Image Enhancement in Spatial Domain

- **Image Enhancement:**
  - No Explicit definition

- **Methods**
  - Spatial Domain:
    - Linear
    - Nonlinear
  - Frequency Domain:
    - Linear
    - Nonlinear
Image Enhancement in Spatial Domain

- Spatial Domain Process

\[ g(x, y) = T(f(x, y)) \]

**FIGURE 3.1** A 3 x 3 neighborhood about a point \((x, y)\) in an image.
Image Enhancement in Spatial Domain

- For $1 \times 1$ neighborhood: $s = T(r)$
  - Contrast Enhancement/Stretching/Point process
- For $w \times w$ neighborhood:
  - Filtering/Mask/Kernel/Window/Template Processing

![Graphical representation of image enhancement](image.png)
Image Enhancement in Spatial Domain

FIGURE 3.3 Some basic gray-level transformation functions used for image enhancement.
• **Image Negatives:** \( s = L - 1 - r \)

**FIGURE 3.4**
(a) Original digital mammogram.
(b) Negative image obtained using the negative transformation in Eq. (3.2-1).
(Courtesy of G.E. Medical Systems.)
Image Enhancement in Spatial Domain

- Log Transform: $s = c \log(1 + r)$
Image Enhancement in Spatial Domain

- **Power-Law Transform:**
  \[ s = c (r + \epsilon)^\gamma \]
Image Enhancement in Spatial Domain

**Figure 3.8**
(a) Magnetic resonance (MR) image of a fractured human spine.
(b)-(d) Results of applying the transformation in Eq. (3.2-3) with $c = 1$ and $\gamma = 0.6, 0.4,$ and $0.3,$ respectively. (Original image for this example courtesy of Dr. David R. Pickens, Department of Radiology and Radiological Sciences, Vanderbilt University Medical Center.)
Image Enhancement in Spatial Domain

**FIGURE 3.9**
(a) Aerial image.
(b)–(d) Results of applying the transformation in Eq. (3.2-3) with $c = 1$ and $\gamma = 3.0, 4.0, \text{and } 5.0$, respectively.
(Original image for this example courtesy of NASA.)
Image Enhancement in Spatial Domain

• Another Medical Example:
Image Enhancement in Spatial Domain

Contrast Stretching

Original

FIGURE 3.10
Contrast stretching. (a) Form of transformation function. (b) A low-contrast image. (c) Result of contrast stretching. (d) Result of thresholding. (Original image courtesy of Dr. Roger Heady, Research School of Biological Sciences, Australian National University, Canberra, Australia.)
Image Enhancement in Spatial Domain

Gray Level Slicing

FIGURE 3.11
(a) This transformation highlights range \([A, B]\) of gray levels and reduces all others to a constant level.
(b) This transformation highlights range \([A, B]\) but preserves all other levels.
(c) An image.
(d) Result of using the transformation in (a).
**Image Enhancement in Spatial Domain**

- **Histogram Processing:**
  - Enhancement based on statistical Properties:
    - Local
    - Global
  - Histogram Definition:

\[
h(r_k) = n_k, \quad r_k \in [0, L-1], \quad n_k \in [0, M \times N]
\]

\[
p(r_k) = \frac{n_k}{n} = \frac{1}{M \times N} n_k
\]
Image Enhancement in Spatial Domain

- Histogram Example:
Image Enhancement in Spatial Domain

• Histogram Visual Meaning:
  – Dark
  – Bright
  – Low Contrast
  – High Contrast
Image Enhancement in Spatial Domain

- Histogram Equalization:
  - First assume continuous case.
  - Seek for a suitable transform (Except for negative):

\[
s = T(r) : \begin{cases} 
T(0) = 0 \\
T(1) = 1 \\
T(r) \geq 0 \\
T'(r) \geq 0 
\end{cases}
\]

\[T(0) = 0 \]
\[T(1) = 1 \]
\[T(r) \geq 0 \]
\[T'(r) \geq 0 \]

**FIGURE 3.16** A gray-level transformation function that is both single valued and monotonically increasing.
Image Enhancement in Spatial Domain

**Histogram Equalization**

\[ s = T(r) = \int_0^r p_r(w)dw \Rightarrow P_S(s) \sim U(0,1) \]

\[ S_k = T(r_k) = \sum_{j=0}^{k} P_r(r_j) = \sum_{j=0}^{k} \frac{n_k}{n}, \quad k = 0,1,\cdots,L-1 \]

\[ \hat{S}_k = \text{Int} \left[ \frac{S_k - S_k^{\text{min}}}{1 - S_k^{\text{min}}} (L-1) + 0.5 \right] \]
Image Enhancement in Spatial Domain

