Exercise 1. Determine a state space realization and the corresponding transfer function for the following system. Is this system stable?

![Figure 1: Exercise 1](image)

Exercise 2. Discuss the stability analysis of the following characteristic equations.

(a) $S^{10} + 3S^9 - 6S^8 + 4S^7 + S^5 + 12S^4 + S^3 + 9S^2 - 1 = 0$

(b) $S^4 + 2S^3 + 2S^2 + 4S + 5 = 0$

(c) $S^8 + 3S^7 + 5S^6 + 9S^5 + 9S^4 + 9S^3 + 7S^2 + 3S + 2 = 0$

Exercise 3. For the following system with the reference input $r$ and the output $y$,

$$\ddot{y} + 6\dot{y} + 11y + Ky = \ddot{r} - r$$

determine $K$ such that all roots of the characteristic equation lie inside the region $\Omega$. 
Exercise 4. For both positive and negative gain, sketch (by hand!) the root loci for the pole-zero plots shown in figure 3.

Exercise 5. Sketch (by hand) the loci of the closed-loop poles for $K > 0$ for systems with the following open-loop transfer functions:

(a) $G(s) = \frac{K(s + 2)}{s(s + 1)(s + 3)(s + 4)}$

(b) $G(s) = \frac{K(s + 3)}{s(s + 2)(s^2 + 2s + 2)}$

(c) $G(s) = \frac{K(s + 2)^2}{s^2(s^2 + 2s + 2)}$
**Exercise 6.** The linearized model for the attitude of a rocket (neglecting the moment of inertia of the engine) has a transfer function of the form

\[ G(s) = \frac{K}{s^2 - a^2}. \]

Let \( K = 1 \) and \( a = 1 \). The open-loop system is clearly unstable, so we need to design a controller \( G_c(s) \) to stabilize the system, as shown in Figure 4. The input \( \delta \) is the angle of the thrust vector, and the output \( \theta \) is the rocket attitude angle.

![Figure 4: Exercise 7](image)

(a) Suppose we use a proportional controller: \( G_c(s) = K_c \). Sketch the root locus for the closed-loop system. Can we stabilize the rocket attitude by using this type of controller?

(b) Now suppose we try to cancel the pole in the right half-plane, using a controller of the form \( G_c(s) = \frac{K_c(s-1)}{s+p} \), where \( p \) is some positive real number. Sketch the root locus for \( p = 2 \). Is this type of controller likely to work in real-life? Why or why not?

(c) Now use a PD controller of the form \( G_c(s) = K_p + K_ds \). Using root locus methods, determine the values of \( K_p \) and \( K_d \) so that the closed-loop system will be stable with a 2\% settling time of 4 seconds, and a damping ratio \( \zeta \approx 0.7 \).