Digital Communication

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Overview

Digital Communication

Advatages and Disadvantages

- ✓ Hardware stability.
- ✓ Operational flexibility.
- ✓ Reliable reproduction.
- ✓ Noise immunity.
- ✗ Complex implementation.

Baseband and Passband Transmission

1. Baseband digital transmission
   1. Lowpass channel.
   2. Carrier-less.
   3. Usually short distance.
   4. Usually wired.
2. Passband digital transmission
   1. Bandpass channel.
   2. Carrier-oriented.
   3. Usually long distance.
   4. Usually wireless.

Baseband Digital Transmission

PAM

Statement (Digital Pulse Amplitude Modulation Signal)

A digital pulse amplitude modulation signal is expressed as

$$x(t) = \sum_{k=-\infty}^{\infty} a_k p(t - kD)$$

, where the kth symbol a_k belongs to a set of $M = 2^n$ levels and $p(t)$ is a pulse that satisfies the condition

$$p(KD) = \begin{cases} 1, & K = 0 \\ 0, & K = \pm 1, \pm 2, \cdots \end{cases}$$

✓ Clearly, $x(KD) = \sum_{k=-\infty}^{\infty} a_k p(KD - kD) = a_K$.

Baud Rate and Bit Rate

Definition (Baud Rate)

The baud or symbol rate of a PAM signal is defined as

$$r = \frac{1}{D}$$

Definition (Bit Rate)

The bit rate of a PAM signal is defined as

$$r_b = r \log_2(M) = \frac{\log_2(M)}{D} = \frac{n}{D} = rn$$

Line Codes

Binary PAM formats with rectangular pulses

1. Unipolar return to zero with $p(t) = \sqcap(\frac{t}{D/2})$ and $a_k = b_k A$.
2. Unipolar nonreturn to zero with $p(t) = \sqcap(\frac{t}{D})$ and $a_k = b_k A$.
3. Polar return to zero with $p(t) = \sqcap(\frac{t}{D/2})$ and $a_k = (b_k - 0.5)A$.
4. Polar nonreturn to zero with $p(t) = \sqcap(\frac{t}{D})$ and $a_k = (b_k - 0.5)A$.
5. Twinned binary with $p(t) = \sqcap(\frac{t}{D/2}) - \sqcap(\frac{t-D/2}{D/2})$ and $a_k = (b_k - 0.5)A$.

✓ The formats differ in DC value, power, power spectral density, and synchronization.

Line Codes

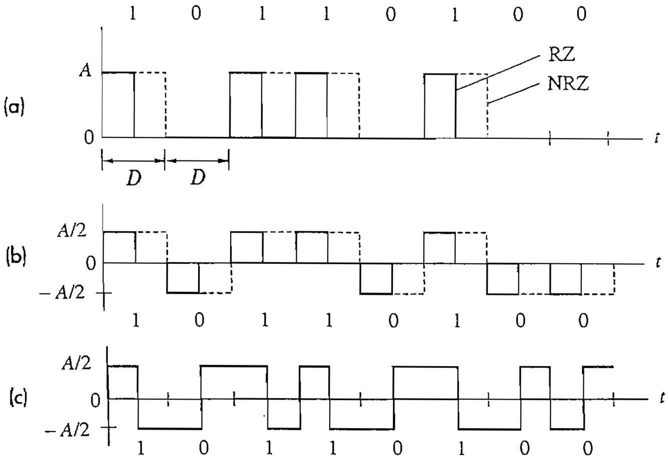

Figure: Binary PAM formats with rectangular pulses (line codes). (a) Unipolar RZ and NRZ (b) Polar RZ and NRZ (c) Twined binary.

M-ary Line Codes

Polar quaternary nonreturn to zero with $p(t) = \sqcap(\frac{t}{D})$ and symbols a_k as

a_k	NBC Code	Gray Code
$3A/2$	11	10
$A/2$	10	11
$-A/2$	01	01
$-3A/2$	00	00

Table: Symbols in polar quaternary NRZ.

