Question 1

For the circuit shown in Fig. 1,

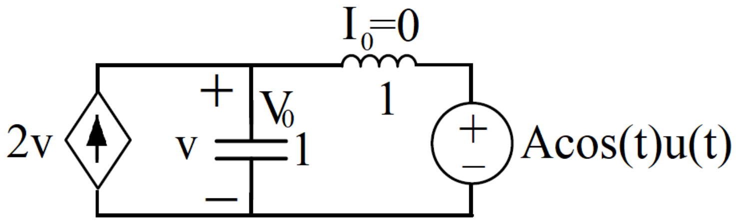

Figure 1: An LTI circuit.

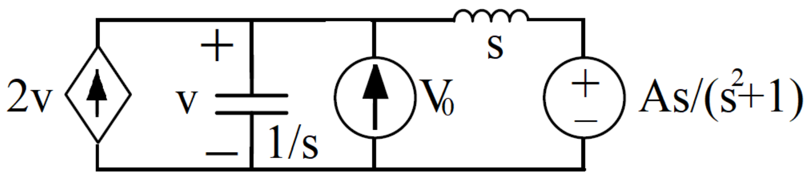

Figure 2: Laplace-domain model of the circuit shown in Fig. 1.

(a) Find the time-domain complete response of $v(t)$.

The circuit can be modeled in the Laplace domain as Fig. 2. Writing a KCL at the upper node of the capacitor,

$$\frac{V}{\frac{1}{s}} + \frac{V - A\frac{s}{s^2+1}}{s} - 2V - V_0 = 0$$

$$V(s + \frac{1}{s} - 2) = V_0 + \frac{A}{s^2+1}$$

$$V = \frac{s}{(s-1)^2}V_0 + \frac{As}{(s-1)^2(s^2+1)}$$

$$V = V_0\frac{s-1+1}{(s-1)^2} + A[\frac{\alpha}{s-1} + \frac{\beta}{(s-1)^2} + \frac{\gamma}{s-j} + \frac{\gamma^*}{s+j}]$$

$$V = V_0\frac{1}{s-1} + V_0\frac{1}{(s-1)^2} + A[\frac{\alpha}{s-1} + \frac{\beta}{(s-1)^2} + \frac{\gamma}{s-j} + \frac{\gamma^*}{s+j}]$$

$$V = V_0\frac{1}{s-1} + V_0\frac{1}{(s-1)^2} + A[\frac{0}{s-1} + \frac{\frac{1}{2}}{(s-1)^2} + \frac{\frac{j}{4}}{s-j} + \frac{-\frac{j}{4}}{s+j}]$$

$$V = V_0\frac{1}{s-1} + V_0\frac{1}{(s-1)^2} + A[\frac{1}{2}\frac{1}{(s-1)^2} - \frac{1}{2}\frac{1}{s^2+1}]$$

So,

$$v(t) = [V_0e^t + V_0te^t + \frac{A}{2}te^t - \frac{A}{2}\sin(t)]u(t)$$

Question 1 continued on next page...

(b) Find the sinusoidal steady state response of $v(t)$.

Since the time-domain response is not bounded as $t \to \infty$, the sinusoidal steady state does not exist.