Question 1

In the balanced three-phase circuit of Fig. 1, when the compensator non-resistive load $Z_C = jX_C$ is disconnected, the readings of the wattmeters are W_1 and W_2 W, and the reading of the voltmeter is V V rms. Find the reactance X_C of the compensator load such that the loss in the transmission line having the impedance Z_T is minimized while the power delivered to the load remains unchanged. Assume that the phase sequence is positive.

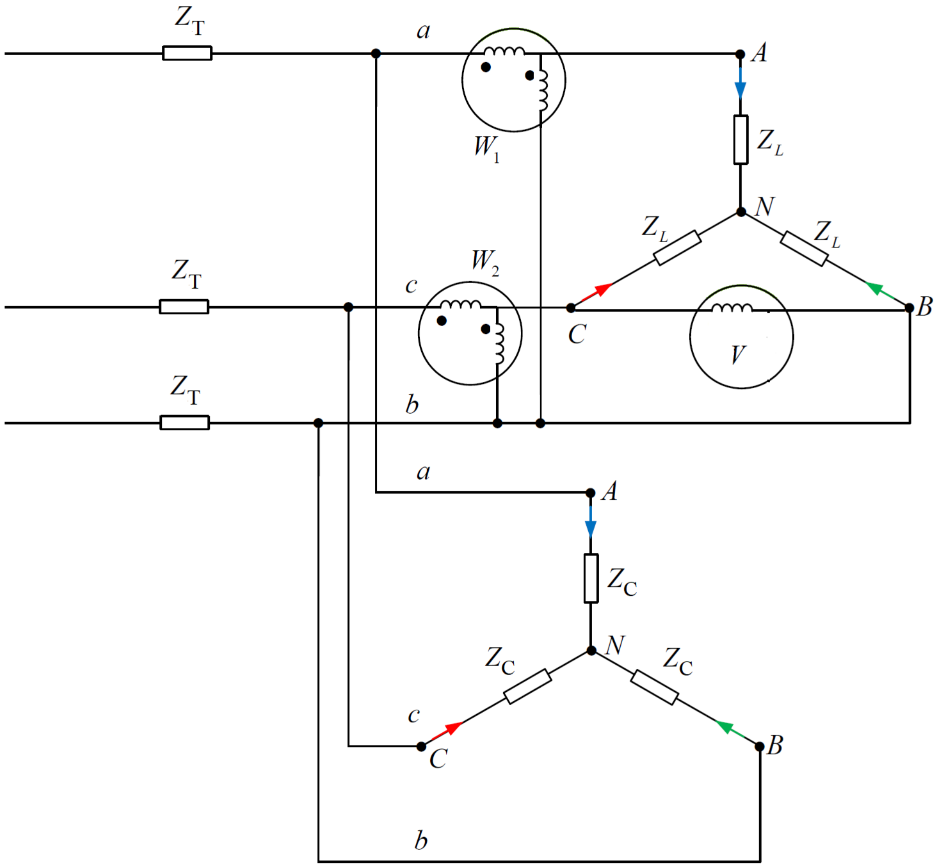

Figure 1: A three-phase balanced circuit.

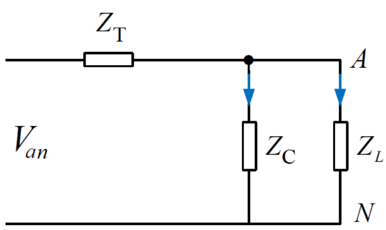

Figure 2: Equivalent single-phase circuit of the three-phase balanced circuit of Fig. 1 when the compensator load is connected.

Question 1 continued on next page...

For the positive phase sequence, we know from the two-wattmeter method that

$$Q_L = \sqrt{3}(W_2 - W_1)$$

Further,

$$Q_L = 3X_L|I_L|^2 = 3X_L|\frac{\frac{V}{\sqrt{3}}}{Z_L}|^2 = X_L\frac{V^2}{|Z_L|^2} = V^2\frac{X_L}{R_L^2 + X_L^2}$$

Thus,

$$\frac{R_L^2 + X_L^2}{X_L} = \frac{V^2}{Q_L} = \frac{V^2}{\sqrt{3}(W_2 - W_1)}$$

The equivalent single-phase circuit when the compensator load $Z_C = jX_C$ is connected is drawn in Fig. 2. We have,

$$P_T = 3R_T|I_T|^2, \quad P_L = 3R_L|I_L|^2 = 3R_L|\frac{Z_C}{Z_C + Z_L}|^2|I_T|^2$$

So,

$$P_T = \frac{3R_T P_L}{3R_L}|\frac{Z_C + Z_L}{Z_C}|^2$$

To minimize P_T while keeping P_L constant, we should minimize

$$|\frac{Z_C + Z_L}{Z_C}|^2 = \frac{R_L^2 + (X_L + X_C)^2}{X_C^2}$$

Therefore,

$$\frac{d}{dX_C}[\frac{R_L^2 + (X_L + X_C)^2}{X_C^2}] = 0 \Rightarrow 2(X_C + X_L)X_C^2 - 2X_C[R_L^2 + (X_L + X_C)^2] = 0$$

$$-R_L^2 - X_L^2 - X_C X_L = 0 \Rightarrow X_C = -\frac{R_L^2 + X_L^2}{X_L} = -\frac{V^2}{\sqrt{3}(W_2 - W_1)}$$

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