Mammographic mass segmentation: Embedding multiple features in vector-valued level set in ambiguous regions

Ying Wang, Dacheng Tao, Xinbo Gao, Xuelong Li, Bin Wang

1. Introduction

Breast cancer is the second most frequently diagnosed cancer in women all around the world [1]. Numerous studies have shown that early detection saves lives and increases treatment options. Currently, mammography is the most reliable and cost-effective tool for detecting the breast cancers at an early stage. Therefore, a dozen of mammographic computer-aided diagnosis (CAD) systems have been developed for assisting doctors in finding the symptoms earlier by using mammograms.

Mass, which always indicates the malignancy, is one of the major abnormalities in mammograms. However, clinical studies show that only a minority of biopsied masses are malignant [2]. Patient information and characteristics of the symptoms are the most effective features to detect masses in CAD systems.

Therefore, accurately segmentation is the most important step for diagnosing the malignancy of the symptoms, because it severely affects the performance of the feature analysis and the subsequent recognition. Masses are always of poor contrast, highly connect to surrounding parenchymal tissues, and possess various scales, complex shapes, and ambiguous margins. Thus, mass segmentation is a big challenge.

There are numerous studies on mass segmentation [3]. For example, pixel based methods [4–6], such as region growing and its extensions; region based methods [7,8], e.g., filter based methods; and simple edge based methods [9–12], e.g., the gradient filters, are employed widely in the early stage for mass segmentation. These methods usually integrate pixel information and other characteristics to obtain better segmentation results. Though these types of methods are easily to implement, it is still difficult to acquire satisfied segmentation results for masses of ambiguous boundaries. This is because simple features cannot handle the complex density distributions and topologies of the masses. To find more accurate boundaries of masses, the active contour method, which is flexible and effective on capturing complex topologies, is introduced to the CAD systems [13–19].

Mammographic mass segmentation plays an important role in computer-aided diagnosis systems. It is very challenging because masses are always of low contrast with ambiguous margins, connected with the normal tissues, and of various scales and complex shapes. To effectively detect true boundaries of mass regions, we propose a feature embedded vector-valued contour-based level set method with relaxed shape constraint.

In particular, we initially use the contour-based level set method to obtain the initial boundaries on the smoothed mammogram as the shape constraint. To prevent the contour leaking and meanwhile preserve the radiative characteristics of specific malignant masses, afterward, we relax the obtained shape constraint by analyzing possible valid regions around the initial boundaries. The relaxed shape constraint is then used to design a novel stopping function for subsequent vector-valued level set method. Since texture maps, gradient maps, and the original intensity map can reflect different characteristics of the mammogram, we integrate them together to obtain more accurate segmentation by incorporating the new stopping function into the newly proposed feature embedded vector-valued contour-based level set method.

The experimental results suggest that the proposed feature embedded vector-valued contour-based level set method with relaxed shape constraint can effectively find ambiguous margins of the mass regions. Comparing against existing active contours methods, the new scheme is more effective and robust in detecting complex masses.

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Sahiner et al. [13] incorporated the curvature, homogeneity, and the smoothed image gradient magnitude to the active contour method; Xiao et al. [14] employed the geometric active contour model with the fusion of color and intensity priors to segment masses; Ball and Bruce [17] presented an adaptive level set segmentation method, which segments the suspicious masses in the polar domain and adaptively adjusts the border threshold to find the boundaries; and Yuan et al. [19] proposed a dual-stage segmentation method for lesion segmentation. The method detects the initial contour by using the radial gradient index method and then uses the level set method to refine the initial segmentation result based on a dynamic stopping criterion. Although these algorithms improve the performance of mass segmentation, they are still not ready for practical utilization.

In particular, general segmentation algorithms find boundaries mainly dependent on the gradient information within regions. It may not work well on mammograms, because masses are always of ambiguous margins. Though the Chan–Vese model considers the density distributions of the foreground and background, it cannot handle the case conveniently. This is because the normal regions around the masses always present so similar characteristics with masses. Thus, if we can properly combine the contour- and region-based methods, more accurate margins of masses can be captured.

In addition, original mammograms are the only material employed by the general segmentation methods. Existing methods only use the density information of the mass regions, but ignore other high-level features embedded in mammograms. It is well known that mass regions and normal ones have different textures and gradient variational features. To effectively detect masses boundaries, it is necessary to integrate texture maps, gradient maps and intensity maps.

Finally, contour leaking is a serious problem in mass segmentation, because some mass margins are ambiguous. For automatically preventing the evolving curve leaking from the true boundaries, we propose an adaptive shape constraint method [20]. This constraint strictly restrains the evolving curve within the shape constraint, and thus tend to lose some radiative characteristics or its growing tendency of masses, especially for malignant cases. Furthermore, the inside regions of masses always possess complex density distributions, which induce the evolving curve shrinking to the highlight core regions. Therefore, a flexible constraint should be considered for preventing the evolving curve shrinking and preserving the radiative characteristics of the malignant masses.

