Question 1

Consider the block diagram of a DSB communication system in Fig. 1, where

1. $m(t)$ **is a lowpass message with the bandwidth** W.
2. **The bandwidth of the bandpass filters and noise equivalent bandwidth of the amplifiers are set to the minimum possible value. Assume an ideal rectangular frequency response for the filters and amplifiers.**
3. **There are** K **stages of amplification, each having the same gain** $\mathcal{G}$ **and noise figure** F.
4. **The carrier is** $A_c \cos(2\pi f_c t)$ **and the local oscillator is phase-locked to it.**
5. **The bandwidth of the lowpass filter is set to its minimum value.**
6. **The mixers are ideal.**
7. $n_1(t)$ **and** $n_2(t)$ **are independent AWGN with the power spectral densities** $\frac{N_1}{2}$ **and** $\frac{N_2}{2}$.

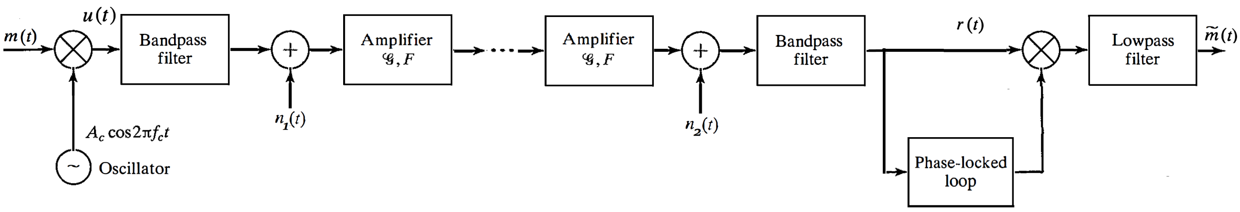

Figure 1: DSB modulation in a channel with cascade of amplifiers.

(a) Find the SNR at the input to the first stage amplifier.

$$(\frac{S}{N})_i = \frac{P_u}{N_1 B_{neq}} = \frac{A_c^2 P_m}{4N_1 W}$$

(b) Obtain a closed-form expression for the total noise figure of the amplifiers in terms of K, $\mathcal{G}$, and F.

According to Fries's formula,

$$F_{tot} = F_1 + \frac{F_2 - 1}{\mathcal{G}_1} + \frac{F_3 - 1}{\mathcal{G}_1 \mathcal{G}_2} + \cdots + \frac{F_K - 1}{\mathcal{G}_1 \mathcal{G}_2 \cdots \mathcal{G}_{K-1}} = 1 + F - 1 + \frac{F-1}{\mathcal{G}} + \frac{F-1}{\mathcal{G}^2} + \cdots + \frac{F-1}{\mathcal{G}^{K-1}}$$

, which is a geometric sum and equals

$$F_{tot} = 1 + (F-1)\sum_{k=0}^{K-1} (\frac{1}{\mathcal{G}})^k = 1 + (F-1)\frac{1 - (\frac{1}{\mathcal{G}})^K}{1 - \frac{1}{\mathcal{G}}}$$

(c) Find the SNR at the output of the noise limiting bandpass filter.

Clearly, the received signal power is

$$P_s = \frac{A_c^2 P_m}{2} \mathcal{G}^K$$

We now that the SNR at the output of the amplifiers is

$$\frac{1}{F_{tot}} (\frac{S}{N})_i = \frac{P_s}{P_{n_1}}$$

, where P_{n_1} is the noise power at the output of the amplifiers. Noting the independency of the noise sources, the total noise power after the noise-limiting bandpass filter equals

$$P_n = P_{n_1} + P_{n_2} = \frac{P_s F_{tot}}{(\frac{S}{N})_i} + 2WN_2 = 2WN_1 \mathcal{G}^K F_{tot} + 2WN_2$$

Finally,

$$(\frac{S}{N})_r = \frac{P_s}{P_n} = \frac{A_c^2 P_m \mathcal{G}^K}{4W(N_1 \mathcal{G}^K F_{tot} + N_2)} = \frac{A_c^2 P_m}{4W} \frac{\mathcal{G}^K}{N_1[\mathcal{G}^K + (F-1)\frac{\mathcal{G}^K - 1}{1 - \frac{1}{\mathcal{G}}}] + N_2}$$

(d) Find the SNR at the output of the demodulator.

The filtering in amplifiers is ideal and rectangular with the gain $H_{max} = \sqrt{\mathcal{G}}$. So, the received signal is

$$r(t) = \sqrt{\mathcal{G}}^K A_c m(t) \cos(2\pi f_c t) + n_c(t) \cos(2\pi f_c t) - n_s(t) \cos(2\pi f_c t)$$

After the coherent detection and lowpass filtering,

$$\tilde{m}(t) = \frac{1}{2} \sqrt{\mathcal{G}}^K A_c m(t) + \frac{1}{2} n_c(t)$$

We have,

$$P_{s_o} = \frac{1}{4} \mathcal{G}^K A_c^2 P_m = \frac{1}{2} P_s$$

and

$$P_{n_o} = \frac{1}{4} P_{n_c} = \frac{1}{4} P_n$$

So,

$$(\frac{S}{N})_o = \frac{P_{s_o}}{P_{n_o}} = 2\frac{P_s}{P_n} = 2(\frac{S}{N})_r = \frac{A_c^2 P_m}{2W} \frac{\mathcal{G}^K}{N_1[\mathcal{G}^K + (F-1)\frac{\mathcal{G}^K - 1}{1 - \frac{1}{\mathcal{G}}}] + N_2}$$