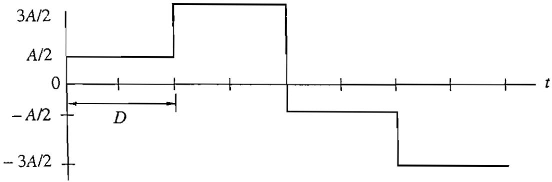

Figure: Polar quaternary NRZ.

PSD of PAM

Statement (Power Spectral Density of PAM)

Power spectral density of the pulse amplitude modulation signal $x(t) = \sum_{k=-\infty}^{\infty} a_k p(t - kD)$ *is*

$$S_x(f) = \frac{1}{D}|P(f)|^2 \sum_{n=-\infty}^{\infty} R_a[n]e^{-j2\pi nfD}$$

, where $P(f)$ *is the Fourier transform of* $p(t)$ *and* $R_a[n] = E\{a_{n+k}a_k\}$ *is the autocorrelation of the stationary discrete random process* a_k*.*

✓ If a_k is a zero-mean uncorrelated discrete random process, $S_x(f) = \frac{R_a[0]}{D}|P(f)|^2$.

PSD of PAM

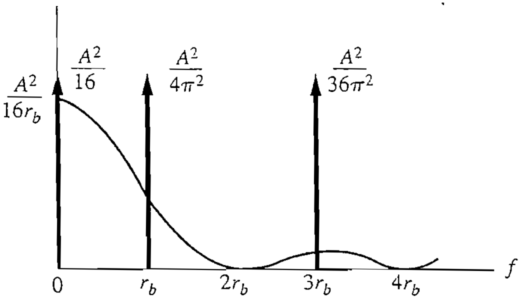

Figure: Power spectra density of unipolar binary RZ.

Transmission Limitations

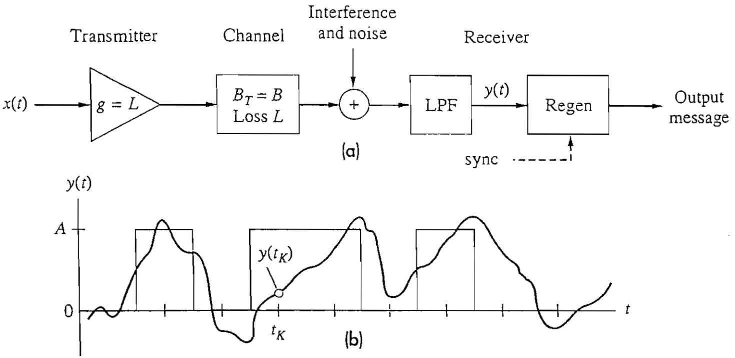

Figure: (a) Baseband transmission system (b) Signal-pluse-noise waveform.

$$y(t) = \sum_{k=-\infty}^{\infty} a_k \tilde{p}(t - t_d - kD) + n(t)$$

$$y(t_K) = y(KD + t_d + t_s) = a_K \tilde{p}(t_s) + \sum_{k \neq K}^{\infty} a_k \tilde{p}(KD + t_s - kD) + n(t_K)$$

Eye Diagram

$$y(t_K) = a_K \underbrace{\tilde{p}(t_s)}_{\text{Synchronization Mismatch}} + \underbrace{\sum_{k \neq K}^{\infty} a_k \tilde{p}(KD + t_s - kD)}_{\text{Inter-Symbol Interference}} + \underbrace{n(t_K)}_{\text{Noise}}$$

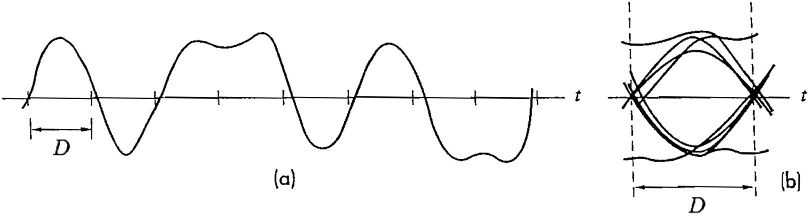

Figure: (a) Distorted polar binary signal (b) Eye diagram.