To capture more accurate mass boundaries, we propose a feature embedded vector-valued contour-based level set method with relaxed shape constraint. The new method firstly detects the initial shape constraint by employing contour-based level set method. Then, a new relaxed shape constraint is designed by analyzing the stopping function of the shape constraint, and a new stopping function is developed subsequently. Finally, to use more types of characteristics of the mammogram, we collect the texture and gradient variational features for subsequent vector-valued level set method. The experimental results on different mammograms demonstrate that the proposed method can effectively detect boundaries of masses with complex shapes. The proposed method has the following properties:

1. The contour of the initial focal region is combined with the original shape constraint to form the new annular constraint region. It effectively avoids the evolving curve shrinking into the initial contour and prevents the contour leaking. In addition, the relaxed constraint considers the surrounding region of the original shape constraint, which can effectively preserve the radiative characteristics of malignant masses and capture more accurate boundaries.

2. The extension to the vector-valued level set method induces the evolving curve by combining multiple features (the texture maps, the gradient maps, and the intensity map) of mammograms. It can effectively handle the instance that masses have ambiguous margins or partially connected with the parenchymal tissues, because it considers different channels, each of which represents a specific characteristic of the mammograms.

3. The stopping function, inherited from the contour-based level set, is incorporated with the region distribution information to determine the segmentation results. By considering both the gradient information and the distribution of feature values, the embedded model can cause the evolving curve to the real boundaries of masses.

The rest of this paper is organized as follows. Section 2 briefly reviews the contour- and region-based level set methods. In Section 3, the proposed feature embedded vector-valued contour-based level set scheme is detailed; and the experimental results are presented in Section 4; finally, Section 5 concludes the whole paper.

2. Previous work

Osher and Sethian [21] proposed the level set method to detect object boundaries. For practical image segmentation, Malladi et al. [22] introduced a stopping function to handle the contour velocity [43]. It is a monotonically decreasing function of the gradient magnitude of the image. Generally, the stopping function is defined by

\[
g(x) = \frac{1}{1 + |\nabla g(x) + f(x)|^2}, \tag{1}
\]

where \(\nabla g(x) + f(x)\) is the absolute gradient of the convoluted image, which is obtained by convolving the original image by the derivative of a Gaussian function with a pre-defined standard deviation \(\sigma\).

The contour-based level set method cannot always perform well for mass segmentation. This is because masses are always of ambiguous margins, which cannot be precisely defined by the gradient. Therefore, we employ the Chan–Vese region-based level set method to integrate the intensity distributions within/without the mass regions.

The Chan–Vese model [23,24,40–42], i.e., active contours without edges, can segment objects whose boundaries are not defined by gradients. The model assumes the image is formed by two approximately piecewise-constant intensities of distinct values \(u_i\) and \(u_0\). Let \(C\) be a parameterized curve, the energy functional of the model is defined by

\[
F(c_1,c_2,C) = \mu \text{length}(C) + v \text{Area}(\text{inside}(C)) \\
+ \lambda_1 \int_{\text{inside}(C)} |u_0(x,y) - c_1|^2 \, dx \, dy \\
+ \lambda_2 \int_{\text{outside}(C)} |u_0(x,y) - c_2|^2 \, dx \, dy, \tag{2}
\]

where \(\mu \geq 0, v \geq 0, \lambda_1, \lambda_2 > 0\) are fixed parameters, and \(c_1, c_2\) are the mean values inside and outside the curve \(C\), respectively. The corresponding level set function \(\phi\) is defined by

\[
\frac{\partial \phi}{\partial t} = \delta_{\epsilon}(\phi) \left[ \mu \text{div} \left( \frac{\nabla \phi}{|\nabla \phi|} \right) - \varphi - \lambda_1 (u_0 - c_1)^2 + \lambda_2 (u_0 - c_2)^2 \right], \tag{3}
\]

where \(\delta_{\epsilon}\) is a slightly regularized one-dimensional Dirac measure \(\delta_0\).
3. The proposed scheme

In this paper, we propose the feature embedded vector-valued contour-based level set method shown in Fig. 1. It contains four stages: the shape constraint initialization, the shape constraint relaxation (or relaxed SC for short) and the new stopping function construction, feature map construction, and the embedded vector-valued level set segmentation.

3.1. Shape constraint initialization

In our previous work [20,25], the morphological component analysis (MCA) is introduced for eliminating the structure noises and providing a piecewise-smooth image for subsequent segmentation [26]. Then, the initial contour of the evolution is automatically detected by using an improved MCL method [25]. To obtain approximate shapes of various masses, the contour-based level set method, without the costly re-initialization procedure [27], is applied to this smoothed image.

