Student Name: .....
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Question 1

Consider the circuit shown in Fig. 1 that is made by interconnection of several two-ports and one-ports. Let the transmittance matrix of the two-port N_0 be $\boldsymbol{T}_0 = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$. Further, assume that the interconnection of the two-ports does not violate two-port current conditions.

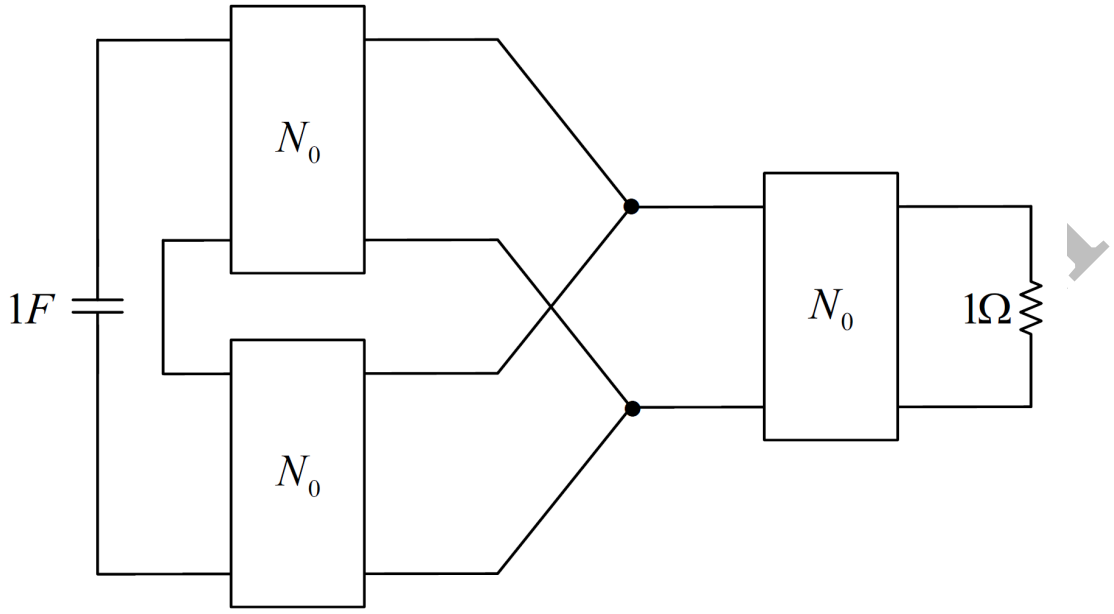

Figure 1: A circuit made from several internal two-ports.

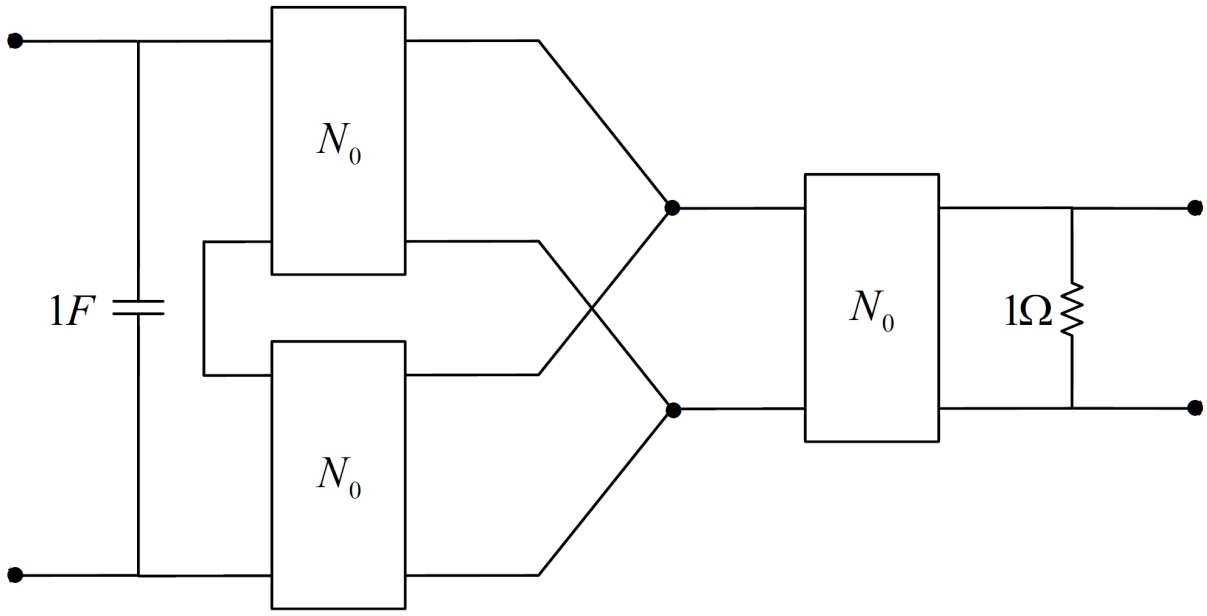

Figure 2: Two-port extension of the circuit shown in Fig. 1.

Question 1 continued on next page...

(a) Find as much as you can the natural frequencies of the circuit.

Clearly, the poles of the impedance matrix elements of the two-port shown in Fig. 2 give the natural frequencies of the circuit. So, we should find the impedance matrix of the two-port in Fig. 2.

$$\boldsymbol{T}_0 = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} \Rightarrow \boldsymbol{H}_0 = \begin{bmatrix} 1 & 0 \\ -1 & 1 \end{bmatrix}$$

Noting the series-parallel interconnection,

$$\boldsymbol{H}_1 = \boldsymbol{H}_0 + \boldsymbol{H}_0 = \begin{bmatrix} 2 & 0 \\ -2 & 2 \end{bmatrix} \Rightarrow \boldsymbol{T}_1 = \begin{bmatrix} 2 & 1 \\ 1 & 0.5 \end{bmatrix}$$

Now, we have a cascade interconnection.

$$\boldsymbol{T}_2 = \boldsymbol{T}_1\boldsymbol{T}_0 = \begin{bmatrix} 3 & 3 \\ -1.5 & 1.5 \end{bmatrix} \Rightarrow \boldsymbol{Y}_2 = \begin{bmatrix} \frac{1}{2} & 0 \\ -\frac{1}{3} & 1 \end{bmatrix}$$

Finally, the desired two-port is created by adding the parallel admittances to the ports. So,

$$\boldsymbol{Y} = \begin{bmatrix} \frac{1}{2} + s & 0 \\ -\frac{1}{3} & 1+1 \end{bmatrix} \Rightarrow \boldsymbol{Z} = \boldsymbol{Y}^{-1} \begin{bmatrix} \frac{2}{2s+1} & 0 \\ \frac{1}{3(2s+1)} & \frac{s+0.5}{2s+1} \end{bmatrix}$$

So, the natural frequency is

$$2s + 1 = 0 \Rightarrow s = -0.5$$

(b) Now, consider the two-port shown in Fig. 2. Thw two-port is made from the circuit of Fig. 1 by adding the shown ports. How many descriptions does the two-port have? Determine which descriptions exist and which does not exist. Obtain two descriptions if the two-port has more than two descriptions.

We have already obtained the impedance and admittance descriptions in the previous part. Noting the table of "descriptions interrelation", since the element z_{12} of the impedance matrix is zero, the A′B′C′D′ description does not exist. The determinant of the impedance matrix and three of its elements are nonzero. So, the two-port has 5 descriptions among which the impedance and admittance descriptions are calculated.