Analog to Digital Conversion

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Overview

Analog to Digital Conversion

✓ In sampling, a discrete-time continuous-valued signal from an analog signal is obtained.

✓ In quantization, a discrete-time discrete-amplitude signal from a discrete-time continuous-valued signal is obtained.

✓ In encoding, a sequence of bits is assigned to different quantized values of a discrete-time discrete-amplitude signal.

Figure: Block diagram of analog to digital converter.

Sampling

Nyquist Sampling

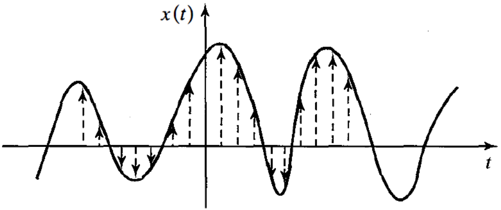

Figure: Nyquist sampling of a signal.

Nyquist Sampling

Theorem (Sampling Theorem)

Let the signal $x(t)$ have a bandwidth W, i.e., let $X(f) = 0$ for $|f| \geq W$. Let $x(t)$ be sampled at multiples of some basic sampling interval T_s, where $T_s \leq \frac{1}{2W}$, to yield the sequence $x_\delta(t) = \sum_{n=-\infty}^{\infty} x(nT_s)\delta(t - nT_s)$. Then it is possible to reconstruct the original signal $x(t)$ from the samples values by the reconstruction formula

$$\begin{aligned} x(t) &= h(t) * x_\delta(t) = 2W' T_s sinc(2W' t) * x_\delta(t) \\ &= \sum_{n=-\infty}^{\infty} 2W' T_s x(nT_s) sinc[2W'(t - nT_s)] \end{aligned}$$

, where W' is any arbitrary number satisfying the condition $W \leq W' \leq \frac{1}{T_s} - W$.

Nyquist Sampling

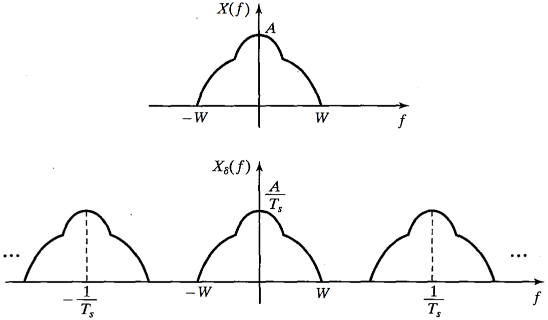

Figure: Frequency-domain representation of the nyquist sampled signal.

$$X_\delta(f) = \frac{1}{T_s} \sum_{n=-\infty}^{\infty} X(f - \frac{n}{T_s})$$

$$H(f) = T_s \sqcap (\frac{f}{2W'})$$

Zero-Order Hold Sampling

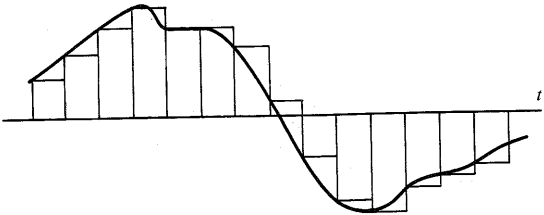

Figure: Flat-top sampling (zero-order hold sampling, sample and hold) of a signal.

$$x_p(t) = x_\delta(t) * p(t) \Rightarrow X_p(f) = X_\delta(f)P(f)$$

$$P_{eq}(f) = \frac{Ke^{-j2\pi f t_d}}{P(f)}$$

$$x(t) = x_p(t) * h(t) * p_{eq}(t)$$

Aliasing

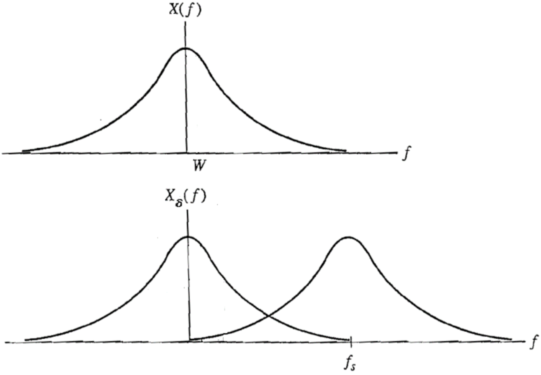

Figure: Aliasing in sampling.