Let the smoothed component decomposed by MCA be $S_{\text{MCA}}$. Following Eq. (1), the stopping function of the contour-based level set method can be written as [20]

$$g(x) = 1/(1 + |\nabla S_{\text{MCA}}|^2).$$  

(4)

According to [27], the corresponding evolution equation of the level set function is

$$\frac{\partial \phi}{\partial t} = \mu \Delta \phi - \text{div} \left( \frac{\nabla \phi}{\|\nabla \phi\|} \right) + \lambda \delta(\phi) \text{div} \left( g \frac{\nabla \phi}{\|\nabla \phi\|} \right) + v g(\delta(\phi)),$$  

(5)

where $\phi$ denotes the level set function, $\mu > 0$ is a parameter controlling the effect of penalizing the deviation of $\phi$, $\lambda > 0$ and $v$ are constants.

The MCA effectively removes noises, weakens the highlight structures, preserves the major intensity distributions and approximates shapes of masses. Therefore, the evolving curve can adaptively and effectively find the peripheries of the different mass regions, which are employed as the shape constraints in [20].

However, experimental results showed that the detected margins approach more to the focal areas of masses. The phenomenon shows that the method sometimes suppresses the evolving curves within the true boundaries of masses and ignores the radiative characteristics of some malignant masses. In the future, we will consider combining MCA with active appearance model [28] to further improve the proposed scheme.

3.2. Relaxed shape constrain and the new stopping function

At present, the most effective and available ways for preventing the contour leaking and inaccuracy is the shape constraint and the shape prior of the object. Masses are always of different scales, various shapes, and ambiguous margins, and thus the general shape constraint or prior methods cannot work well. Though the manual shape prior can improve the segmentation performance, the cost of the delineation is unacceptably high. Moreover, the accuracy of the delineation is affected by many factors, such as different understandings of masses, experience, and the fatigue of eyes. Since the accuracy of the prior severely affects the final segmentation results, the above problems limit the applications of the shape prior methods for mass segmentation.

The shape constraint proposed in [20] is defined on the smooth image obtained by MCA. This piecewise smooth sub-image alleviates the effects of the structure and equipment noises and the highlight blood vessels within the mass regions. Fig. 2 shows a mass region and the sketch map of its stopping function according to the smooth component decomposed by MCA. However, the final shape constraint cannot contain the whole mass region.

The black area of Fig. 2(c) indicates the large gradient values of the smooth image. The gradients around the mass boundary possess larger values, which form an annular zone contained the true mass boundaries. Thus, the evolving curve easily stops earlier around the focal region of the mass as shown in Fig. 2(d). It restrains the final segmentation result within the inadequate constraint, and the detected boundary is not the true boundary. The black area also contains the radiative characteristics of the masses, which is weakened by MCA. As shown in Fig. 3(a), the inner dashes contour denotes the original shape constraint, the outer dashed contour stands for the borderline of the possible valid region, and the black annular region between two dashed contours can be considered as the relaxed shape constraint. Therefore, it is possible to obtain the true mass boundaries by considering the regions of this black annular zone.

The relaxed region contains not only the true boundary but also background regions that possess gradient variations similar to the true boundaries. Thus, the gradient distributions of the black annular region should be re-estimated for eliminating negative effects of the background regions. A pixel that belongs to mass regions obeys the Gaussian distribution. To estimate the

![Fig. 1. Feature embedded vector-valued contour-based level set method.](image-url)

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pixels distributions within the relaxed annular region, we can calculate the sample mean and variance of focal regions of masses

\[ \mu_g = \frac{1}{|F|} \sum_{(x,y) \in F} I(x,y) \sqrt{(x-m_x)^2 + (y-m_y)^2}, \quad (6) \]

\[ \sigma_g = \frac{1}{|F|} \sum_{(x,y) \in F} \left[ I(x,y) \sqrt{(x-m_x)^2 + (y-m_y)^2} - \mu_g \right]^2, \quad (7) \]

where \( F \) is a focal region, \(|F|\) is the area of \( F \), and \( m_x \) and \( m_y \) are the mean of \( x \) and \( y \), respectively.

The distributions are defined based on the distance from the pixel \((i, j)\) to the focal region. It is defined by considering the average intensity weighted distance as

\[ d(i,j) = \frac{1}{|F|} \sum_{(x,y) \in F} I(x,y) \sqrt{(x-i)^2 + (y-j)^2}. \quad (8) \]

Then the distributions of pixels within the relaxed annular region can be obtained by employing the above estimated mean and variance

\[ p(d) = \frac{1}{\sqrt{2\pi}} \sigma_g \exp \left( -\frac{(d-\mu_g)^2}{2\sigma_g^2} \right). \quad (9) \]

If we denote the original shape constraint as \( \partial \Omega \), the region within the original shape constraint is \( \Omega \), the region out of the constraint but within the black annular zone as \( \Omega_R \), the outer boundary of the relaxed annular region is \( \partial \Omega_R \), and the new relaxed shape constraint is

\[ R(x) = \begin{cases} 1 & x \in \Omega - F \\ p(d) & x \in \Omega_R \\ 0 & x \not\in (\Omega - F) \cup \Omega_R \end{cases} \quad (10) \]

Under the relaxed shape constraint, the gradient variations within the original shape constraint can be preserved and the variations out of the relaxed annular region can ensure the evolving curve within valid regions.

