MATHEMATICAL QUESTIONS

Question 1

In transmission of telephone signals over line-of-sight microwave links, a combination of FDM-SSB and FM is often employed. A block diagram of such a system is shown in Fig. 1. Each of the signals $m_i(t)$ is bandlimited to W Hz, and these signals are USSB modulated on carriers $c_i(t) = A_i \cos(2\pi f_i t)$, where $f_i = (i-1)W, 1 \leq i \leq K$, and $m(t)$ is the sum of all USSB-modulated signals. This signal FM modulates a carrier with frequency f_c with a modulation index of β and an amplitude of A_c.

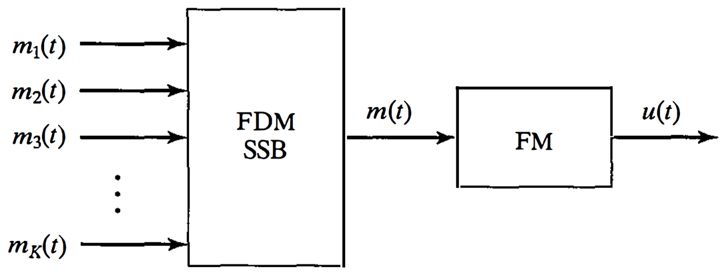

Figure 1: Combined FDM-SSB and FM.

(a) Plot a typical spectrum of the USSB-modulated signal $m(t)$.

(b) Determine the bandwidth of $m(t)$.

(c) At the receiver side, the received signal $r(t) = u(t) + n_W(t)$ is first FM demodulated and then passed through a bank of USSB demodulators, where $n_W(t)$ is an AWGN with a power spectral density of $\frac{N_0}{2}$. Show that the noise power entering these demodulators depend on i.

(d) Determine an expression for the ratio of the noise power entering the demodulator, whose carrier frequency is f_i to the noise power entering the demodulator with the carrier frequency $f_j, 1 \leq i, j \leq K$.

Question 1 continued on next page...

(e) How should the carrier amplitudes A_i be chosen to guarantee that, after USSB demodulation, the SNR for all channels is the same?

Question 2

A superheterodyne FM receiver operates in the frequency range of 88-108 MHz. The IF and local oscillator frequencies are chosen such that $f_{IF} < f_{LO}$. We require that the image frequency f_c' fall outside of the 88-108 MHz region. Determine the minimum required f_{IF} and the range of variation in f_{LO}.

Question 3

Show that the overall noise figure of a cascade of n amplifiers with gains $\mathcal{G}_i$ and noise figures $\mathcal{F}_i$, as shown in Fig. 2, is

$$F = F_1 + \frac{F_2 - 1}{\mathcal{G}_1} + \frac{F_3 - 1}{\mathcal{G}_1 \mathcal{G}_2} + \cdots + \frac{F_n - 1}{\mathcal{G}_1 \mathcal{G}_2 \cdots \mathcal{G}_{n-1}}$$

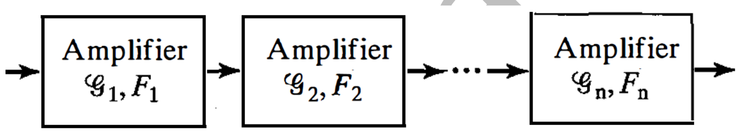

Figure 2: Cascade of several amplifiers.

Question 4

The lowpass signal $x(t)$ with a bandwidth of W is sampled with a sampling interval of T_s, and the signal

$$x_p(t) = \sum_{n=-\infty}^{\infty} x(nT_s)p(t - nT_s)$$

is reconstructed from the samples, where $p(t)$ is an arbitrary-shaped pulse (not necessarily time limited to the interval $[0, T_s]$).

(a) Find the Fourier transform of $x_p(t)$.

(b) Find the conditions for perfect reconstruction of $x(t)$ from $x_p(t)$.

(c) Determine the required reconstruction filter.

Question 5

Show that power spectral density of the pulse amplitude modulation signal $x(t) = \sum_{k=-\infty}^{\infty} a_k p(t-kD)$ is

$$S_x(f) = \frac{1}{D}|P(f)|^2 \sum_{n=-\infty}^{\infty} R_a[n]e^{-j2\pi nfD}$$

, where $P(f)$ is the Fourier transform of $p(t)$ and $R_a[n] = E\{a_{n+k}a_k\}$ is the autocorrelation of the stationary discrete random process a_k.
Hint: Use the definition of the power spectral density of a random process on page 68 of the slides on "Probability and Random Processes".

SOFTWARE QUESTIONS

Question 6

A quantizer with 2^ν quantized levels working over the input range $[-1, 1]$ is fed with a zero-mean Gaussian random process having the power σ^2. Develop a MATLAB function to calculate the signal to quntization noise ratio when the quntization intervals are uniformly distributed and when the quantization intervals are nonuniformly distributed according to μ-law companding method with the parameter μ. Discuss the results for different values of ν, σ^2, and μ. Feel free to plot any suitable curve to better describe the observations.

BONUS QUESTIONS

Question 7 continued on next page...

Question 7

Mean-ergodicity is a useful feature, which allows to replace statistical means with time averages in communication analysis.

(a) Consider the real stationary process $X(t)$ with the statistical average $E\{X(t)\} = \eta$. Define the random variable η_T as

$$\eta_T = \frac{1}{2T}\int_{-T}^{T} X(t)dt$$

, which is called time average random variable. Show that $E\{\eta_T\} = \eta$.

(b) The process $X(t)$ is called mean-ergodic if and only if

$$\lim_{T\to\infty} \mathit{Var}\{\eta_T\} = 0$$

. show that this condition is equivalent to

$$\lim_{T\to\infty} \frac{1}{T}\int_{0}^{2T} C(\alpha)(1-\frac{\alpha}{2T})d\alpha = 0$$

, where the autocovariance $C(\tau) = R(\tau) - \eta^2$ and $R(\tau)$ is the autocorrelation function of $X(t)$.

(c) Show that a zero-mean white process with a power spectral density of $\frac{N_0}{2}$ is mean-ergodic.

(d) How can we find the statistical mean of a zero-mean white process from the time average of its any given sample function?

Question 8

Return your answers by filling the LaTeXtemplate of the assignment.

EXTRA QUESTIONS

Question 9

Feel free to solve the following questions from the book *Fundamentals of Communication Systems* by J. Proakis and M. Salehi.

1. **Chapter 4, question 19.**
2. **Chapter 6, question 14.**
3. **Chapter 6, question 15.**
4. **Chapter 7, question 6.**