MATHEMATICAL QUESTIONS

Question 1

Use the definitions of the unit step, unit impulse, and unit doublet function to prove the following identities.
Hint: Obviously, if $\int_{-\infty}^{+\infty} f(t)x(t)dt = \int_{-\infty}^{+\infty} g(t)x(t)dt$ for any test function $x(t)$, the singular functions $f(t)$ and $g(t)$ are equal.

(a) $u'_{-1}(t) = u_0(t)$.

(b) $u'_0(t) = u_1(t)$.

(c) $\delta(at) = \frac{1}{|a|}\delta(t), a \neq 0$.

Question 2

Take the Fourier transform of $x(t) = Ae^{-\frac{t^2}{\sigma^2}}$, where A and σ are given real values.

Question 3

The analytic signal $x_a(t)$ of the real signal $x(t)$ is a signal with the spectrum $2X(f)u(f)$, where $X(f)$ is the Fourier transform of $x(t)$.

(a) Show that the real and imaginary parts of $x_a(t)$ relates to $x(t)$ and its Hilbert transform $\hat{x}(t)$.

(b) Find the analytic signal of $x(t) = A\cos(2\pi f_0 t + \theta)$.

(c) How does the analytic signal generalize the concept of phasors?

Question 4 continued on next page. . .

Question 4

Let $\{\phi_i(t)\}_{i=1}^{N}$ be an orthogonal set of N signals, i.e.,

$$\int_{-\infty}^{\infty} \phi_i(t)\phi_j^*(t)dt = 0, \quad 1 \leq i, j \leq N, \quad i \neq j$$

and

$$\int_{-\infty}^{\infty} |\phi_i(t)|^2 = 1, \quad 1 \leq i \leq N$$

. Let $\hat{x}(t) = \sum_{i=1}^{N} \alpha_i \phi_i(t)$ be the linear approximation of an arbitrary signal $x(t)$ in terms of $\{\phi_i(t)\}_{i=1}^{N}$, where α_i's are chosen such that

$$\epsilon^2 = \int_{-\infty}^{\infty} |x(t) - \hat{x}(t)|^2 dt$$

is minimized.

(a) Show that the minimizing α_i's satisfy

$$\alpha_i = \int_{-\infty}^{\infty} x(t)\phi_i^*(t)dt$$

(b) Show that

$$\epsilon_{\mathit{min}}^2 = \int_{-\infty}^{\infty} |x(t)|^2 dt - \sum_{i=1}^{N} |\alpha_i|^2$$

(c) How does this general linear approximation relate to the Fourier series expansion?

Question 5

The generalized Fourier transform of the singular function $y(t)$ is defined as the function $Y(f)$ satisfying the integral equation

$$\int_{-\infty}^{\infty} Y(\alpha)x(\alpha)d\alpha = \int_{-\infty}^{\infty} y(\beta)X(\beta)d\beta$$

, where $x(t)$ is any test function such that the existence of its Fourier transform $X(f)$ is guaranteed under Dirichlet sufficient conditions.
Hint: It can be shown that the properties of the normal Fourier transform remain valid for the generalized Fourier transform.

(a) Discuss the reasons behind the definition.

Question 5 continued on next page…

(b) Use the definition to find the Fourier transform of $\delta(t)$.

(c) Use the definition to find the Fourier transform of $u(t)$.

SOFTWARE QUESTIONS

Question 6

Validate the performance of the tapped delay-line microwave equalizer using MATLAB simulation. To do this,

(a) Develop a function, which simulates the point-to-point microwave radio channel.

(b) Develop a function, which simulates the taped delay line microwave equalizer.

(c) Observe the output of the channel before and after the equalizer and discuss the observations for different number of taps.

(d) How can we measure the distortion before and after the equalizer. Do you know any suitable metric?

BONUS QUESTIONS

Question 7 continued on next page. . .

Question 7

Return your answers by filling the LaTeXtemplate of the assignment.