Question 1

For the coupled circuit of Fig. 1, the reciprocal inductance matrix is

$$\Gamma = \begin{bmatrix} \Gamma_{11} & \Gamma_{12} & \Gamma_{13} \\ \Gamma_{12} & \Gamma_{22} & \Gamma_{23} \\ \Gamma_{13} & \Gamma_{23} & \Gamma_{33} \end{bmatrix}$$

and the initial conditions are

$$v_c(0^-1) = V_0, \quad i_{L_1}(0^-) = I_1, \quad i_{L_2}(0^-) = I_2, \quad i_{L_3}(0^-) = I_3$$

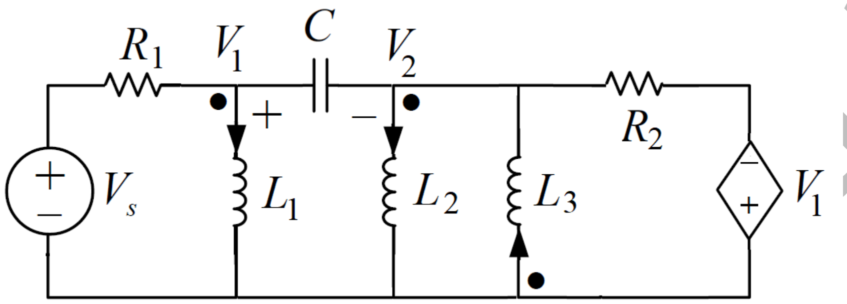

Figure 1: A coupled circuit.

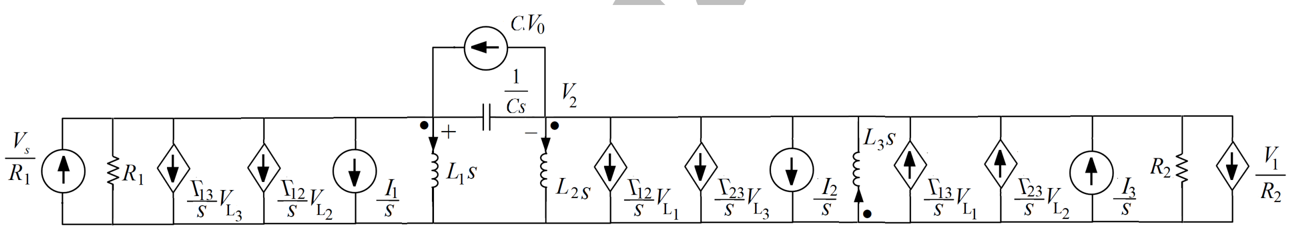

Figure 2: The equivalent circuit in Laplace domain.

(a) Write the Laplace-domain node equations.

Considering the equivalent circuit shown in Fig. 2,

$$\begin{bmatrix} \frac{1}{R_1} + \frac{\Gamma_{11}}{s} + Cs & -Cs \\ -Cs & \frac{1}{R_2} + \frac{\Gamma_{22}}{s} + \frac{\Gamma_{33}}{s} + Cs \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} =$$

$$\begin{bmatrix} \frac{V_s}{R_1} - \frac{\Gamma_{12}V_2}{s} + \frac{\Gamma_{13}V_2}{s} - \frac{I_1}{s} + CV_0 \\ -\frac{V_1}{R_2} - \frac{\Gamma_{12}V_1}{s} + \frac{\Gamma_{23}V_2}{s} - \frac{I_2}{s} + \frac{\Gamma_{13}V_1}{s} + \frac{\Gamma_{23}V_2}{s} + \frac{I_3}{s} - CV_0 \end{bmatrix}$$

Question 1 continued on next page...

So,

$$\begin{bmatrix} \frac{1}{R_1} + \frac{\Gamma_{11}}{s} + Cs & -Cs + \frac{\Gamma_{12}}{s} - \frac{\Gamma_{13}}{s} \\ -Cs + \frac{\Gamma_{12}}{s} - \frac{\Gamma_{13}}{s} + \frac{1}{R_2} & \frac{1}{R_2} + \frac{\Gamma_{22}}{s} + \frac{\Gamma_{33}}{s} + Cs - \frac{\Gamma_{23}}{s} - \frac{\Gamma_{23}}{s} \end{bmatrix} \begin{bmatrix} V_1(s) \\ V_2(s) \end{bmatrix} = \begin{bmatrix} \frac{V_s(s)}{R_1} - \frac{I_1}{s} + CV_0 \\ -\frac{I_2}{s} + \frac{I_3}{s} - CV_0 \end{bmatrix}$$

(b) Write the time-domain node equations.

Using the Laplace to time domain conversion rules,

$$\begin{bmatrix} \frac{1}{R_1} + \frac{\Gamma_{11}}{D} + CD & -CD + \frac{\Gamma_{12}}{D} - \frac{\Gamma_{13}}{D} \\ -CD + \frac{\Gamma_{12}}{D} - \frac{\Gamma_{13}}{D} + \frac{1}{R_2} & \frac{1}{R_2} + \frac{\Gamma_{22}}{D} + \frac{\Gamma_{33}}{D} + CD - \frac{\Gamma_{23}}{D} - \frac{\Gamma_{23}}{D} \end{bmatrix} \begin{bmatrix} v_1(t) \\ v_2(t) \end{bmatrix} = \begin{bmatrix} \frac{v_s(t)}{R_1} - I_1 \\ -I_2 + I_3 \end{bmatrix}$$

(c) Write the phasor-domain node equations.

Using the Laplace to phasor domain conversion rules,

$$\begin{bmatrix} \frac{1}{R_1} + \frac{\Gamma_{11}}{j\omega} + Cj\omega & -Cj\omega + \frac{\Gamma_{12}}{j\omega} - \frac{\Gamma_{13}}{j\omega} \\ -Cj\omega + \frac{\Gamma_{12}}{j\omega} - \frac{\Gamma_{13}}{j\omega} + \frac{1}{R_2} & \frac{1}{R_2} + \frac{\Gamma_{22}}{j\omega} + \frac{\Gamma_{33}}{j\omega} + Cj\omega - \frac{\Gamma_{23}}{j\omega} - \frac{\Gamma_{23}}{j\omega} \end{bmatrix} \begin{bmatrix} V_1(j\omega) \\ V_2(j\omega) \end{bmatrix} = \begin{bmatrix} \frac{V_s(j\omega)}{R_1} \\ 0 \end{bmatrix}$$