MATHEMATICAL QUESTIONS

Question 1

A vestigial-sideband modulation system is shown in Fig. 1. The bandwidth of the message signal $m(t)$ is W, and the transfer function of the bandpass filter is shown in the figure.

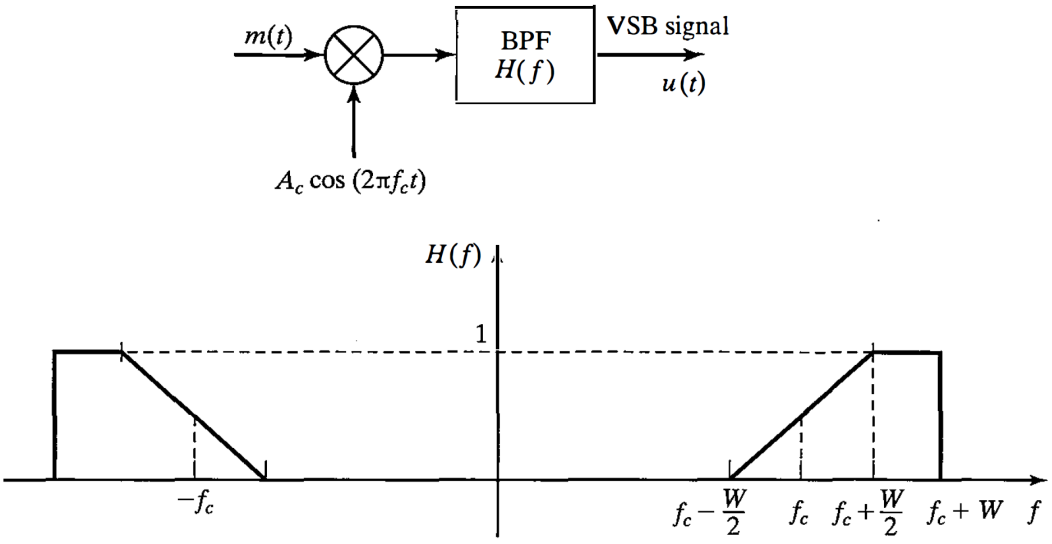

Figure 1: A VSB modulation system.

(a) Determine $h_l(t)$, which is the lowpass equivalent of $h(t)$, where $h(t)$ represents the impulse response of the bandpass filter.

(b) Derive an expression for the modulated signal $u(t)$.

Question 2

Follow the steps below to show the power of the FM signal $u(t) = A_c \cos(2\pi f_c t + \phi(t))$ is $\frac{A_c^2}{2}$.

(a) Write the power expression for the FM signal and show that the power equals $P = \frac{A_c^2}{2} + I$, where

$$I = \lim_{T \to \infty} \frac{A_c^2}{2T} \int_{-T/2}^{T/2} \cos(4\pi f_c t + 2\phi(t)) dt$$

Question 2 continued on next page...

(b) Show that

$$I_\infty = \int_{-\infty}^{\infty} \cos(4\pi f_c t + 2\phi(t))dt$$

relates to the Fourier transforms $\mathcal{F}\{e^{j2\phi(t)}\}$ and $\mathcal{F}\{e^{-j2\phi(t)}\}$ at the frequencies $-2f_c$ and $2f_c$, respectively.

(c) Use Taylor series expansion to show that I_∞ depends to the Fourier transforms $\mathcal{F}\{\phi^n(t)\}, n \in \mathbb{W}$ at the frequency $\pm 2f_c$.

(d) Show that if $f_c \gg W$, where W is the bandwidth of the message-related phase $\phi(t)$, $I_\infty \approx 0$.

(e) Show that the power is approximately equal to $\frac{A_c^2}{2}$.

Question 3

Find the spectrum of the narrowband FM signal

$$u(t) = A_c \cos(2\pi f_c t) - A_c\big[2\pi k_f \int_{-\infty}^{t} m(\tau)d\tau\big] \sin(2\pi f_c t)$$

in terms of the message spectrum $M(f)$.

Question 4

The signal $m(t)$, whose Fourier transform $M(f)$ is shown in Fig. 2, is to be communicated. We know that the signal is normalized, meaning that $-1 \leq m(t) \leq 1$.

(a) If USSB is employed, what is the bandwidth of the modulated signal?.

Question 4 continued on next page...

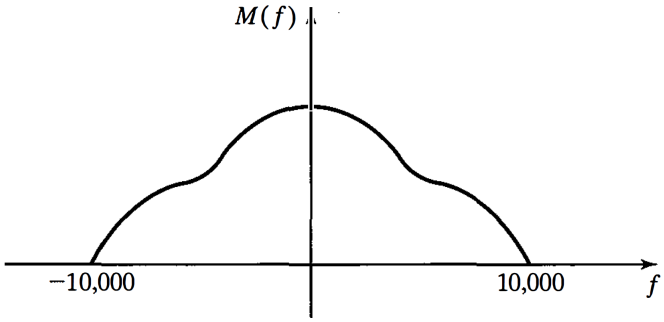

Figure 2: A sample message signal.

(b) If DSB is employed, what is the bandwidth of the modulated signal?

(c) If an AM scheme with the index $a = 0.8$ is used, what is the bandwidth of the modulated signal?

(d) If an FM scheme with the deviation $k_f = 50000$ is used, what is the bandwidth of the modulated signal?

Question 5

Determine the in-phase and quadrature components, $x_c(t)$ and $x_s(t)$, as well as the envelope and the phase, $V(t)$ and $\Theta(t)$ of an FM-modulated signal.

SOFTWARE QUESTIONS

Question 6

Develop a MATLAB mfile that returns the required number of harmonics M_c that includes $x\%$ of the total power content of a sinusoidally-modulated FM signal with the modulation index β. Use the mfile to obtain the number of required harmonics M_c for $x = 98$ and $\beta = 0.5, 1, 2, \cdots, 10$.

BONUS QUESTIONS

Question 7

Return your answers by filling the LaTeXtemplate of the assignment.

EXTRA QUESTIONS

Question 8

Feel free to solve the following questions from the book *Fundamentals of Communication Systems* by J. Proakis and M. Salehi.

1. **Chapter 3, question 7.**
2. **Chapter 3, question 14.**
3. **Chapter 3, question 18.**
4. **Chapter 4, question 5.**
5. **Chapter 4, question 8.**
6. **Chapter 6, question 5.**