Question 1

Consider the block diagram of Fig. 1, where

1. $m(t)$ **is a lowpass message with the bandwidth** W_m**.**
2. **The zero-order hold ADC has the sampling rate** f_s **and quantization level** 2^ν**.**
3. **In the PAM modulator, the bits are mapped to the bipolar NRZ signal**

$$u(t) = \sum_{k=-\infty}^{\infty} a_k p(t - kT)$$

, where the pulse $p(t) = \mathrm{sinc}(\frac{t}{T})$ **and the binary polar symbols** $a_k = 2(b_k - 0.5)A$**. For simplicity, the symbols are assumed to be independent and identically distributed with** $P\{a_k = A\} = P\{a_k = -A\} = 0.5$**, the mean** 0**, and the autocorrelation function**

$$R_a[n] = E\{a_{n+k}a_k\} = \begin{cases} A^2, & n = 0 \\ 0, & n \neq 0 \end{cases}$$

4. **The FM modulator has the index** β_f **and its output is** $v(t) = A_c \cos(2\pi f_c t + 2\pi k_f \int_{-\infty}^{t} u(\tau)d\tau)$**.**
5. **The channel adds an AWGN noise** $n_W(t)$ **with the power spectral density** $\frac{N_0}{2}$ **to its input signal.**
6. **The FM demodulator is an ideal FM receiver.**
7. **The PAM demodulator works perfectly without any ISI or synchronization mismatch. The comparator threshold is set to its optimal value in the PAM demodulator.**
8. **The conditions of the perfect reconstruction are held in the DAC.**
9. **The required filters in the modulators and demodulators are ideal.**

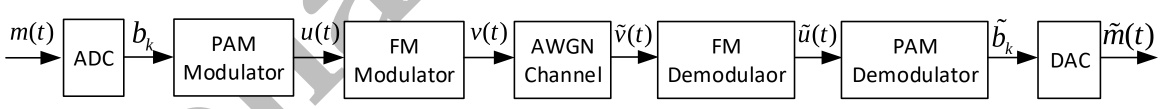

Figure 1: A mixed analog-digital communication system.

(a) Write an expression for the power spectral density $S_u(f)$ *of* $u(t)$ *and determine the power content* P_u *of* $u(t)$.

(b) Determine 100% *power bandwidth of* $u(t)$*, i.e., the bandwidth* $[0, W_u]$ *containing half of the power content of* $u(t)$.

(c) *Determine the power P_v and bandwidth W_v of $v(t)$.*

(d) *Determine the output bit rate B_b of the ADC.*

(e) *Determine the bit rate B_u and symbol rate S_u of the PAM modulator.*

(f) *Specify the required conditions for perfect reconstruction of $\widetilde{m}(t)$ at the DAC.*

(g) *Assuming high SNR conditions for the FM demodulator, find the SNR of $\tilde{u}(t)$.*

(h) *Assuming high SNR conditions for the FM demodulator, find the BER of $\tilde{b}_k$.*

(i) *Assuming that the channel bandwidth is W_c, find the maximum bandwidth of $m(t)$ for which the system works properly.*

(j) *Find the maximum bandwidth of $m(t)$ for which the BER of $\tilde{b}_k$ is less than a given value of $P_{e_{th}}$.*