Question 1

Find the input impedance of the coupled π matching circuit shown in Fig. 1.

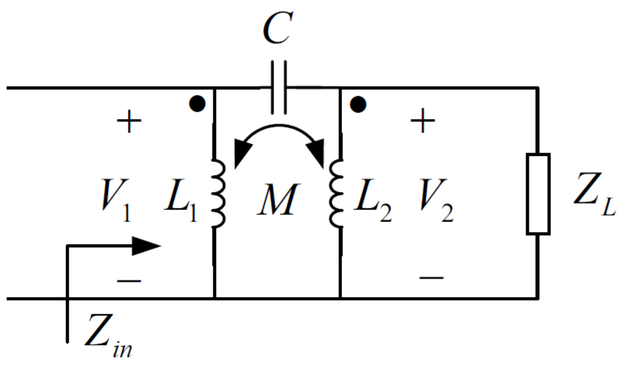

Figure 1: Coupled π matching circuit.

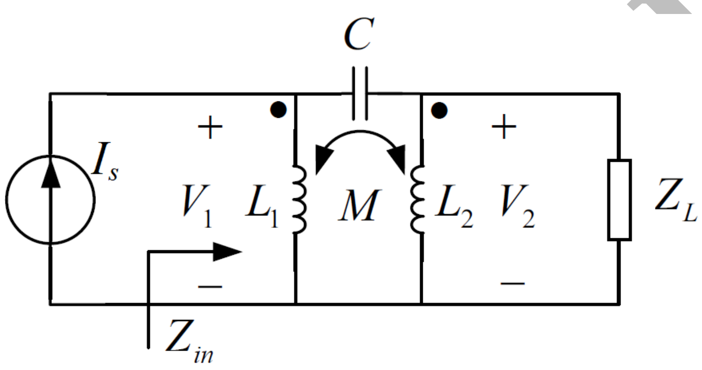

Figure 2: Impedance seen from the input of the coupled π matching circuit.

Using node analysis for Fig. 2,

$$\begin{cases} \frac{V_2}{Z_L} + j\omega C(V_2 - V_1) + \frac{\Gamma_{21}}{j\omega}V_1 + \frac{\Gamma_{22}}{j\omega}V_2 = 0 \\ +j\omega C(V_1 - V_2) + \frac{\Gamma_{11}}{j\omega}V_1 + \frac{\Gamma_{12}}{j\omega}V_2 - I_s = 0 \end{cases}$$

, where the reciprocal inductance values are obtained form

$$\Gamma = \begin{bmatrix} \Gamma_{11} & \Gamma_{12} \\ \Gamma_{21} & \Gamma_{22} \end{bmatrix} = L^{-1} = \begin{bmatrix} L_1 & M \\ M & L_2 \end{bmatrix}^{-1}$$

We have,

$$V_2 = \frac{j\omega c - \frac{\Gamma_{21}}{j\omega}}{\frac{1}{Z_L} + j\omega C + \frac{\Gamma_{22}}{j\omega}} V_1$$

So,

$$[j\omega C + \frac{\Gamma_{11}}{j\omega} + (\frac{\Gamma_{12}}{j\omega} - j\omega c)\frac{j\omega c - \frac{\Gamma_{21}}{j\omega}}{\frac{1}{Z_L} + j\omega C + \frac{\Gamma_{22}}{j\omega}}]V_1 = I_s$$

Question 1 continued on next page...

Finally,

$$Z_{in} = \frac{V_1}{I_s} = \frac{1}{j\omega C + \frac{\Gamma_{11}}{j\omega} + (\frac{\Gamma_{12}}{j\omega} - j\omega c)\frac{j\omega c - \frac{\Gamma_{21}}{j\omega}}{\frac{1}{Z_L} + j\omega C + \frac{\Gamma_{22}}{j\omega}}}$$

Since $\Gamma_{12} = \Gamma_{21}$,

$$Z_{in} = \frac{1}{j\omega C + \frac{\Gamma_{11}}{j\omega} - \frac{(j\omega c - \frac{\Gamma_{21}}{j\omega})^2}{\frac{1}{Z_L} + j\omega C + \frac{\Gamma_{22}}{j\omega}}}$$

Question 2

Find the input impedance of the circuit shown in Fig. 3.

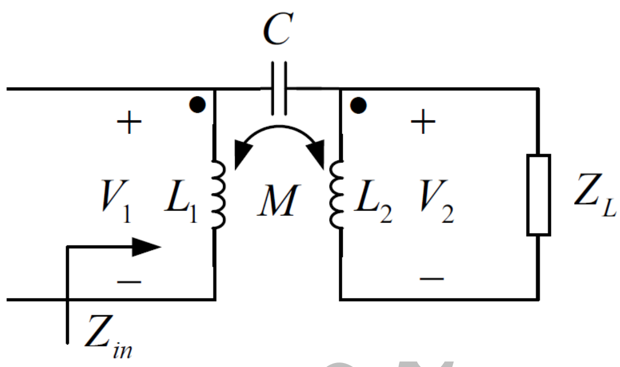

Figure 3: A coupled circuit.

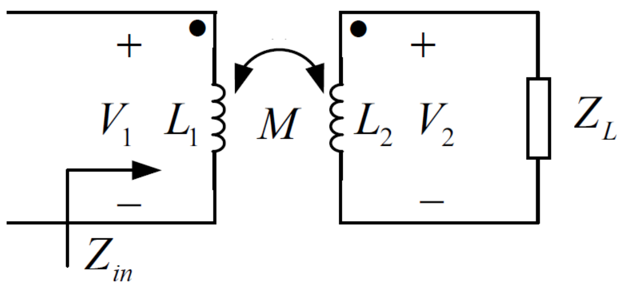

Figure 4: Simple coupled circuit.

Question 2 continued on next page...

Clearly, no current flows through the capacitor. So, the capacitor acts like an open circuit, and therefore, this circuit is the special case of the circuit in Fig. 1 for $C = 0$. So,

$$Z_{in} = \frac{1}{\frac{\Gamma_{11}}{j\omega} + \frac{\frac{\Gamma_{21}^2}{\omega^2}}{\frac{1}{Z_L} + \frac{\Gamma_{22}}{j\omega}}}$$

On the other hand, this circuit is the same as the simple coupled circuit shown in Fig. 4 with the input impedance

$$Z_{in} = j\omega L_1 + \frac{M^2\omega^2}{j\omega L_2 + Z_L}$$

, which is an equivalent expression for Z_{in}.