can then be tailored by the introduction of stubs, which are easily created by removing a couple of rods along the line perpendicular to the axis of PCW. This is illustrated in Fig. 1, where a typical PCW with four stubs is shown. Note that stubs are placed on both sides of the PCW to avoid unwanted cross talk between neighboring stubs. In accordance with the figure, the structure is here modeled by a transmission line with an appropriate shunt admittance corresponding to each stub. In the proposed model, the propagation constant and the characteristic impedance of the transmission line are denoted by $\beta_W$ and $Z_W$, respectively. $Y_{\text{stub}}$ denotes the shunt admittance representing each created stub, and $L_i$ stands for the distance between the $i$th and the $(i + 1)$th stub.

Since the transmission spectrum of the structure is controlled by normalized impedances, we assume that the characteristic impedance of the transmission line is $Z_W = 1$. Its propagation constant, on the other hand, is set equal to the propagation constant of the E-polarized mode supported by the PCW and can therefore be easily determined using any of the many methods available for extraction of PCW modes [8,9].

The stub shunt admittance is not very easy to obtain. It should correspond to the input admittance of a terminated transmission line representing the introduced stub. Since the stub is created by removing a row of rods in the same photonic crystal where the PCW was originally carved, the electrical characteristics of the transmission line representing the introduced stub (i.e., its characteristic impedance and propagation constant) are not different from those of the transmission line representing the

![Fig. 1. A typical PCW with four stubs and its corresponding transmission line model.](image-url)
PCW (i.e., $Z_W$ and $\beta_W$). The length of the transmission line corresponding to the introduced stub is expected to match the geometrical length of the stub, which is indicated by $L_{\text{stub}}$. Although each stub is created by removing rods along a column of rods in the original photonic crystal, $L_{\text{stub}}$ is not necessarily an integer multiple of the lattice constant $a$, because the column of rods can be shifted by an arbitrary value of $\delta$ within the interval $(-0.5a, 0.5a)$. This is demonstrated in Fig. 2(a), where $L_{\text{stub}} = 2a + \delta$. It should, however, be noted that the in- and outcoupling of light back and forth between the PCW and the stub imposes a certain delay. The effective length of the stub should therefore be slightly larger than its geometrical length and is set to $L_{\text{stub}} + L_{\text{extra}}$, where $L_{\text{extra}}$ is hereafter referred to as the extra length of the stub and takes the effects of the abovementioned delay into account. While the geometrical length of the typical stub shown in Fig. 2(a) is clearly $L_{\text{stub}} = 2a + \delta$, its extra length depends on the electromagnetic characteristics of the wave propagating within the structure and cannot be easily determined.

Inasmuch as the extra length is to account for the time delay incurred because of the coupling of the electromagnetic energy in and out of the stub, a logical suggestion for $L_{\text{extra}}$ is to set it proportional to the inverse of the group velocity. It is therefore written as $L_{\text{extra}} = \zeta a/v_g$, where $\zeta$ is a constant to be numerically obtained, $v_g = v_0/2\pi c$, $c$ is the speed of light, and $v_g$ is the group velocity. The independence of the $L_{\text{extra}}$ from the geometrical length of the stub, however, makes a certain correction necessary because the overall shunt admittance of the stub is expected to be $Y_{\text{stub}} = 0$ whenever the geometrical length of the stub is $L_{\text{stub}} = 0$. If $Y_1$ is the admittance of the transmission line whose length is $L_{\text{stub}} + L_{\text{extra}}$ and its characteristic impedance and propagation constant are $Z_W$ and $\beta_W$ in accordance with Fig. 2(b), the overall shunt admittance of the stub should be $Y_{\text{stub}} = Y_1 - Y_2$, where $Y_2$ is the input admittance of the transmission line shown in Fig. 2(c). It is in this fashion that the abovementioned necessary correction is applied.

The only remaining parameter is the load impedance, $Z_L$, which is needed to determine $Y_1$ and $Y_2$. Given that there is a strong similarity between the sought-after load impedance, $Z_L$, and the scalar impedance of terminated PCWs given in [10], the zeroth-order reflection coefficient, $R(\omega_n, \beta_W(\omega_n))$, of an appropriate semi-infinite photonic crystal can be used to determine the load impedance, $Z_L$:

$$Z_L = \frac{1 + R(\omega_n, \beta_W(\omega_n))}{1 - R(\omega_n, \beta_W(\omega_n))} Z_W. \quad (1)$$

The appropriate semi-infinite photonic crystal whose zeroth-order reflection coefficient, $R(\omega_n, \beta_W(\omega_n))$, yields load impedance, $Z_L$, is shown in Fig. 3. It does not differ from the original photonic crystal in which the PCW has been carved. It is illuminated by an E-polarized uniform plane wave having the $z$ component of its wave number set at $\beta_W(\omega_n))$. In this letter, the Legendre polynomial expansion method is employed to extract diffraction orders and their reflection coefficients [11]. The MATLAB code for this method is available at http://ee.sharif.edu/~khavasi/index_files/LPEM.zip.

