Question 1

For the circuit shown in Fig. 1,

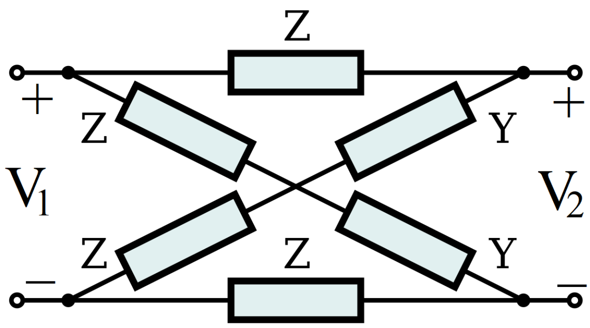

Figure 1: Lattice network.

(a) Find the transfer function $H(s) = \frac{V_2(s)}{V_1(s)}$*, where* $Z(s)$ *and* $Y(s)$ *denote impedance and admittance of two single-port passive network.*

$$V_2(s) = \frac{Y^{-1} + Z}{Y^{-1} + Z + Z} V_1(s) - \frac{Z}{Y^{-1} + Z + Z} V_1(s) = \frac{Y^{-1}}{Y^{-1} + 2Z} V_1(s) = \frac{1}{1 + 2ZY} V_1(s)$$

So,

$$H(s) = \frac{V_2(s)}{V_1(s)} = \frac{1}{1 + 2Z(s)Y(s)}$$

(b) Let $Z = s + \frac{1}{s}$ *and* $Y = s + \frac{1}{s}$ *be a series and a parallel LC network, respectively. Find the simplified transfer function* $H(s) = \frac{V_2(s)}{V_1(s)}$ *and the corresponding frequency response* $H(j\omega) = \frac{V_2(j\omega)}{V_1(j\omega)}$.

$$H(s) = \frac{V_2(s)}{V_1(s)} = \frac{1}{1 + 2Z(s)Y(s)} = \frac{1}{1 + 2(s + \frac{1}{s})(s + \frac{1}{s})} = \frac{s^2}{2s^4 + 5s^2 + 2}$$

$$H(j\omega) = H(s)|_{s=j\omega} = \frac{-\omega^2}{2\omega^4 - 5\omega^2 + 2}$$

(c) Draw the zero-pole diagram of $H(s)$ *calculated in part (b).*

The transfer function has two repeated zeros at $z_{1,2} = 0$ and four simple imaginary conjugate poles at $p_{1,2} = \pm j\sqrt{2}$ and $p_{3,4} = \pm j\frac{1}{\sqrt{2}}$. The zero-pole diagram is shown in Fig. 2.

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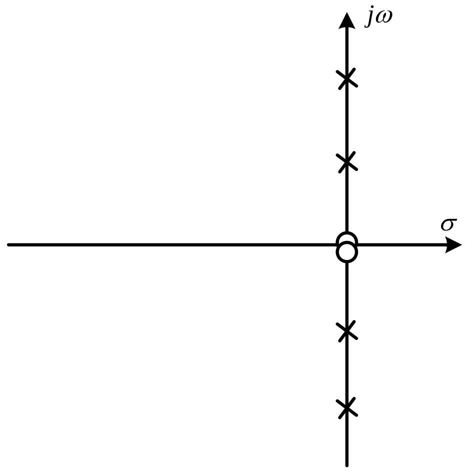

Figure 2: Zero-pole diagram.

(d) Draw the approximated amplitude and phase response of $H(j\omega)$ calculated in part (b).

Magnitude and phase responses are shown in Fig. 3 and Fig. 4, respectively. The magnitude has two vertical asymptote lines at $\omega = \frac{1}{\sqrt{2}}$ and $\omega = \sqrt{2}$ due to the poles. The magnitude is zero at $\omega = 0$ due to the zeros. The magnitude also approaches zero for $\omega \to \infty$ since the number of poles is more than the number of zeros. The phase experiences sudden jumps at the pole frequencies $\omega = \frac{1}{\sqrt{2}}$ and $\omega = \sqrt{2}$.