We then design a new stopping function for the contour-based method based on the aforementioned relaxed shape constraint. By inheriting from the stopping function of the original contour-based method, the new stopping function with the relaxed shape constraint is

\[ g_R(x) = \frac{1}{1 + R(x) \left| \nabla G_p(x) \right| I(x)^2}. \quad (11) \]

Fig. 3(b) shows that part of true mass boundary exceeds the shape constraint. By using the relaxed shape constraint, the evolving curve can keep evolving to the true mass boundary. When the evolving curve exceeds the original constraint, gradient variations of valid regions strictly restrain the stopping force, which makes the evolving curve effectively evolved to boundaries of mass regions, as shown in Fig. 3(c).
3.3. Feature maps and vector-valued level set

It has been verified that besides of intensity information, texture information and gradient variations are effective for mass recognition [3,29–31]. In this paper, the texture information is encoded in four maps, where the first map is the texture image decomposed by MCA and the rest three maps are elements of the Hessian matrix of local regions. Our experiments show that these texture maps can effectively reveal the local texture discrimination between masses and background tissues.

The gradients with respect to coordinates x and y are used to strengthen the gradient variational information. In addition, the feature maps include the magnitudes and orientations of the gradients. If we denote the gradients in x and y directions as $g_x$ and $g_y$, respectively, denote the magnitudes and orientations of the gradients as $|g|$ and $\arctan(g_x/g_y)$, by combining the original mammograms (the intensity information), the feature maps used for subsequent segmentation can be composed by nine feature images

$$F = \{l, T_{MCA}, l_{Ixx}, l_{Iyy}, l_{Ixy}, l_{g_x}, l_{g_y} \}.$$

(12)

Fig. 4 presents all the feature images of a cropped mammogram. Fig. 4(a) shows the original cropped mammogram with a mass on it. The original image is then decomposed and the textural component can be obtained and shown in Fig. 4(b). By calculating the Hessian matrix, we can obtain $l_{Ixx}$, $l_{Iyy}$, and $l_{Ixy}$, and corresponding feature maps are shown in Fig. 4(c)–(e), respectively. Since the gradient of the original mammogram is important for the edge-based level set, the gradients along x and y directions are obtained and shown in Fig. 4(f) and (g), respectively. Besides, we derive the magnitude and directions of the image gradient, which are shown in Fig. 4(h) and (i). Recently, the biologically inspired features [32] and subspace learning [33–35] have shown potential for image based scene recognition and we will consider applying these strategies to improve the segmentation performance.

3.4. Feature embedded vector-valued contour-based level set method

The vector-valued level set method is originally proposed for object detection in color/multispectral images [36]. Let $\Omega$ be a bounded open subset of $\mathbb{R}^2$, let $\partial \Omega$ be the corresponding boundary, $u_i$ be the ith channel of an image on $\Omega$ with $i = 1, \ldots, N$ channels, and $\phi$ be the level set function. Let $c^+ = (c_1^+, \ldots, c_N^+)$ and $c^- = (c_1^-, \ldots, c_N^-)$ be two unknown constant vectors, and then the energy of the vector-valued Chan–Vese model in level set format is

$$F(c^+, c^-, \phi) = \mu \int_{\Omega} \delta(\phi(x,y)) \left| \nabla \phi(x,y) \right| \, dx \, dy$$

+ $\frac{1}{\mu N} \sum_{i=1}^{N} \lambda_i^+ \left| u_i(x,y) - c_i^+ \right|^2 \mathcal{H}(\phi(x,y)) \, dx \, dy$

+ $\frac{1}{\lambda_i^+} \sum_{i=1}^{N} \lambda_i^- \left| u_i(x,y) - c_i^- \right|^2 \left(1 - \mathcal{H}(\phi(x,y))\right) \, dx \, dy.$

(13)

We replace the original channels by the nine feature maps defined above to represent different characteristics of mammograms. By using the vector-valued Chan–Vese model, these feature maps can be integrated together to detect the unified contours.

