Question 1

A zero-mean white Gaussian WSS noise process $n_w(t)$ with the power spectral density $S_{n_w}(f) = \frac{N_0}{2}$ passes the bandpass filter shown in Fig. 1 to generate the colored noise $n(t)$.

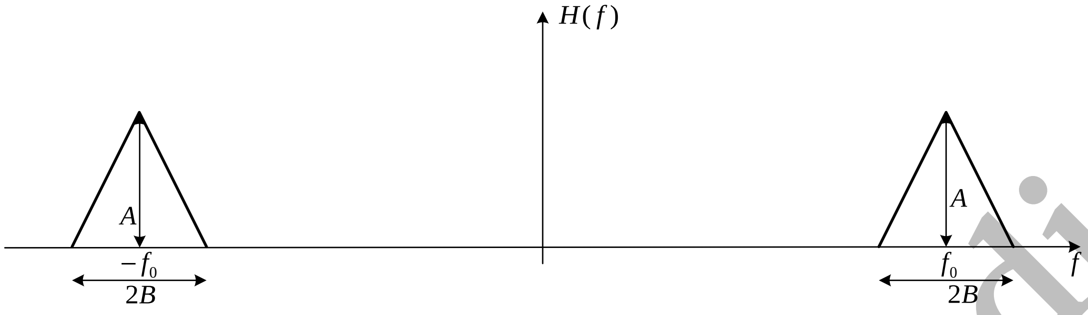

Figure 1: Triangle bandpass signal.

(a) Find the noise equivalent bandwidth of $H(f)$.

$$B_{neq} = \frac{\int_{-\infty}^{\infty} |H(f)|^2 df}{2H^2_{\text{max}}} = \frac{4\int_0^B (A - \frac{A}{B}f)^2 df}{2A^2} = \frac{4A^2\int_0^B (1 - \frac{2f}{B} + \frac{f^2}{B^2}) df}{2A^2} = 2(f - \frac{f^2}{B} + \frac{f^3}{3B^2})\Big|_0^B$$

, which equals

$$B_{neq} = 2(B - B + \frac{B}{3}) = \frac{2B}{3}$$

(b) Derive and draw the power spectral density and calculate the power content of the in-phase process $n_c(t)$ and quadrature process $n_s(t)$ of $n(t)$.

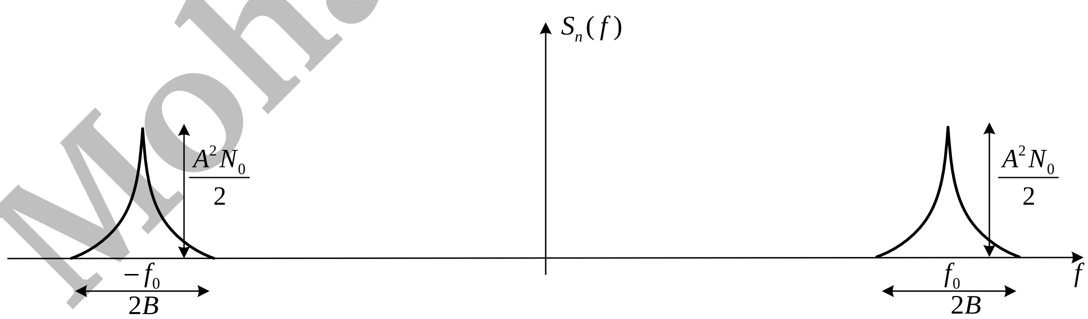

Figure 2: Power spectral density of the colored noise.

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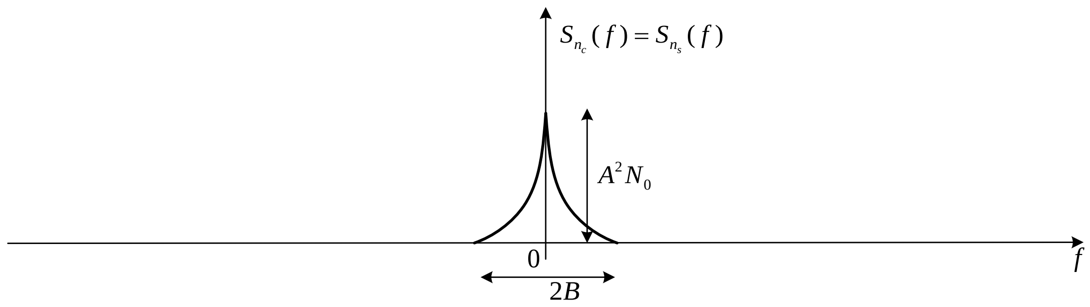

Figure 3: Power spectral density of the in-phase and quadrature processes.

The power spectral density of the colored noise is

$$S_n(f) = \frac{N_0}{2}|H(f)|^2 = \frac{N_0 A^2}{2}[\Lambda^2(\frac{f-f_0}{B}) + \Lambda^2(\frac{f+f_0}{B})]$$

, which is shown in Fig. 2. The power spectral density of the in-phase and quadrature components are equal to

$$S_{n_s}(f) = S_{n_c}(f) = \big[S_n(f+f_0) + S_n(f-f_0)\big] \sqcap (\frac{f}{2f_0}) = N_0 A^2 \Lambda^2(\frac{f}{B})$$

, as drawn in Fig. 3. The power of the colored, in-phase, and quadrature noises are the same and equal to

$$P_{n_s} = P_{n_c} = P_n = N_0 B_{neq} H^2_{\text{max}} = \frac{2N_0 B A^2}{3}$$

(c) Find the joint probability density function of $n_c(t_0)$ and $n_s(t_1)$.

Since f_0 and $-f_0$ are the symmetry axis of the positive and negative sections of $S_n(f)$, the in-phase and quadrature components are independent. Further, each component is Gaussian process with the mean $\mu = 0$ and variance $\sigma^2 = P_{n_s} = P_{n_c} = P_n$. So,

$$f_{n_c(t_0),n_s(t_1)}(n_c, n_s) = f_{n_c(t_0)}(n_c) f_{n_s(t_1)}(n_s) = \frac{1}{\sqrt{2\pi P_n}} e^{-\frac{n_c^2}{2P_n}} \frac{1}{\sqrt{2\pi P_n}} e^{-\frac{n_s^2}{2P_n}}$$

, which simplifies to

$$f_{n_c(t_0),n_s(t_1)}(n_c, n_s) = \frac{1}{2\pi P_n} e^{-\frac{n_c^2+n_s^2}{2P_n}} = \frac{3}{4\pi N_0 B A^2} e^{-\frac{3n_c^2+3n_s^2}{4N_0 B A^2}}$$