✗ The unlimited bandwidth of messages creates aliasing.

Anti-aliasing

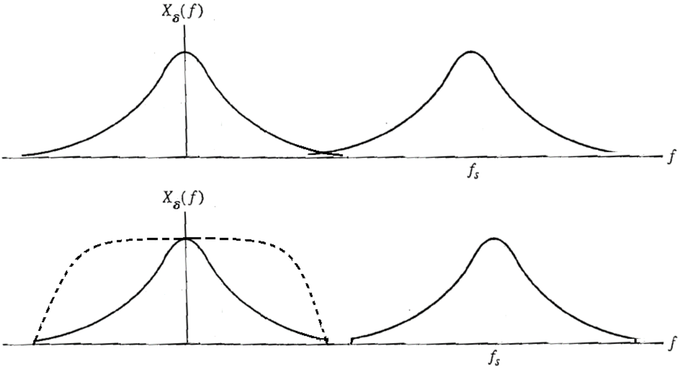

Figure: Anti-aliasing techniques in sampling.

✓ Increase sampling frequency and/or use anti-aliasing filter to mitigate aliasing effect.

Quantization

Quantization

Theorem (Quantization)

Quantization is a function defined as

$$Q(x) = \hat{x}_i : \quad x \in \mathbb{R}_i$$

where the sets $\mathbb{R}_i$ *partition the set of real numbers* $\mathbb{R}$.

Definition (Signal to Quantization Noise Ratio)

If the random variable X is quantized to $Q(X)$, the signal to quantization noise is defined as

$$\text{STQN} = \frac{E\{X^2\}}{E\{(X - Q(X))^2\}}$$

Uniform Quantization

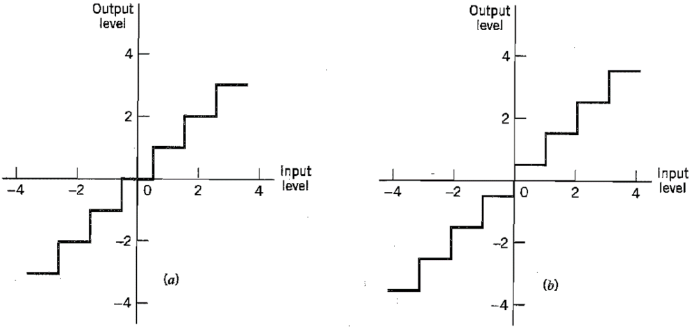

Figure: Two types of uniform quantization. (a) midtread and (b) midrise.

Nonuniform Quantization

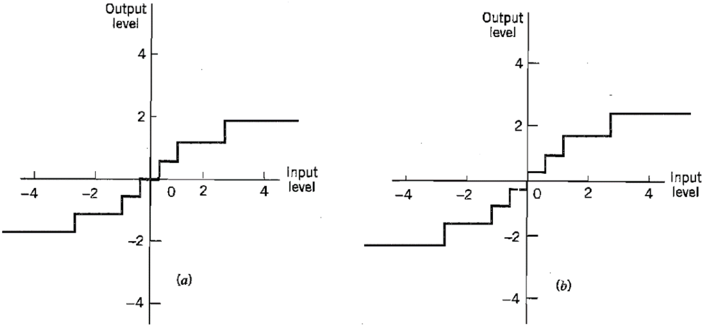

Figure: Two instances of nonuniform quantization.

Quantization Performance

Example (SQNR)

The source $X(t)$ is a stationary Gaussian source with mean zero and power spectral density $S_x(f) = 2\sqcap(f/200)$. The source is sampled at the Nyquist rate and each sample is quantized using an eight-level quantizer with $a_1 = -60$, $a_2 = -40$, $a_3 = -20$, $a_4 = 0$, $a_5 = 20$, $a_6 = 40$, $a_7 = 60$, and $\hat{x}_1 = -70$, $\hat{x}_2 = -50$, $\hat{x}_3 = -30$, $\hat{x}_4 = -10$, $\hat{x}_5 = 10$, $\hat{x}_6 = 30$, $\hat{x}_7 = 50$, $\hat{x}_7 = 70$. The SQNR for this quantization is $11.98 \equiv 10.78$ dB.