Since different feature maps contribute to the segmentation procedure in different ways, the scale coefficient $1/N$ in Eq. (14) is redefined as a weighted normalized one. Assuming that $c^+$ and $c^-$ are constant vectors that represent the mean value of foreground and background in each feature map, respectively, and then we have

$$\frac{\partial \phi}{\partial t} = \delta \div \left( \nabla \phi \right) - \sum_{i=1}^{N} \lambda_i^+ \left( u_i(x,y) - c_i^+ \right)^2 + \sum_{i=1}^{N} \lambda_i^- \left( u_i(x,y) - c_i^- \right)^2.$$

(14)

where $\lambda_i^+$ is the weighted scale coefficient of a feature map and $\sum \lambda_i^+ = 1$. It can be acquired by employing iterative optimization methods. In this paper, the optimal weighted scale coefficient is obtained when the segmentation result is stationary after iterated about 500 times.

The region-based level set method can effectively detect some ambiguous boundaries on mammograms, but it cannot accurately detect boundaries defined by gradients. Since most of the mass boundaries are defined by both ambiguous boundaries and gradient defined boundaries, neither the region-based level set nor the contour-based one can capture the all boundaries of mass regions independently.

According to the Chan–Vese model [23], if $C$ is an evolving curve, the following “fitting” term can be extracted from the model represented by Eq. (2)

$$F_1(C) + F_2(C) = \int_{\text{inside}(C)} \left| u_0(x,y) - c_1 \right|^2 \, dx \, dy$$

+ $\int_{\text{outside}(C)} \left| u_0(x,y) - c_2 \right|^2 \, dx \, dy.$

(15)

The curve C is the boundary of an objects, if and only if $F_1(C) \approx 0$ and $F_2(C) \approx 0$.

However, it is difficult to obtain the minimum of the above term for mammograms. This is because the region-based method does not consider the gradient information of regions. Since masses always possess partially gradient defined boundaries, we propose the vector-valued contour-based level set method for detecting the ambiguous boundaries and gradient defined boundaries, simultaneously. In addition, the stopping function defined by the relaxed shape constraints are introduced to the “fitting” term of Eq. (16), i.e.,

$$F_{31}(C) + F_{32}(C) = \int_{\text{inside}(C)} \left| u_0(x,y) - c_1 \right|^2 \, dx \, dy + \int_{\text{outside}(C)} \left| u_0(x,y) - c_2 \right|^2 \, dx \, dy.$$

(16)

By using this new “fitting” term, the resultant force will be restricted to be small when the gradients take large values, and vice versa, i.e., the resultant force will be restricted to be large when the gradients take small values. Therefore, the new level set

Fig. 4. Feature maps. (a) The original image; (b) the textural component; (c) the element of Hessian matrix about x; (d) the element of Hessian matrix about y; (e) the element of Hessian matrix about x and y; (f) the gradient in x direction; (g) the gradient in y direction; (h) the magnitude of gradient; (i) the direction of gradient.
function is
\[
\frac{\partial \phi}{\partial t} = \delta \left[ \mu \text{div} \left( \nabla \phi \right) - g_{\text{R}} \left( \sum_{i=1}^{N} c_i \phi (u_{0i} - c_i)^2 \right) - \sum_{i=1}^{N} c_i \phi (u_{0i} - c_i)^2 \right].
\]  
(17)

Fig. 5 shows that feature maps can detect different contours by using evolving curves, and the final segmentation results will be obtained by integrating results in different maps.

The integration of the stopping functions inevitably brings some local extremes that exhibit as the noisy points or tiny circles in the final results. Thus, we conduct a post-processing step to eliminate local extremes. Masses are always one-piece and solid regions that have no holes within them and also no noisy tiny regions outside of them. Therefore, we keep the longest boundary among the detected results, while eliminate the isolated noisy tiny regions or points within and outside of it. Then more meaningful mass boundaries will be obtained.

The feature embedded vector-valued contour-based level set method can be summarized in Table 1.

The time cost of the proposed method is \(O(n)\), because the only consumption operation is the iteration of Eq. (17). The space cost of the proposed method includes three parts, the input image, the feature maps, and the output segmentation result. If the image size is \(N \times m \times m\), the space for storing these images are eleven times of \(N\). According to the complexity analysis, it can be written as \(O(N)\).

### 4. Experimental results and analysis

Experiments have been done on 150 sub-mammograms containing mass regions, which are cropped from the original mammograms for saving computational costs. All the mammograms are selected from the DDSM database [37] provided by University of South Florida, of which the mammograms are digitized with a LUMISYS and HOWTEK laser scanner at a pixel size of 0.5 mm or 0.45 mm and 12-bits or 16-bit per pixel. All the masses are marked by the radiologists, while only the approximate peripheries of masses are described. Among the 150 sub-mammograms, there are 150 masses. To test the effectiveness of the proposed scheme, masses possess various characteristics, such as the radiative characteristics, the complex intensity distributions, the ambiguous margins, et al., are used in the following experiments.

