Question 1

Consider the block diagram of Fig. 1, where

1. **$m(t)$ is a lowpass message with the bandwidth W.**
2. **The output of the USSB modulator is $u(t) = A_{c_{USSB}}\big[m(t)\cos(2\pi f_{c_{USSB}}t) - \hat{m}(t)\sin(2\pi f_{c_{USSB}}t)\big]$.**
3. **The FM modulator has the index β_f and its output is $v(t) = A_{c_{FM}}\cos(2\pi f_{c_{FM}}t + 2\pi k_f\int_{-\infty}^{t} u(\tau)d\tau)$.**
4. **The USSD demodulator is an ideal coherent receiver.**
5. **The FM demodulator is an ideal FM receiver.**
6. **The required filters in the modulators and demodulators are ideal.**
7. **$n_W(t)$ is an AWGN noise with the power spectral density $\frac{N_0}{2}$.**

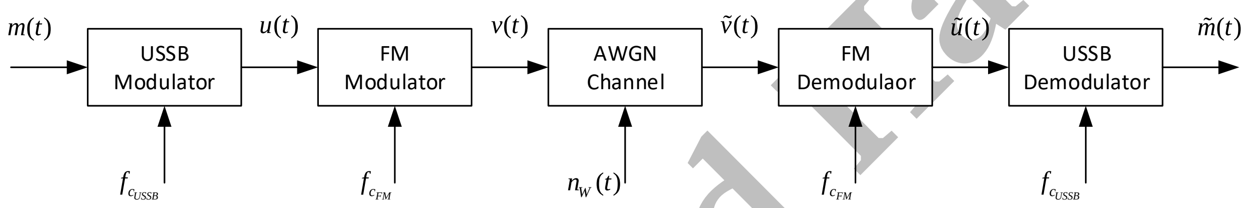

Figure 1: Cascade of USSB and FM modulations.

(a) Calculate the occupied bandwidth on the channel.

Here $u(t)$ acts a message with the bandwidth $B_u = f_{c_{USSB}} + W$ for the FM modulator. Therefore, the occupied bandwidth equals

$$B_v = 2(\beta_f + 1)B_u = 2(\beta_f + 1)(f_{c_{USSB}} + W)$$

(b) Calculate the power transmitted to the channel.

The power injected to the channel is simply

$$P_v = \frac{A_{c_{FM}}^2}{2}$$

(c) Assuming high SNR conditions, find the SNR of $\tilde{u}(t)$.

We know that $B_u = f_{c_{USSB}} + W$ and $P_u = A_{c_{USSB}}^2 P_m$. So,

$$(\frac{S}{N})_{\tilde{u}} = \frac{3k_f^2 A_{c_{FM}}^2 P_u}{2N_0 B_u^3} = \frac{3k_f^2 A_{c_{FM}}^2 A_{c_{USSB}}^2 P_m}{2N_0(f_{c_{USSB}} + W)^3}$$

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(d) Assuming high SNR conditions, find the SNR of $\tilde{m}(t)$.

The output of the FM demodulator is

$$\hat{u}(t) = k_f u(t) + \frac{1}{2\pi}\frac{dY_n(t)}{dt}$$
$$= k_f A_{c_{USSB}} m(t)\cos(2\pi f_{c_{USSB}} t) - k_f A_{c_{USSB}} \hat{m}(t)\sin(2\pi f_{c_{USSB}} t) + \frac{1}{2\pi}\frac{dY_n(t)}{dt}$$

, where the PSD of the noise part $\frac{1}{2\pi}\frac{dY_n(t)}{dt}$ is $\frac{N_0}{A_{c_{FM}}^2} f^2$ for $f \in [-f_{c_{USSB}} - W, f_{c_{USSB}} + W]$. When $\hat{u}(t)$ passes the USSB demodulator, the output is

$$\tilde{m}(t) = \frac{A_{c_{USSB}} k_f}{2} m(t) + \frac{1}{2} n_c(t)$$

, where $n_c(t)$ is the in-phase component of a colored noise, generated from bandpass filtering $\frac{1}{2\pi}\frac{dY_n(t)}{dt}$, with the power spectral density

$$\begin{cases} \frac{N_0}{A_{c_{FM}}^2} f^2, & f_{c_{USSB}} \leq f \leq f_{c_{USSB}} + W \\ \frac{N_0}{A_{c_{FM}}^2} f^2, & -f_{c_{USSB}} - W \leq f \leq -f_{c_{USSB}} \\ 0, & \text{otherwise} \end{cases}$$

. The power of in-phase component of the noise is

$$P_{n_o} = \frac{1}{4} P_{n_c} = \frac{1}{4} P_n$$
$$= \frac{2}{4} \int_{f_{c_{USSB}}}^{f_{c_{USSB}}+W} \frac{N_0}{A_{c_{FM}}^2} f^2 df$$
$$= \frac{N_0}{6A_{c_{FM}}^2} \left[(f_{c_{USSB}} + W)^3 - f_{c_{USSB}}^3\right]$$
$$= \frac{N_0}{6A_{c_{FM}}^2} \left[3f_{c_{USSB}}^2 W + 3f_{c_{USSB}} W^2 + W^3\right]$$
$$= \frac{N_0 W}{6A_{c_{FM}}^2} \left[3f_{c_{USSB}}^2 + 3f_{c_{USSB}} W + W^2\right]$$

Further,

$$P_{s_o} = \frac{1}{4} A_{c_{USSB}}^2 k_f^2 P_m$$

Finally,

$$(\frac{S}{N})_{\tilde{m}} = \frac{P_{s_o}}{P_{n_o}} = \frac{3A_{c_{FM}}^2 A_{c_{USSB}}^2 k_f^2 P_m}{2N_0 W \left[3f_{c_{USSB}}^2 + 3f_{c_{USSB}} W + W^2\right]}$$