Eye Diagram

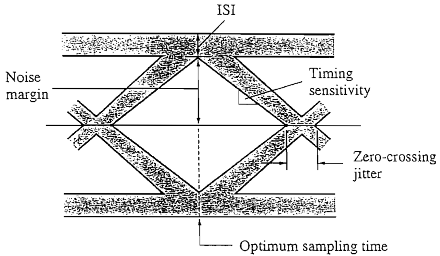

Figure: Binary eye pattern.

1. Inter-symbol interference
2. Optimum sampling time
3. Zero-crossing jitter
4. Noise margin
5. Timing sensitivity
6. Nonlinear distortion

Synchronization Mismatch and ISI

Statement (Suitable Synchronization)

A suitable synchronization can mitigate or eliminate synchronization mismatch. Zero-crossing in PAM signal has a key role in synchronization.

Statement (ISI Cancellation)

Given an ideal lowpass channel of bandwidth B, it is possible to transmit independent symbols at a rate $r \leq 2B$ baud without ISI. It is not possible to transmit independent symbols at $r > 2B$.

✓ Signaling at the maximum rate $r = 2B$ requires since pulse shaping $p(t) = \text{sinc}(rt)$.

Bit Error Probability

Statement (Bit Error Probability of Unipolar NRZ)

Assuming perfect ISI cancellation and synchronization, the bit error probability for unipolar NRZ binary signaling in zero-mean Gaussian noise with variance σ^2 *is* $Pe = Q(A/(2\sigma))$ *, where* $Q(x)$ *is the tail distribution function of the standard normal distribution, and* 0 *and* A *are the symbols corresponding to the equally-probable binary digits* 0 *and* 1.

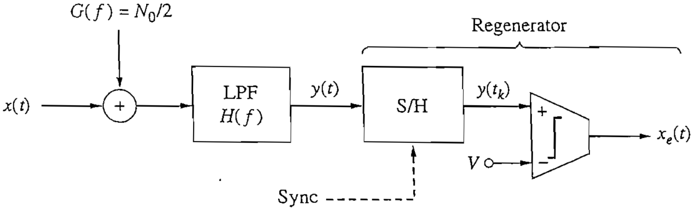

Figure: Baseband binary receiver.

Bit Error Probability

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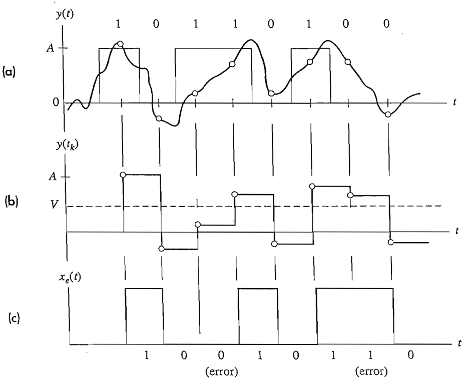

Figure: A unipolar NRZ signal (a) signal plus noise (b) S/H output (c) Comparator output.

Bit Error Probability

Statement (Bit Error Probability of Unipolar NRZ)

Assuming perfect ISI cancellation and synchronization, the bit error probability for unipolar NRZ binary signaling is $Pe = Q(A/(2\sigma))$.

$$
\begin{aligned}
P_e &= P_0 P_{e|0} + P_1 P_{e|1} = \frac{1}{2}(P_{e|0} + P_{e|1}) \\
&= \frac{1}{2}(P[y(t_K) > V | a_K = 0] + P[y(t_K) \leq V | a_K = A]) \\
&= \frac{1}{2}(P[n(t_K) > V | a_K = 0] + P[n(t_K) + A \leq V | a_K = A]) \\
&= \frac{1}{2}(Q(\frac{V}{\sigma}) + Q(\frac{A - V}{\sigma}))
\end{aligned}
$$

$$
\frac{dP_e}{dV} = 0 \Rightarrow V = \frac{A}{2} \Rightarrow P_{e_{\min}} = Q(\frac{A}{2\sigma})
$$