To initialize the evolving curves and detect the original shape constraints, MCA and improved MCL method are conducted first. The contours of focal regions detected by [25] are served as the initial contours of evolving curves in the whole segmentation process. Then the original shape constraints, which evolve on the smooth mammograms decomposed by MCA, are obtained by using the contour-based level set method [20]. And the constraints are also analyzed in this paper for finding the relaxed

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**Table 1**

The proposed segmentation algorithm for mammographic mass.

<table>
<thead>
<tr>
<th><strong>Input:</strong> the original image I</th>
<th><strong>Output:</strong> the evolving curve C and the segmentation result R</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Initialization:</strong> the initial position of evolving curve</td>
<td></td>
</tr>
<tr>
<td>1. Extract the shape constraint by [20]: Calculate (g(x) = 1/(1 +</td>
<td>\nabla \text{MCA}</td>
</tr>
<tr>
<td>2. Find the relaxed annular region.</td>
<td></td>
</tr>
<tr>
<td>3. Re-estimate the gradient distributions of pixels within the relaxed extension regions</td>
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<tr>
<td>Calculate (\mu_p = 1/\sum_{x,y} f(x,y)(x-m_x)^2 + (y-m_y)^2), (\sigma_g = \sqrt{1/\sum_{x,y} f(x,y)(x-m_x)^2 + (y-m_y)^2 - \mu_p})</td>
<td></td>
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<tr>
<td>and (d(l,j) = 1/\sum_{x,y} f(x,y)(l-x)^2 + (j-y)^2), estimate (p(d) = 1/2\sigma_g \exp(-\frac{d^2}{2\sigma_g^2})), then calculate (g_s(x) = \frac{1}{1+\exp(-\mu_p)})</td>
<td></td>
</tr>
<tr>
<td>4. Calculate the feature maps: (l, t_{\text{MCA}}, l_{\text{MC},}, l_{\text{MCL}}, f_{\text{MC},}, f_{\text{MCL}},</td>
<td>\nabla \phi</td>
</tr>
<tr>
<td>5. Incorporate the multi-channel feature maps: (\frac{\partial \phi}{\partial t} = \delta \left[ \mu \text{div} \left( \nabla \phi \right) - \sum_{i=1}^{N} c_i \phi (u_{0i} - c_i)^2 + \sum_{i=1}^{N} c_i \phi (u_{0i} - c_i)^2 \right] )</td>
<td></td>
</tr>
<tr>
<td>6. Iterate (\frac{\partial \phi}{\partial t} = \delta \left[ \mu \text{div} \left( \nabla \phi \right) - g_{\text{R}} \left( \sum_{i=1}^{N} c_i \phi (u_{0i} - c_i)^2 + \sum_{i=1}^{N} c_i \phi (u_{0i} - c_i)^2 \right) \right] )</td>
<td></td>
</tr>
<tr>
<td>7. Calculate the evolving curve (C={(l,j): \Omega(l,j)=0}) and the segmentation result (R = m^2 H(\phi) + m^2 (1-H(\phi))).</td>
<td></td>
</tr>
</tbody>
</table>
shape constraints of mass regions. To validate the proposed segmentation scheme, we conduct three types of experiments, that are the segmentation of the malignant and benign cases, the comparison tests of the proposed scheme with the classical active contour methods, and finally, the comparison experiments of the proposed method with the existing mass segmentation methods on mammograms possessed different characteristics.

First of all, the detail procedures of the whole segmentation scheme are shown in Fig. 6, which makes the proposed scheme more understandable. The original mammogram in Fig. 6(a) is firstly decomposed by MCA, and the smooth component of the original mammogram is obtained (in Fig. 6(b)). Meanwhile, we could also get the sketch map of the original stopping function as Fig. 6(c), and the original shape constraint as Fig. 6(d). Then, according to the proposed relaxed shape constraint and the new stopping function described in Section 3.2, we obtain the sketch map of the relaxed shape constraint in Fig. 6(e). The lighter annular region in Fig. 6(e) represents the relaxed extension of the original shape constraint, which is lightened for the requirement of visibility. With the relaxed shape constraint and the proposed feature embedded vector-valued contour-based level set method, the initial contour in Fig. 6(f) will be evolved on the multi-channel feature images. Some iteration steps are shown in Fig. 6(g)–(i), and Fig. 6(j) gives the final segmentation result. The iteration will stop when the difference between present contour and the former one is less than three pixels, or the iteration times exceed 5000. In Fig. 6(j), the ultimate iteration time is 160.

4.1. Segmentation tests on malignant and benign cases

The masses mainly divided as the malignant cases which indicate the cancers and benign ones. Therefore, the segmentation scheme should be tested on the two cases firstly for preliminarily proving the availability of the scheme.

