Student Name: .....
Student Number: .....

Question 1

The circuit shown in Fig. 1 is in initial rest, i.e., the initial voltage of the capacitor and the initial current of the inductor are zero.

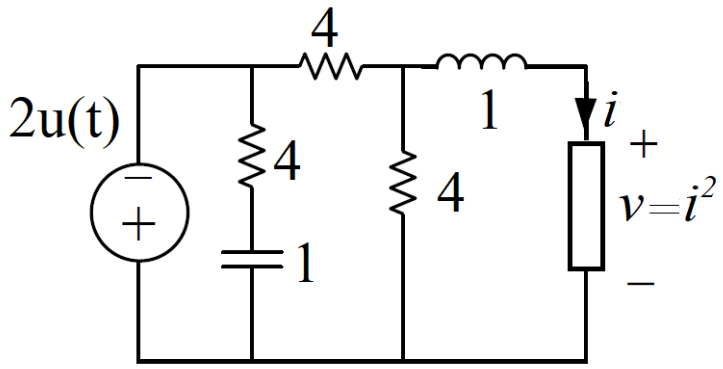

Figure 1: A circuit with nonlinear load.

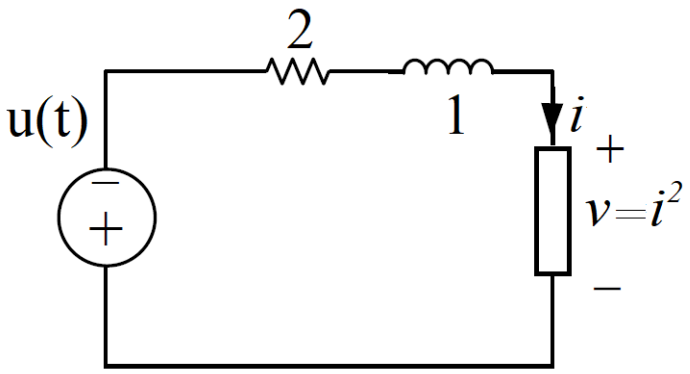

Figure 2: Thevenin equivalent circuit seen from the nonlinear load for the circuit of Fig. 1.

(a) Find the Thevenin equivalent circuit seen from the port connected to the nonlinear load.

Killing the voltage source, the RC shunt part is removed and the impedance seen from the load port is $Z_{th} = s + 4||4 = s + 2$. The open circuit voltage equals $V_{oc}(s) = \frac{2}{2+2}\frac{2}{s} = \frac{1}{s}$. So, in the time domain, we get the equivalent circuit shown in Fig. 2.

(b) Find $i(t)$, the current of the nonlinear load having the characteristic curve $v = i^2$.

Writing a KVL for the equivalent circuit shown in Fig. 2,

$$2i + i' + i^2 + 1 = 0 \Rightarrow i' + i^2 + 2i + 1 = i' + (i+1)^2 = 0, \quad i(0) = 0$$

Question 1 continued on next page...

This is a separable differential equation and can be simply solved. We have

$$\frac{di}{dt} = -(i+1)^2 \Rightarrow \frac{di}{-(i+1)^2} = dt \Rightarrow \frac{1}{i+1} = t + C \Rightarrow i(t) = \frac{-t+1-C}{t+C}$$

Applying the initial condition,

$$i(0) = 0 \Rightarrow C = 1 \Rightarrow i(t) = -\frac{t}{t+1}$$