Passband Digital Transmission

Transmission Techniques

Common passband digital transmission techniques

1. Amplitude Shift Keying (ASK)
2. Phase Shift Keying (PSK)
3. Frequency Shift Keying (FSK)

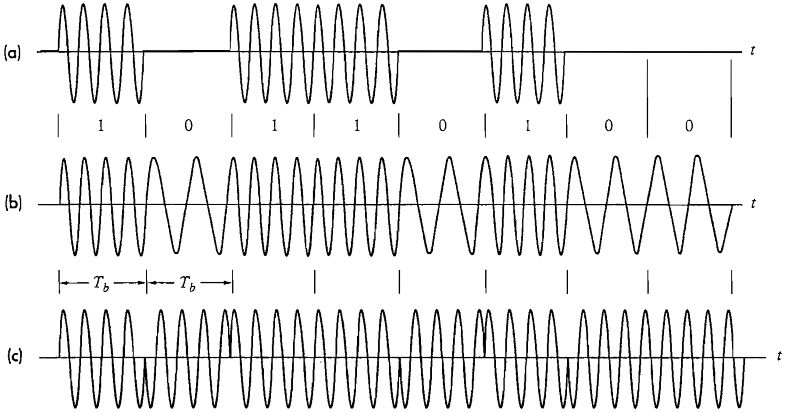

Figure: Binary modulated waveforms (a) ASK (b) FSK (c) PSK.

Passband Digital Signal

Statement (Passband Digital Signal)

Any modulated passband signal may be expressed in the quadrature-carrier form

$$x(t) = A_c[x_i(t)\cos(2\pi f_c t + \theta) - x_q(t)\sin(2\pi f_c t + \theta)]$$

. The carrier frequency f_c, amplitude A_c, and phase θ are constant, while the time-varying in-phase $x_i(t)$ and quadrature $x_q(t)$ components contain the message.

PSD of Passband Digital Signal

Statement (PSD of Passband Digital Signal)

Power spectral density of the passband digital signal $x(t) = A_c[x_i(t)\cos(2\pi f_c t + \theta) - x_q(t)\sin(2\pi f_c t + \theta)]$ *is*

$$S_x(f) = \frac{A_c^2}{4}[S_{x_i}(f - f_c) + S_{x_i}(f + f_c) + S_{x_q}(f - f_c) + S_{x_q}(f + f_c)]$$

, where $S_{x_i}(f)$ *and* $S_{x_q}(f)$ *are the power spectral density of the in-phase and quadrature components, respectively.*

PSD of Passband Digital Signal

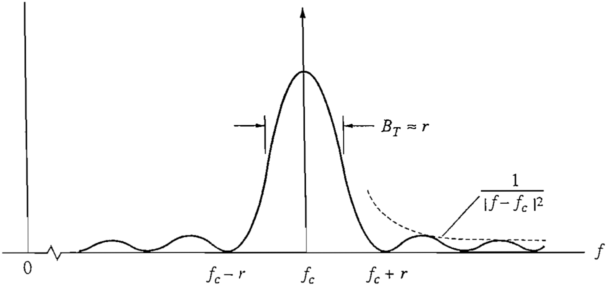

Figure: Power spectra density of ASK.

ASK

Statement (ASK)

In ASK,

$$x_i(t) = \sum_k a_k p(t - kD), \quad x_q(t) = 0$$

$$a_k = 0, 1, \cdots, M - 1$$

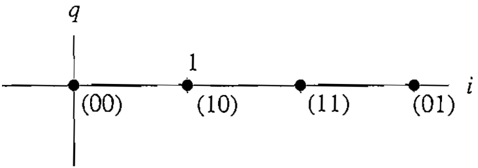

Figure: ASK constellation.

PSK

Statement (PSK)

In PSK,

$$x_i(t) = \sum_k \cos(\phi_k)p(t-kD), \quad x_q(t) = \sum_k \sin(\phi_k)p(t-kD)$$

$$\phi_k = \pi(2a_k+1)/M, \quad a_k = 0, 1, \cdots, M-1$$

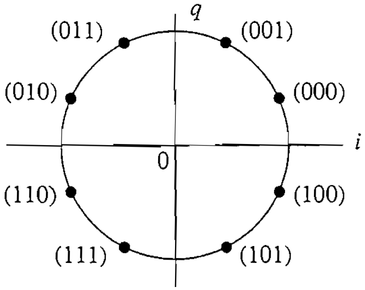

Figure: PSK constellation.

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