- Transformation Function:

**FIGURE 3.18**
Transformation functions (1) through (4) were obtained from the histograms of the images in Fig.3.17(a), using Eq. (3.3-8).
Image Enhancement in Spatial Domain

- Medical Examples:
Image Enhancement in Spatial Domain

- Example (Mars Image and its Histogram):

![Mars Image](image1.png)

**FIGURE 3.20** (a) Image of the Mars moon Photos taken by NASA's *Mars Global Surveyor*. (b) Histogram. (Original image courtesy of NASA.)
Image Enhancement in Spatial Domain

- **Histogram Equalization:**

![Graph showing histogram equalization](image)

**FIGURE 3.21**
(a) Transformation function for histogram equalization. (b) Histogram-equalized image (note the washed-out appearance). (c) Histogram of (b).
**Image Enhancement in Spatial Domain**

**FIGURE 3.22**
(a) Specified histogram.
(b) Curve (1) is from Eq. (3.3-14), using the histogram in (a); curve (2) was obtained using the iterative procedure in Eq. (3.3-17).
(c) Enhanced image using mappings from curve (2).
(d) Histogram of (c).

**Initial CDF**

**Modified CDF**

**Desired**
Image Enhancement in Spatial Domain

- **Local Enhancement:**

![Image](image.png)

**Figure 3.23** (a) Original image. (b) Result of global histogram equalization. (c) Result of local histogram equalization using a $7 \times 7$ neighborhood about each pixel.
Image Enhancement in Spatial Domain

- Histogram Statistics For Image Enhancement:
  - Uses of Statistical Parameters

\[
\mu_n(r) = \sum_{i=0}^{L-1} (r_i - m)^n p(r_i)
\]

\[
m = \sum_{i=0}^{L-1} r_i p(r_i)
\]

\[
\begin{align*}
\mu_0 &= 1 \\
\mu_1 &= 0 \\
\mu_2 &= \text{variance}
\end{align*}
\]
Image Enhancement in Spatial Domain

- Histogram Statistics For Image Enhancement:
  - Global mean and global variance:
    - Gross adj. of overall intensity and contrast
  - Local mean and local variance:
    - Changes dependents on predefined region about pixels
    - Local Definitions:

\[
\begin{align*}
    m_S(x,y) &= \sum_{(s,t) \in S(x,y)} r(s,t) p(r(s,t)) \\
    \sigma^2(x,y) &= \sum_{(s,t) \in S(x,y)} (r(s,t) - m_S(x,y))^2 p(r(s,t))
\end{align*}
\]
Image Enhancement in Spatial Domain

- Measure of relative darkness/lightness:
  - Compare local mean vs. global mean.

- Measure of relative contrast:
  - Local variance vs. global variance.

- Rules:

\[
g(x, y) = \begin{cases} 
E.f(x, y) & m_s(x, y) \leq K_0 M_G \text{ and } k_1 D_G \leq \sigma_s(x, y) \leq k_2 D_G \\
O.W & \text{otherwise}
\end{cases}
\]
Medical Image Analysis and Processing

Image Enhancement in Spatial Domain

- A SEM sample images:

**FIGURE 3.24** SEM image of a tungsten filament and support, magnified approximately 130×. (Original image courtesy of Mr. Michael Shaffer, Department of Geological Sciences, University of Oregon, Eugene).
Image Enhancement in Spatial Domain

FIGURE 3.25 (a) Image formed from all local means obtained from Fig. 3.24 using Eq. (3.3-21). (b) Image formed from all local standard deviations obtained from Fig. 3.24 using Eq. (3.3-22). (c) Image formed from all multiplication constants used to produce the enhanced image shown in Fig. 3.26.
Medical Image Analysis and Processing

Image Enhancement in Spatial Domain

• Enhanced Images:

FIGURE 3.26
Enhanced SEM image. Compare with Fig. 3.24. Note in particular the enhanced area on the right side of the image.
Image Enhancement in Spatial Domain

• Image Subtraction:

\[ g(x,y) = f(x,y) - h(x,y) \]

• Application in medical imaging:
  – Angiography
  – DSA
Image Enhancement in Spatial Domain