Quantization Performance

Example (SQNR (cont.))

The source $X(t)$ is a stationary Gaussian source with mean zero and power spectral density $S_x(f) = 2\sqcap(f/200)$. The source is sampled at the Nyquist rate and each sample is quantized using an eight-level quantizer. The SQNR for this quantization is $11.98 \equiv 10.78$ dB.

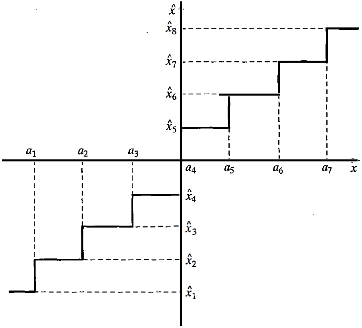

Figure: Eight-level quantizer.

Quantization Performance

Example (SQNR (cont.))

The source $X(t)$ is a stationary Gaussian source with mean zero and power spectral density $S_x(f) = 2\sqcap(f/200)$. The source is sampled at the Nyquist rate and each sample is quantized using an eight-level quantizer. The SQNR for this quantization is $11.98 \equiv 10.78$ dB.

$$E\{X^2\} = \sigma^2 = R_X(0) = \int_{-\infty}^{\infty} S_X(f)df = 400$$

$$E\{(X - Q(X))^2\} = \int_{-\infty}^{a_1} (x - \hat{x}_1)^2 f_X(x)dx + \sum_{i=2}^{7} \int_{a_{i-1}}^{a_i} (x - \hat{x}_i)^2 f_X(x)dx$$

$$+ \int_{a_7}^{\infty} (x - \hat{x}_8)^2 f_X(x)dx = 33.38, f_X(x) = \frac{1}{\sqrt{800\pi}} \exp(-x^2/800)$$

$$\text{SQNR} = \frac{400}{33.38} = 11.98$$

Encoding

Encoding

Statement (Encoding)

In encoding, a unique sequence of ν bits is assigned to each $N = 2^\nu$ quantization level.

Natural Binary Coding and Gray Coding

Quantization Level	Level Order	NBC Code	Gray Code
$\hat{x}_1$	0	0000	0000
$\hat{x}_2$	1	0001	0010
$\hat{x}_3$	2	0010	0011
$\hat{x}_4$	3	0011	0001
$\hat{x}_5$	4	0100	0101
$\hat{x}_6$	5	0101	0100
$\hat{x}_7$	6	0110	0110
$\hat{x}_8$	7	0111	0111
$\hat{x}_9$	8	1000	1111
$\hat{x}_{10}$	9	1001	1110
$\hat{x}_{11}$	10	1010	1100
$\hat{x}_{12}$	11	1011	1101
$\hat{x}_{13}$	12	1100	1001
$\hat{x}_{14}$	13	1101	1000
$\hat{x}_{15}$	14	1110	1010
$\hat{x}_{16}$	15	1111	1011

Table: NBC and gray codes for 16-level quantization.

Pulse Code Modulation

PCM Transmitter

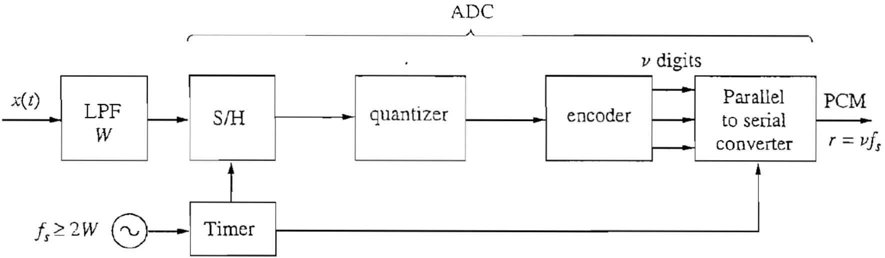

Figure: PCM transmitter.

✓ Output data rate is $r = \nu f_s$ bit/s.