In summary, three parameters are to be calculated: $Z_L$, $\beta_W$, and $\zeta$. Evaluation of $Z_L$ and $\beta_W$ does not pose difficulty, as these parameters are extracted in fully periodic structures that can be very efficiently analyzed. Calculation of $\zeta$ is the most time-consuming step, as it requires a rigorous numerical simulation. This point is further expounded below.

Now that the parameters of the transmission line model have all been provided, the transmission spectrum of the PCW can be tailored by the introduction of stubs. In accordance with the schematic diagram of the stub-tuned PCW shown in Fig. 1, four stubs are employed as an example to design bandpass and bandstop filters. Here, without losing the generality, we assume that the PCW is made by removing a row of rods in a photonic crystal made of GaAs rods with relative permittivity $\varepsilon_r = 11.56$ and radius $r = 0.2a$. The transmission characteristics of the stub-tuned PCW are then studied within the frequency range of $0.35 < \omega < 0.42$, where the group velocity dispersion is minimum [1]. The center frequency and the bandwidth of the desired filters are chosen to be 0.38, and 0.03, respectively. Extraction of $Z_L$ and $\beta_W$ on a machine with a dual Core 2 GHz Intel Processor takes about 0.1 sec and 4 sec, respectively, at each single frequency. To determine the unknown factor $\zeta$, which is needed for calculation of $L_{\text{extra}}$, the PCW with a single stub of an arbitrary geometrical length is first analyzed by applying a rigorous numerical analysis such as the finite difference time domain (FDTD) method. Using a $50 \times 50$ grid

![Fig. 2. (a) A typical stub whose length is $L_{\text{stub}} = 2a + \delta$. The admittance of the stub is $Y_{\text{Stub}} = Y_1 - Y_2$. (b) The loaded transmission line whose input admittance is $Y_{\text{in}} = Y_1$, and (c) the loaded transmission line whose input admittance is $Y_{\text{in}} = Y_2$.](image)

![Fig. 3. A semi-infinite photonic crystal whose reflection coefficient; $R$, yields the load impedance $Z_L$.](image)
point for each unit cell and 100 000 time steps, a FORTRAN code [12] is employed. It is in this fashion that \( \zeta = 0.021 \) is obtained by fitting the zeros of the transmission spectrum of the transmission line model to those of the stub-tuned PCW. This step takes about an hour on the same machine; fortunately, this step has to be taken only once. For simplicity’s sake, we set \( L_1 = L_2 = L_3 = L = na \), where \( n \) is an arbitrary integer.

It is known that stubs result in a set of zeros in the transmission spectrum of the stub-tuned PCW [6]. Now, since the frequency at which the transmission of the stub-tuned PCW becomes zero can be easily determined using the here-proposed transmission line model, bandpass or bandstop filters can be straightforwardly designed. The length of each stub is chosen to have the zeros of the transmission spectrum of the transmission line model lying outside/inside of the desired bandwidth to design a bandpass/bandstop filter. Applying the standard transmission line theory reveals that there is a multitude of choices for the appropriate geometrical length of each stub. For instance, the zeros of the stub-tuned PCW with \( L_{\text{stub}} = 3.75a \) and \( L_{\text{stub}} = 2.85a \) are all lying outside and inside, respectively, of the desired bandwidth. The most fitting choice for the length of each stub is found by optimization of the transmission spectrum, which is quickly performed thanks to the availability of closed-form expressions for the transmission spectrum of the transmission line model. The geometrical lengths of the four stubs of the optimum bandpass filter are \( 3.775a, 3.659a, 3.7107a, \) and \( 3.8123a \) for the first, second, third, and fourth stubs, respectively. The geometrical lengths of the four stubs of the optimum bandstop filter are \( 2.827a, 3.365a, 2.989a, \) and \( 2.687a \) for the first, second, third, and fourth stubs, respectively. The optimum distance between adjacent stubs is \( L = 3a \) for both filters.

The transmission spectrum of the designed bandpass and bandstop filters are plotted in Figs. 4 and 5, respectively. The results are obtained by the proposed model (solid line) and the FDTD method (circles). These figures illustrate the fact that the results of the proposed model are in very good agreement with the FDTD results. Interestingly, rigorous analysis of the designed bandpass and bandstop filters is more time-consuming than the design procedure.

In conclusion, we have proposed a transmission line model for a stub-tuned PCW. Each stub is considered as a shunt admittance. It is shown that the length of each stub is not necessarily an integer multiple of the lattice constant. This is proved to be quite important since optimum bandstop and bandpass filters have stubs whose lengths are not integer multiples of the lattice constant.

References