• Contrast Agent Enhancement:

FIGURE 3.29
Enhancement by image subtraction. (a) Mask image. (b) An image (taken after injection of a contrast medium into the bloodstream) with mask subtracted out.
Image Enhancement in Spatial Domain

- Windows Processing

\[
g(x, y) = \sum_{s=-a}^{+a} \sum_{t=-b}^{+b} w(s, t) f(x + s, y + t)
\]

\text{FIGURE 3.32} The mechanics of spatial filtering. The magnified drawing shows a 3 \times 3 mask and the image section directly under it; the image section is shown displaced out from under the mask for ease of readability.
### Medical Image Analysis and Processing

#### Image Enhancement in Spatial Domain

- **Smoothing Linear Filter:**

\[
g(x, y) = \frac{\sum_{s=-a}^{+a} \sum_{t=-b}^{+b} w(s, t) f(x + s, y + t)}{\sum_{s=-a}^{+a} \sum_{t=-b}^{+b} w(s, t)}
\]

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**Figure 3.34** Two 3 × 3 smoothing (averaging) filter masks. The constant multiplier in front of each mask is equal to the sum of the values of its coefficients, as is required to compute an average.
Medical Image Analysis and Processing

Image Enhancement in Spatial Domain

Blurring vs. Denoising

FIGURE 3.35 (a) Original image, of size 500 × 500 pixels. (b)–(f) Results of smoothing with square averaging filter masks of sizes $n = 3, 5, 9, 15, 35$, and 55, respectively. The black squares at the top are of sizes 3, 5, 9, 15, 25, 35, 45, and 55 pixels, respectively; their borders are 25 pixels apart. The letters at the bottom range in size from 10 to 24 points, in increments of 2 points; the large letter at the top is 60 points. The vertical bars are 5 pixels wide and 100 pixels high; their separation is 20 pixels. The diameter of the circles is 25 pixels, and their borders are 15 pixels apart; their gray levels range from 0% to 100% black in increments of 20%. The background of the image is 10% black. The noisy rectangles are of size 50 × 120 pixels.
Image Enhancement in Spatial Domain

- Order Statistic filters:
  - Median
  - Max
  - min
  - Good for Salt-Pepper/impulse noise.
Image Enhancement in Spatial Domain

• Median and LPF Comparison:

**FIGURE 3.37** (a) X-ray image of circuit board corrupted by salt-and-pepper noise. (b) Noise reduction with a $3 \times 3$ averaging mask. (c) Noise reduction with a $3 \times 3$ median filter. (Original image courtesy of Mr. Joseph E. Pascenti, Lixi, Inc.)
Image Enhancement in Spatial Domain

• Sharpening Spatial Filter:
  – First Derivative:
    • Discrete Implementation: \( \frac{\partial f}{\partial x} = f(x + 1, y) - f(x, y) \)
  – Second Derivative:
    • Discrete Implementation:
      \[ \frac{\partial^2 f}{\partial x^2} = f(x + 1, y) - 2f(x, y) + f(x - 1, y) \]
Image Enhancement in Spatial Domain

FIGURE 3.38
(a) A simple image. (b) 1-D horizontal gray-level profile along the center of the image and including the isolated noise point. (c) Simplified profile (the points are joined by dashed lines to simplify interpretation).
Image Enhancement in Spatial Domain

- **1st and 2nd Derivative Comparison:**
  - **First Derivative:**
    - Thicker Edge;
    - Strong Response for step changes;
  - **Second Derivative:**
    - Strong response for fine details and isolated points;
    - Double response at step changes.
Image Enhancement in Spatial Domain

- Laplacian as an isotropic Edge Enhancer:

\[ \nabla^2 f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} \]

- Discrete Implementation:

\[ \nabla^2 f = \left[ f(x+1,y) + f(x,y+1) + f(x-1,y) + f(x,y-1) - 4f(x,y) \right] \]

\[
\begin{bmatrix}
0 & 1 & 0 \\
1 & -4 & 1 \\
0 & 1 & 0
\end{bmatrix} \quad 90^\circ \text{ isotropic} \quad \begin{bmatrix}
1 & 1 & 1 \\
1 & -8 & 1 \\
1 & 1 & 1
\end{bmatrix} \quad 45^\circ \text{ isotropic}
\]
Medical Image Analysis and Processing

Image Enhancement in Spatial Domain

Practically use:

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FIGURE 3.39
(a) Filter mask used to implement the digital Laplacian, as defined in Eq. (3.7-4).
(b) Mask used to implement an extension of this equation that includes the diagonal neighbors. (c) and (d) Two other implementations of the Laplacian.
Image Enhancement in Spatial Domain

- **Background Recovering:**

\[
g(x, y) = \begin{cases} 
  f(x, y) - \nabla^2 f(x, y) - \text{sign} \\
  f(x, y) + \nabla^2 f(x, y) + \text{sign}
\end{cases}
\]

\[
\begin{bmatrix}
  0 & -1 & 0 \\
  -1 & +5 & -1 \\
  0 & -1 & 0 \\
\end{bmatrix} \quad 90^\circ \text{ isotropic} \\
\begin{bmatrix}
  -1 & -1 & -1 \\
  -1 & +9 & -1 \\
  -1 & -1 & -1 \\
\end{bmatrix} \quad 45^\circ \text{ isotropic}
\]
Image Enhancement in Spatial Domain

\[
g(x, y) = \begin{cases} 
  f(x, y) - \nabla^2 f(x, y) & -\text{sign} \\
  f(x, y) + \nabla^2 f(x, y) & +\text{sign}
\end{cases}
\]

**FIGURE 3.40**
(a) Image of the North Pole of the moon.
(b) Laplacian-filtered image.
(c) Laplacian image scaled for display purposes.
(d) Image enhanced by using Eq. (3.7-5).
(Original image courtesy of NASA.)
Image Enhancement in Spatial Domain

Two L. Mask

SEM image

a. Mask Result

b. Mask Result (Sharper)

FIGURE 3.41 (a) Composite Laplacian mask, (b) A second composite mask, (c) Scanning electron microscope image, (d) and (e) Results of filtering with the masks in (a) and (b), respectively. Note how much sharper (e) is than (d). (Original image courtesy of Mr. Michael Shaffer, Department of Geological Sciences, University of Oregon, Eugene.)
• First Derivative-Gradient:

\[ \nabla f = \begin{bmatrix} G_x \\ G_y \end{bmatrix} = \begin{bmatrix} \frac{\partial f}{\partial x} \\ \frac{\partial f}{\partial y} \end{bmatrix} \]

\[ |\nabla f| = \left[ \left( \frac{\partial f}{\partial x} \right)^2 + \left( \frac{\partial f}{\partial y} \right)^2 \right]^{1/2} \approx \left| \frac{\partial f}{\partial x} \right| + \left| \frac{\partial f}{\partial y} \right| \]
Image Enhancement in Spatial Domain

- **First Derivative-Gradient:**
  - Discrete Implementation:

\[
G_x = (z_9 - z_5) \quad G_y = (z_8 - z_6)
\]

\[
|\nabla f| \approx \left[ (z_9 - z_5)^2 + (z_8 - z_6)^2 \right]^{1/2}
\]

\[
|\nabla f| \approx |z_9 - z_5| + |z_8 - z_6|: \quad \textit{Roberts Cross-Gradient}
\]
Medical Image Analysis and Processing

Image Enhancement in Spatial Domain

**Figure 3.44**
A 3 × 3 region of an image (the z's are gray-level values) and masks used to compute the gradient at point labeled \( z_5 \). All masks coefficients sum to zero, as expected of a derivative operator.

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**Roberts Cross Gradient**

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**Sobel**

(2 → 1 for prewitt)
Medical Image Analysis and Processing

Image Enhancement in Spatial Domain

• An Example (Contact Lens) of Sobel:

![Contact Lens Image and Sobel Gradient](image)

**FIGURE 3.45**
Optical image of contact lens (note defects on the boundary at 4 and 5 o’clock).
(b) Sobel gradient.
(Original image courtesy of Mr. Pete Sites, Perceptics Corporation.)
Image Enhancement in Spatial Domain

- Combination

**Bone Scan**  
Original + Laplacian  
Soble of Original
Image Enhancement in Spatial Domain

Smoothed Sobel

(Orig. + L.)*S.Sobel

Orig.+

(Orig. + L.)*S.Sobel

Apply Power-Law
• MATLAB Command:
  – Image Statistics:
    • means2, std2, corr2, imhist, regionprops
  – Image Intensity Adjustment:
    • imadjust, histeq, adapthisteq, imnoise
  – Linear Filter:
    • imfilter, fspecial, conv2, corr2,
  – Nonlinear filter:
    • medfilt2, ordfilt2,