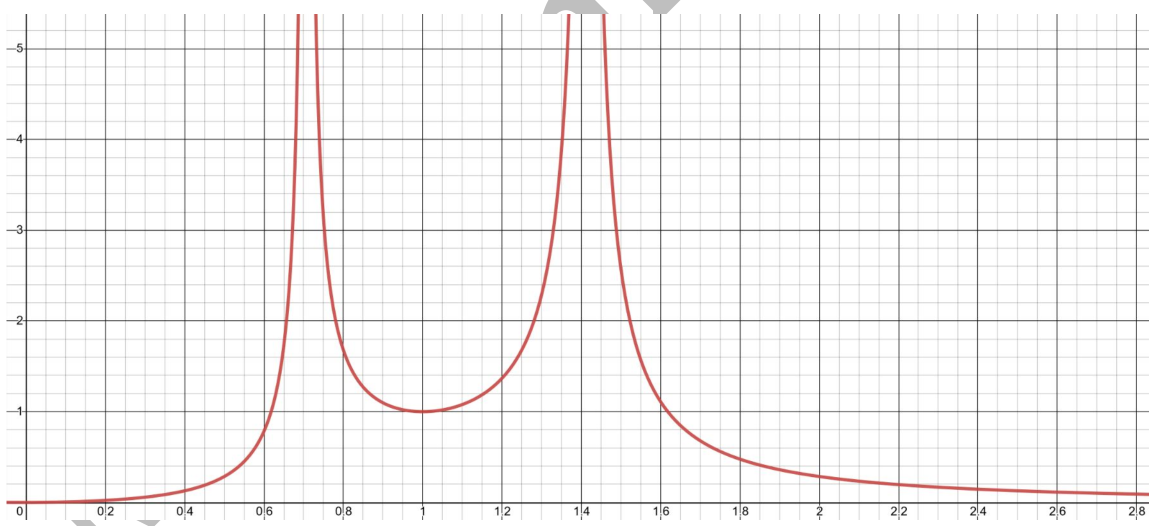

Figure 3: Magnitude response curve.

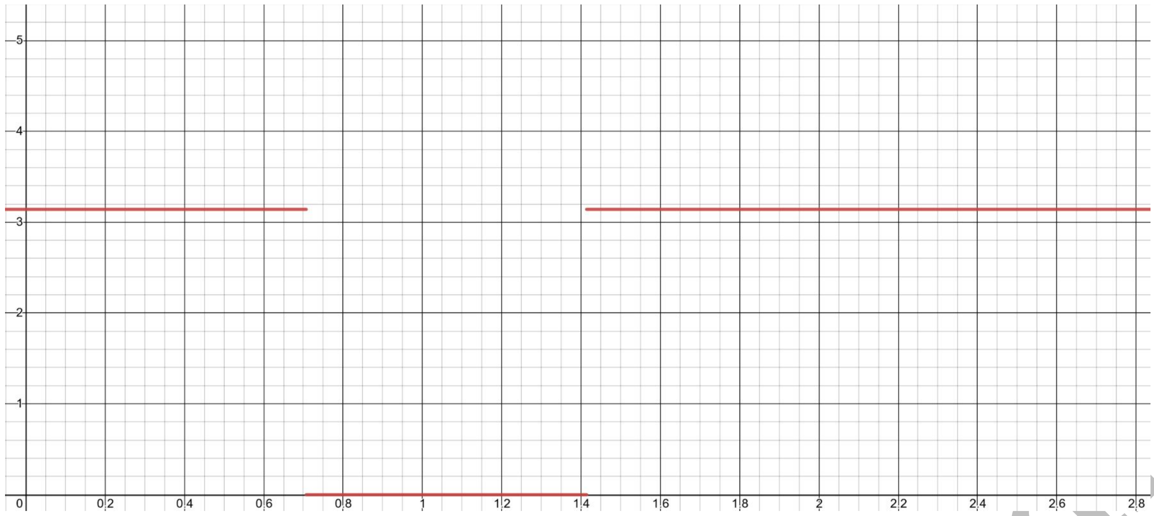

Figure 4: Phase response curve.