MATHEMATICAL QUESTIONS

Question 1

Show that at high SNR conditions, the SNR at the output of a PM demodulator in Fig. 1 is

$$(\frac{S}{N})_o = P_R(\frac{\beta_p}{\max\{|m(t)|\}})^2 \frac{P_m}{N_0 W}$$

, where P_R is the received power after the demodulator bandpass filter, β_p is the modulation index, $N_0/2$ is the power spectral density of the AWGN noise, and W is the bandwidth of the message signal $m(t)$.

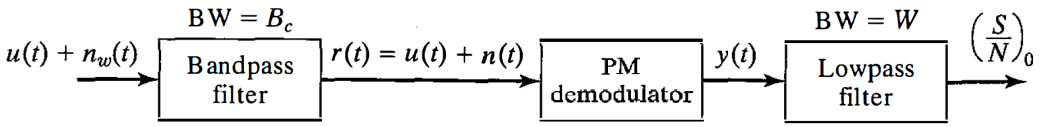

Figure 1: The block diagram of a PM demodulator.

Question 2

As shown in Fig. 2, preemphasis and deemphasis filters may accompany an FM modulation system to improves its achievable SNR.

Figure 2: FM with preemphasis and deemphasis fillers.

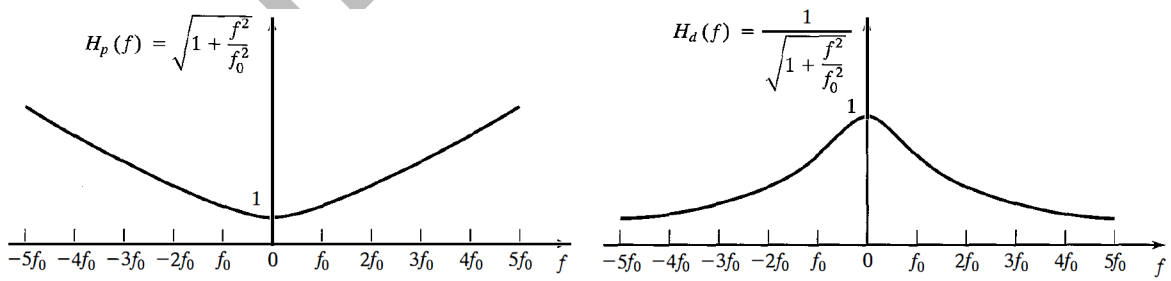

Figure 3: Preemphasis and deemphasis filter characteristics.

Question 2 continued on next page...

(a) Let $H_p(f)$ and $H_d(f)$ in Fig. 3 characterize the preemphasis and deemphasis filters. Find the SNR after the deemphasis filter provided that A_c is the carrier amplitude, k_f is the frequency deviation constant, $N_0/2$ is the power spectral density of the AWGN noise, and W is the bandwidth of the message signal $m(t)$.

(b) How is SNR improved compared to when no emphasis filter is applied?

Question 3

In an analog communication system, demodulation gain is defined as the ratio of the SNR at the output of the demodulator to the SNR at the output of the noise-limiting bandpass filter at the reciver front end. Find expressions for the demodulation gain in each of the following cases.

(a) DSB.

(b) SSB.

(c) Conventional AM with a modulation index a.

(d) FM with a modulation index β_f.

(e) PM with a modulation index β_p.

Question 4

Show that if an FM system and a PM system are employed for transmitting a message signal and these systems have the same output SNR and the same carrier amplitude, then

$$\frac{B_{c_{PM}}}{B_{c_{FM}}} = \frac{\sqrt{3}\beta_f + 1}{\beta_f + 1}$$

, where β_f is the FM index.

SOFTWARE QUESTIONS

Question 5

Develop a MATLAB mfile that takes a message, passes it through an FM modulator, and plots the spectrum of the modulated signal. Plot the spectrum of the modulated signal for several input messages and use the results to validate Carson's bandwidth rule. You may use modulation index, carrier amplitude, and so on as the input argumrnts to your developed mfile.

BONUS QUESTIONS

Question 6

Chernoff bound is a useful tool in communication analysis.

(a) Prove the Chernoff inequality

$$P\{X \geq a\} \leq e^{-ta} E\{e^{tX}\}$$

for any random variable X, $t \geq 0$, and $a \in \mathbb{R}$.

(b) How does the Chernoff bound relate to the characteristic function?

(c) Find the Chernoff bound for the Gaussian random variable $\mathcal{N}(\mu, \sigma^2)$.

Question 6 continued on next page...

(d) Tighten the derived Chernoff bound for the Gaussian random variable $\mathcal{N}(\mu, \sigma^2)$ by selecting a suitable value for t.

Question 7

Return your answers by filling the LaTeXtemplate of the assignment.

EXTRA QUESTIONS

Question 8

Feel free to solve the following questions from the book *Fundamentals of Communication Systems* by J. Proakis and M. Salehi.

1. **Chapter 6, question 2.**
2. **Chapter 6, question 4.**
3. **Chapter 6, question 11.**