PCM Receiver

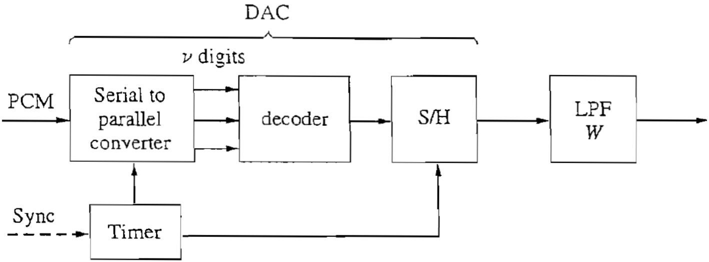

Figure: PCM receiver.

PCM Waveform

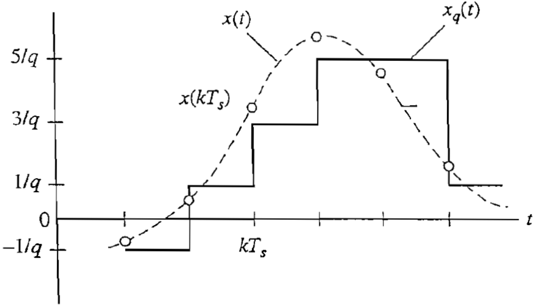

Figure: PCM waveform.

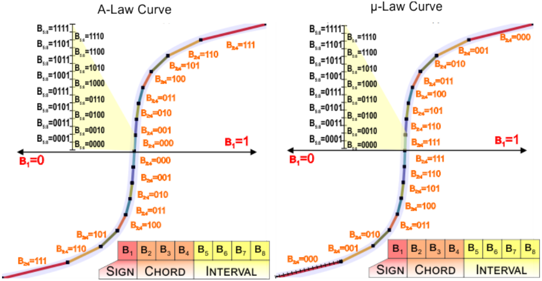

Figure: A-law and μ-law companding.

$$z(x) = \frac{1 + \ln(A|x|)}{1 + \ln(A)}\text{sgn}(x), \quad z(x) = \frac{\ln(1 + \mu|x|)}{\ln(1 + \mu)}\text{sgn}(x)$$

E1 Digital Voice Multiplexing

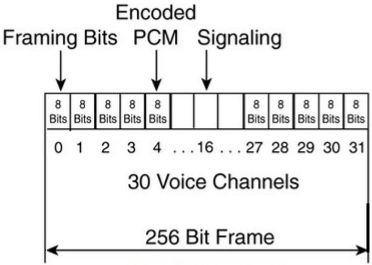

Figure: E1 frame.

✓ Each E1 frame carries 32 PCM channels with $f_c = 8000$ Hz and $\nu = 8$, which results in a net rate of $32 \times 8 \times 8000 \times 10^{-6} = 2.048$ Mb/s.

Delta Modulation

Delta Transmitter

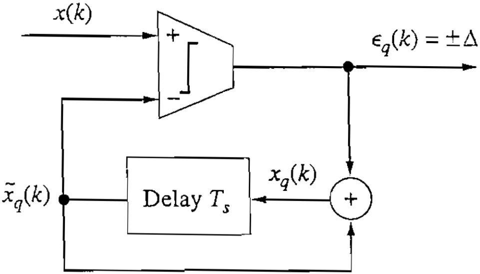

Figure: Delta transmitter.

✓ Output data rate is $r = f_s$ bit/s.

Delta Receiver

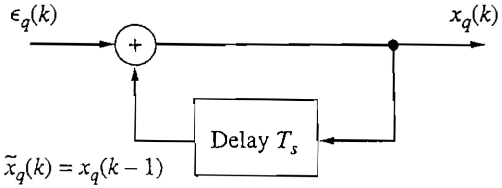

Figure: Delta receiver.

Delta Waveform

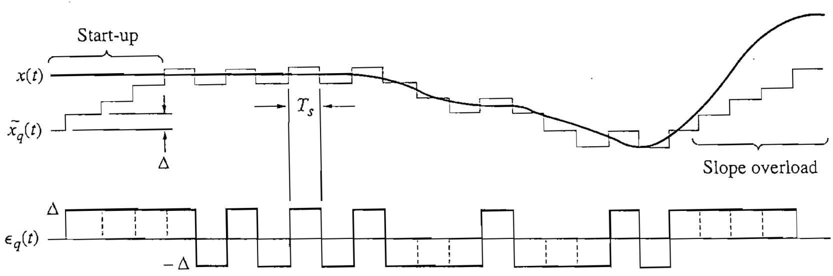

Figure: Delta waveform.

The End