Question 1

For the circuit of Fig. 1, find an expression for $i_L(t)$ valid for all $t0$.

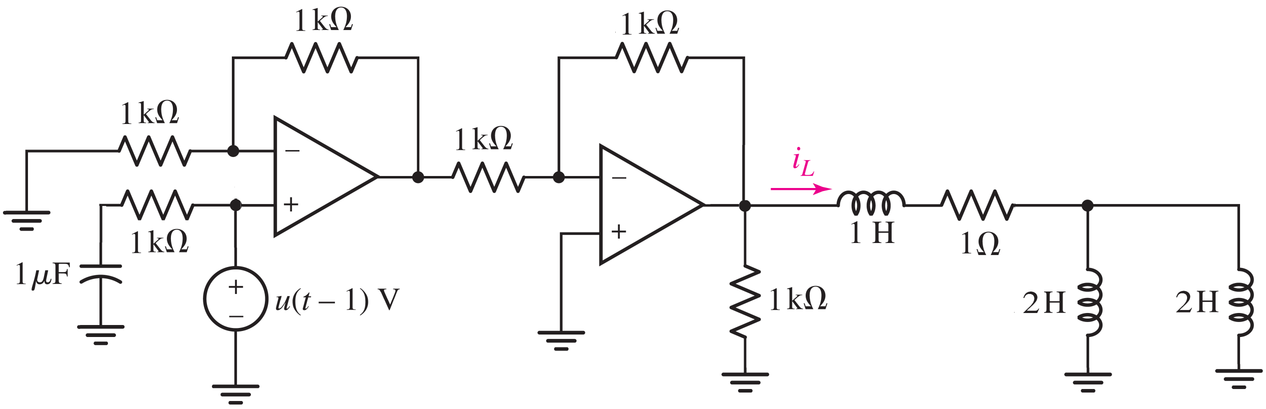

Figure 1: A circuit for which the current $i_L(t)$ is desired.

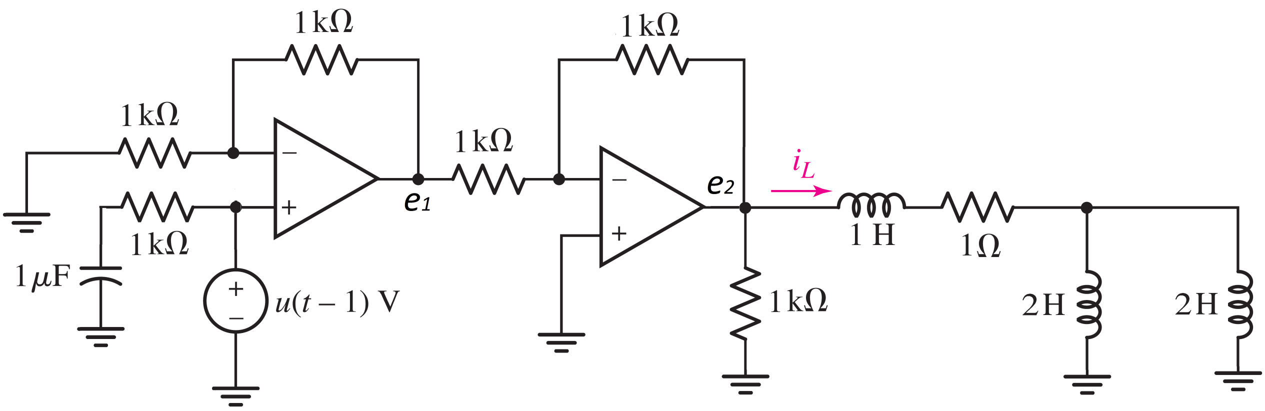

Figure 2: Labeling the nodes in the circuit of Fig. 1.

Label the circuit as shown in Fig. 2. The voltage at non-inverting leg of the left Op Amp is $v(t) = u(t-1)$. Writing a KCL for the inverting input,

$$\frac{v(t)-0}{1} + \frac{v(t)-e_1(t)}{1} = 0 \Rightarrow e_1(t) = 2v(t)$$

. Similarly, we write a KCL for the inverting leg of the next Op Amp, whose voltage is 0 V.

$$\frac{0-e_1(t)}{1} + \frac{0-e_2(t)}{1} = 0 \Rightarrow e_2(t) = -e_1(t) = -2v(t)$$

. Now, we get the equivalent RL circuit of Fig. 3, where we have merged the inductors as $L_{eq} = 2||2+1 = 2$ H. This is a simple series RL circuit driven with the DC voltage $e_2(t) = -2v(t) = -2u(t-1)$. For $t < 1$, the voltage is zero and therefore, $i_L(1^-) = 0$. The inductor keeps the current continuous when the voltage source switches to -2 V. So,

Question 1 continued on next page...

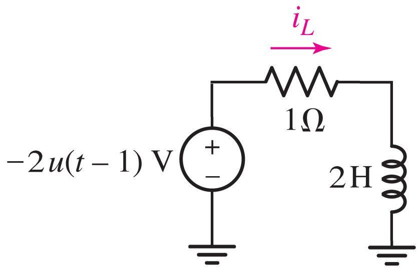

Figure 3: Equivalent circuit of Fig. 1.

$i_L(1^+) = i_L(1^-) = i_L(1) = 0$. When the circuit gets into its steady state, the inductor is short circuit and consequently, $i_L(\infty) = \frac{-2}{1} = -2$ A. Finally, for $t \geq 1$,

$$i_L(t) = i_L(\infty) + (i_L(1) - i_L(\infty))e^{-\frac{t-1}{\tau}} = -2(1 - e^{-\frac{t-1}{2}})$$

, where $\tau = \frac{L_{eq}}{R_{eq}} = \frac{2}{1} = 2$. Overall,

$$i(t) = \begin{cases} 0 & , t < 1 \\ -2(1 - e^{-\frac{t-1}{2}}) & , t \geq 1 \end{cases} = -2(1 - e^{-\frac{t-1}{2}})u(t-1)$$

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