Question 1

Find the output SNR of the demodulator in the Fig. 1, where

1. $m(t)$ **is a lowpass message with the bandwidth** W**.**
2. **The frequency response of the bandpass filters is** $\sqcap(\frac{f-f_c}{2W}) + \sqcap(\frac{f+f_c}{2W})$**.**
3. **The input-output relation in the distortion-less channel is** $y(t) = Lx(t-D)$**.**
4. $n_W(t)$ **is an AWGN noise with the power spectral density** $\frac{N_0}{2}$**.**
5. **The oscillators generate** $A_c \cos(2\pi f_c t)$**.**
6. **The lowpass filter is described by** $\sqcap(\frac{f}{2W})$**.**

. For which values of the attenuation $L \leq 1$ **and delay** $D \geq 0$ **the SNR is maximized?**

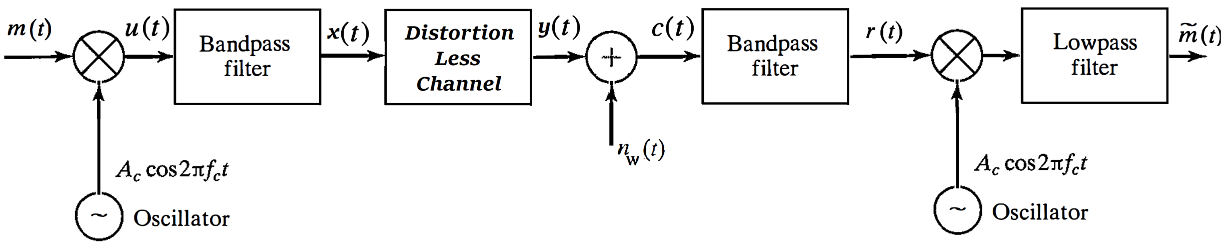

Figure 1: DSB system with a noisy distortion-less channel.

We have $u(t) = A_c m(t)\cos(2\pi f_c t)$. The spectrum of the modulated signal $u(t)$ falls within the passband of the bandpass filter, so $x(t) = u(t)$. Then,

$$y(t) = Lx(t-D) = LA_c m(t-D)\cos(2\pi f_c(t-D))$$

and

$$c(t) = LA_c m(t-D)\cos(2\pi f_c(t-D)) + n_W(t)$$

. Again, the bandpass filter does not change the signal part of $c(t)$ while filters the noise $n_W(t)$ to generate the colored noise $n(t) = n_c(t)\cos(2\pi f_c t) - n_s(t)\sin(2\pi f_c t)$. So,

$$r(t) = LA_c m(t-D)\cos(2\pi f_c(t-D)) + n_c(t)\cos(2\pi f_c t) - n_s(t)\sin(2\pi f_c t)$$

. After the mixer,

$$r(t)A_c\cos(2\pi f_c t) = LA_c^2 m(t-D)\frac{1}{2}[\cos(4\pi f_c t - 2\pi f_c D) + \cos(2\pi f_c D)] + \frac{A_c}{2}n_c(t) + \frac{A_c}{2}n_c(t)\cos(4\pi f_c t) - \frac{A_c}{2}n_s(t)\sin(4\pi f_c t)$$

Finally, the lowpass filter yields

$$\widetilde{m}(t) = LA_c^2 m(t-D)\frac{1}{2}\cos(2\pi f_c D) + \frac{A_c}{2}n_c(t)$$

. Now,

$$P_o = \frac{1}{4}L^2 A_c^4 \cos^2(2\pi f_c D) P_m$$

and

$$P_{n_o} = \frac{1}{4}A_c^2 P_{n_c} = \frac{1}{4}A_c^2 P_n = \frac{1}{4}A_c^2 \frac{N_0}{2} 2W \times 2 = \frac{1}{2}A_c^2 N_0 W$$

. Finally,

$$(\frac{S}{N})_o = \frac{P_o}{P_{n_o}} = \frac{L^2 A_c^2 \cos^2(2\pi f_c D) P_m}{2N_0 W}$$

. The SNR is maximized for $L = 1$ and $2\pi f_c D = n\pi, n \in \mathbb{W}$, or equivalently,

$$L = 1, \quad D = \frac{n}{2f_c}, n \in \mathbb{W}$$

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