As shown in Figs. 7 and 8, the malignant masses always possess complex density distributions with certain contrast to the background tissues, while the benign cases have more homogeneous density distributions compare to the malignant ones with relative low contrast to the background tissue. The second columns of Figs. 7 and 8 show the initial contours of evolving curves, and then the curve evolved as the third columns to the fifth ones to find the true mass boundaries. And the last column in Figs. 7 and 8 give the ultimate segmentation results of the proposed scheme. The results demonstrate that either the complex malignant cases or the low contrast benign ones are all accurately detected from the mammograms by employing the proposed segmentation scheme.

4.2. Comparison results with classical active contour methods

The effectiveness of a method should be also testified by comparing it with the other methods in the literature. Therefore, we then conduct the second type of the experiments, i.e., the comparison tests of the proposed segmentation scheme with the classical active contour methods in the literature. Fig. 9(a) presents a set of original mammograms with masses, which contains both malignant cases and benign ones. These masses are various in their size and shapes, and some of them possess ambiguous boundaries, some are of radiative characteristics, and also some with low contrast to the background tissues. For comparison the segmentation performance, the snake in gradient vector flow (GVF), the region-based level set (LS) method, the Chan–Vese (CV) model, the feature vector-valued CV method, and the proposed scheme are tested. The column Fig. 9(b) shows the segmentation results of GVF snake method. Since the parametric active contour methods have more difficulties to deal with the complex gradient variations and topologies of regions, it cannot capture complex topologies of mass boundaries. And the evolving curves usually stop earlier within mass regions. The contour-based level set method is the geometric active contour, and thus is more effective on capturing the adaptive topologies of mass regions. Fig. 9(c) presents the segmentation results of LS method, which could actually find better segmentation results. However, it is not so accurate on the locations where are of too large
The CV method considers the density distributions of the foreground and background regions, which could effectively detect the homogeneous regions with ambiguous boundaries. But it does not work well when applied on mammograms (Fig. 9(d)), because masses possess very complex density distributions and severely variational background. The segmentation results of the feature vector-valued region-based level set method are shown in Fig. 9(e). With nine feature images, the vector-valued method effectively detects the more accurate boundaries of mass regions. However, when the contrast of masses to the background is too poor, the contour leaking still severely affects the accuracy of segmentation. Therefore, the relaxed shape constraints are proposed in this paper for further improving the segmentation accuracy. As shown in Fig. 9(f), the proposed segmentation...
scheme could effectively detect the boundaries of various mass regions. Furthermore, the radiative characteristics around the masses can be found, and the contour leaking can also be prevented by restraining the evolving curve within the constraints.

4.3. Comparison results with existing mass segmentation methods

There are a number of studies focuses on the mass segmentation, such as the improved active contour (AC) methods in [13], the adaptive level set (LS) method introduced in [17], the dual-stage level set method of [19], and our previous work on shape constraint [20]. Then the proposed segmentation scheme will be tested with all these existing methods on mammograms. They are of different characteristics which are tested for further validating the effectiveness of the proposed scheme. The adjustable parameters are adjusted thoroughly and some necessary operations are conducted for alleviating the impaction of the different database.

The radiative characteristics are very important features of masses, which always indicate the malignancy of symptoms. Fig. 10(a) gives three malignant mass regions with radiative characteristics around them, and then each method mentioned

![Fig. 9. The comparison results of the proposed scheme with the classical methods. (a) The original mammograms; (b) the segmentation results of GVF snake method; (c) the segmentation results of LS method; (d) the segmentation results of CV method; (e) the segmentation results of vector-valued CV method; (f) the segmentation results of the proposed method.](image-url)
above are tested on the mammograms. As Fig. 10(b) shows, the improved active contour method still cannot capture this typical characteristic of the malignant cases. While the adaptive level set method in [17] present more effectiveness on tackling the growing tendency of the marginal characteristics (as shown in Fig. 10(c)). The segmentation results of [19] are shown in Fig. 10(d), which could capture the complex topologies of malignant masses. The result boundaries of this method are smoother than the other methods. Fig. 10(e) gives the segmentation results of the previous work in [20]. With the restrict shape constraints detected by [20], the final boundaries are restrained within the constraints. Though the method actually detects the complex topologies of different mass regions, the segmentation results are not adequate to cover the whole mass regions. The final segmentation results of the proposed segmentation scheme are given in Fig. 10(f). As we can see, the proposed scheme could effectively detect the radiative characteristics around the masses, and meanwhile capture the adaptive topologies of different mass regions.

Another difficulty of the mass segmentation is the partial ambiguous margins and connection with the background tissues. Fig. 11(a) shows two benign masses and a malignant mass that have partial ambiguous margins and also connected with the background tissues. The proposed scheme and four existing methods are then tested on the mammograms. Under this condition, most of the method cannot find the accurate boundaries of masses, because the ambiguous boundaries are hardly detected from the regions where the masses are connected with background tissues. If we adjust the parameters of the existing methods for driving the segmentation results to cover the whole mass regions, the contour leaking will be severely apparent. As shown in Fig. 11(b)–(e), the segmentation results are all inadequate. With the multi-channel feature images and the relaxed shape constraints, the proposed scheme could do the segmentation work satisfactory. In Fig. 11(f), the proposed scheme effectively finds the ambiguous boundaries, and also captures the gradient defined boundaries.

