MATHEMATICAL QUESTIONS

Question 1

Let $\mathbf{X} = (X_1, X_2, \cdots, X_n)$ be a jointly Gaussian random vector. Show that a linear combination of $X_1, \cdots, X_n$ has a univariate normal distribution.
Hint: You may use the characteristic function of the jointly Gaussian random vector $\mathbf{X}$ to prove the statement.
Hint: You became familiar with the univariate characteristic function in the previous assignment. Look at Wikipedia to know how the characteristic function is defined for a random vector, especially, for a jointly Gaussian random vector.

Question 2

The random variable Y is defined as

$$Y = \frac{1}{n}\sum_{i=1}^{n} X_i$$

where $X_i, i = 1, 2, \cdots, n$ are statistically independent and identically distributed random variables.

(a) Determine the characteristic function of Y?

(b) Determine the PDF of Y.

(c) Assume that X_i's have Cauchy PDF given by

$$f_X(x) = \frac{a}{\pi(a^2 + x^2)}, \quad -\infty < x < \infty$$

. Does the central limit theorem hold as $n \to \infty$?
Hint: Find the characteristic function of Y for any given n and then, determine its distribution type.

Question 3 continued on next page...

Question 3

Prove Schwarz's inequality $E^2[XY] \leq E[X^2]E[Y^2]$ for two random variables X and Y. Then, use it to show that for two jointly stationary processes $X(t)$ and $Y(t)$, we have

$$|R_{XY}(\tau)| \leq \sqrt{R_X(0)R_Y(0)} \leq \frac{1}{2}[R_X(0) + R_Y(0)]$$

Question 4

Prove that the power spectral density of the quadrature component of the bandpass random process $X(t)$ equals

$$S_{X_s}(f) = \left[S_X(f+f_c) + S_x(f-f_c)\right] \sqcap (\frac{f}{2f_c})$$

Question 5

In the block diagram shown in Fig. 1, $X(t)$ denotes a zero-mean Gaussian white WSS noise process with the power spectral density $S_X(f) = \frac{N_0}{2}$.

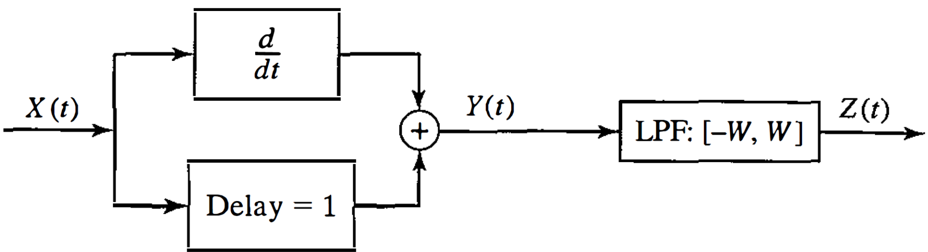

Figure 1: Block diagram of a system with random input.

(a) Is $Z(t)$ a WSS random process? Why?

(b) What is the power spectral density and the mean of $Z(t)$?

(c) What is the power in $Z(t)$?

Question 5 continued on next page...

(d) What is the variance of $Z(t)$?

(e) What is the pdf of $Z(t_0)$?

SOFTWARE QUESTIONS

Question 6

MATLAB provides a function named awgn(), which can add white Gaussian noise to a given signal.

(a) Extend the function that you developed for modeling the point to point microwave radio channel such that the output signal of the channel is polluted by additive while Gaussian noise.

(b) Observe the output of the channel for different levels of the distortion and noise, and discuss the results.

BONUS QUESTIONS

Question 7

Let $X(t)$ be a stationary real normal process with zero mean. Determine the autocorrelation function of the random process $Y(t) = X^2(t)$ in terms of the autocorrelation function of $X(t)$.

Question 8

Return your answers by filling the LaTeXtemplate of the assignment.

EXTRA QUESTIONS

Question 9

Feel free to solve the following questions from the book *Fundamentals of Communication Systems* by J. Proakis and M. Salehi.

1. **Chapter 5, question 38.**
2. **Chapter 5, question 40.**
3. **Chapter 5, question 41.**
4. **Chapter 5, question 45.**
5. **Chapter 5, question 57.**
6. **Chapter 5, question 59.**
7. **Chapter 5, question 61.**