Electromagnetics

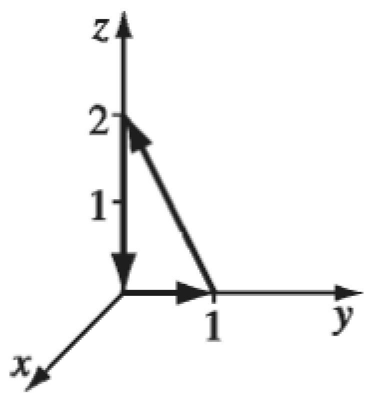
Homework 1

Due:

1- Compute the line integral of

$$\mathbf{v} = r\cos^2(\theta)\hat{\mathbf{r}} - r\cos(\theta)\sin(\theta)\,\hat{\boldsymbol{\theta}} + 3r\hat{\boldsymbol{\varphi}}$$

for the path shown in the following figure



2- Let $\mathbf{A}(\mathbf{r}) = \mathbf{c}\exp(i\mathbf{k}\,.\,\mathbf{r})$ where $\mathbf{c}$ is constant. Show that, in every case, the replacement $\nabla \rightarrow i\mathbf{k}$ produces the correct answer for $\nabla \cdot \mathbf{A}$, $\nabla \times \mathbf{A}$, $\nabla \times (\nabla \times \mathbf{A})$, $\nabla(\nabla \cdot \mathbf{A})$, and $\nabla^2\mathbf{A}$.

3- Show that:

a) $\int_V (\nabla \mathrm{T})\, dv = \oint_{\partial V} \mathrm{T} d\vec{s}$

b) $\int_V (\nabla \times \vec{\mathbf{v}})\, dv = -\oint_{\partial V} \vec{\mathbf{v}} \times d\vec{\boldsymbol{s}}$

c) $\int_V [\mathrm{T}\nabla^2\mathrm{U} + (\nabla T).(\nabla \mathrm{U})]\, dv = \oint_{\partial V} (\mathrm{T}\nabla\mathrm{U}) d\vec{\boldsymbol{s}}$

d) $\int_S \nabla\mathrm{T} \times d\vec{\boldsymbol{s}} = -\oint_{\partial S} \mathrm{T} d\vec{\boldsymbol{l}}$

4- Prove the following identities in Cartesian coordinates:

a) $\nabla.(\mathbf{F}\times\mathbf{G}) = \mathbf{F}.(\nabla\times\mathbf{G}) - \mathbf{G}.(\nabla\times\mathbf{F})$

b) $\nabla\times(\mathbf{F}\times\mathbf{G}) = \mathbf{F}(\nabla.\mathbf{G}) - \mathbf{G}(\nabla.\mathbf{F}) + (\mathbf{G}.\nabla)\mathbf{F} - (\mathbf{F}.\nabla)\mathbf{G}$

5- a) Compute the Laplacian of

$$f_a(r) = -\frac{1}{4\pi}\frac{1}{\sqrt{r^2+a^2}}$$

b) Using part (a) show that

$$-\nabla^2\frac{1}{4\pi r} = \delta(\vec{\boldsymbol{r}})$$