Fig. 12(a) exhibits two benign masses with homogeneous density distributions and well-defined boundaries. As we see from the segmentation results of different existing methods and the proposed one, all these method can effectively capture the mass boundaries. As shown in Fig. 12(f), the proposed scheme is more sensitive to the tiny variation on the boundaries, which could find more concavities. Though the relaxed shape constraint has extended the valid region for including more accurate boundary information, it still prevents the full growing of the radiative characteristics. And the segmentation results in Fig. 12(c), line two, seem more adequate than the proposed one.

We also test the above methods on very complex mass regions in Fig. 13(a). The two masses include a benign case and a malignant one, which all possess very complex density distributions within the mass regions. As shown in Fig. 13(b)–(e), the existing methods are difficult to effectively segment the mass regions even the parameters are adjusted to extremum. In line 1, Fig. 13(c), the adaptive level set method could really find the satisfied boundaries of the benign mass. Fig. 13(f) gives the segmentation results of the proposed scheme, which also detects the comparative boundaries of the two mass regions. It could not only cover the whole mass regions, but also capture the detail variations on the boundaries. And the segmentation results are more accurate in comparison with the existing methods.

The above experimental results validate the effectiveness of the proposed mass segmentation scheme. The scheme clearly detect the most mass boundaries in both malignant and benign cases. Comparison with the classical methods in the literature, the proposed scheme is more sensitive to the complex topologies and concavities of the mass boundaries. It could find more accurate boundaries of the mass regions with various size and shapes. Furthermore, the proposed scheme is tested on some

![Fig. 10](image_url) The comparison results of the proposed scheme with the existing method. (a) The original mammograms; (b) the segmentation result of the improved AC method; (c) the segmentation result of the improved LS method; (d) the segmentation result of the dual-stage LS method; (e) the segmentation result of the original shape constraint method; (f) the segmentation result of the proposed method.
special cases for further validating its effectiveness. The comparison experimental results demonstrate that the proposed scheme could effectively detect the radiative characteristics of the malignant masses, the partial ambiguous versus partial gradient defined boundaries of some masses, the smoothed benign masses, and also the masses with very complex density distributions. Among all the methods, the proposed scheme could always obtain the comparative even more accurate segmentation results, which provide more significant results to the diagnosis procedure.

However, there are still some insufficient aspects of the proposed method. As the iteration results shown in Figs. 7 and 8, the feature images and re-estimation of the gradient distributions inevitably bring some local extremums. These extremums produce some noisy points and tiny circles in the segmentation results, which have to eliminate by employing the post-process operation. Moreover, the detected boundaries by using the proposed scheme are too sensitive to the gradient variations on the boundaries, which make the final contours too rough. These problems will be considered in future for designing more effective and robust segmentation method.

5. Conclusions

A feature embedded vector-valued contour-based level set method with relaxed shape constraint is proposed in this paper.
The scheme firstly finds the adaptive shape constraint by evolving the contours on the smoothed mammograms. Then according to the original stopping function and shape constraint, the possible valid region beyond the original constraint is analyzed and the relaxed shape constraint can be obtained. Based on the relaxed shape constraint, a new stopping function is defined. To incorporate various characteristics of mammograms, the texture and gradient features are combined together with the original mammograms as the multi-slice feature images. Finally, the vector-valued contour-based level set is proposed to integrate different characteristics of mammograms. The experimental results show that the proposed scheme can obtain better segmentation results of various mass regions comparing against the existing methods. In the future, we will consider specific metric [38] to evaluate the segmentation performance.

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Reference

He is a Professor of Pattern Recognition and Intelligent System, and Director of the VIPS Lab, Xidian University. His research interests are computational intelligence, Department of Information Engineering at the Chinese University of Hong Kong. Since 2001, he joined the School of Electronic Engineering at Xidian University. Currently, to 1998, he was a research fellow in the Department of Computer Science at Shizuoka University, Japan. From 2000 to 2001, he was a postdoctoral research fellow in the

Xuelong Li

received the B.Sc. and M.Sc. degrees from Northwest University, Xi’an, China, in 1999 and 2002, respectively. He is currently working toward the Ph.D. degree in pattern recognition and intelligent system at Xidian University, Xi’an, China. His research interest is image segmentation.

Bin Wang

received the B.Sc. and M.Sc. degrees from Northwest University, Xi’an, China, in 1999 and 2002, respectively. He is currently working toward the Ph.D. degree in pattern recognition and intelligent system at Xidian University, Xi’an, China. His research interest is image segmentation.