MATHEMATICAL QUESTIONS

Question 1

The linear rime-invariant network shown in Fig. 1 is in the sinusoidal steady state. The coupling between the inductors is specified by the reciprocal inductance matrix

$$\begin{bmatrix} \Gamma_0 & \Gamma_1 & \Gamma_2 & 0 \\ \Gamma_1 & \Gamma_0 & \Gamma_1 & \Gamma_2 \\ \Gamma_2 & \Gamma_1 & \Gamma_0 & \Gamma_1 \\ 0 & \Gamma_2 & \Gamma_1 & \Gamma_0 \end{bmatrix}$$

Write the cut-set equations $\boldsymbol{Y}_q(j\omega)\boldsymbol{E} = \boldsymbol{I}_s$ and the node equations $\boldsymbol{Y}_n(j\omega)\boldsymbol{E} = \boldsymbol{I}_s$ in matrix form.

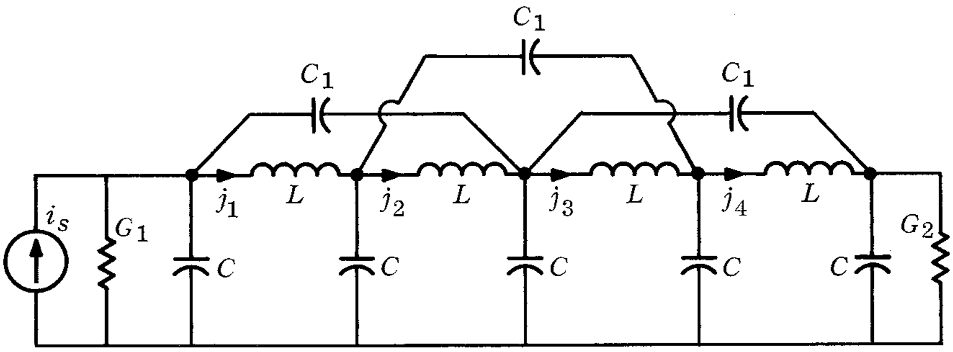

Figure 1: A coupled circuit.

Question 1 continued on next page...

Using the shortcut method,

$$\begin{bmatrix} G_1+\frac{\Gamma_0}{j\omega}+j\omega(C+C_1) & -\frac{\Gamma_0}{j\omega} & -j\omega C_1 & 0 & 0 \\ -\frac{\Gamma_0}{j\omega} & 2\frac{\Gamma_0}{j\omega}+j\omega(C+C_1) & -\frac{\Gamma_0}{j\omega} & -j\omega C_1 & 0 \\ -j\omega C_1 & -\frac{\Gamma_0}{j\omega} & \frac{2\Gamma_0}{j\omega}+j\omega(C+2C_1) & -\frac{\Gamma_0}{j\omega} & -j\omega C_1 \\ 0 & -j\omega C_1 & -\frac{\Gamma_0}{j\omega} & \frac{2\Gamma_0}{j\omega}+j\omega(C+C_1) & -\frac{\Gamma_0}{j\omega} \\ 0 & 0 & -j\omega C_1 & -\frac{\Gamma_0}{j\omega} & G_2+\frac{\Gamma_0}{j\omega}+j\omega(C+C_1) \end{bmatrix} \begin{bmatrix} E_1(j\omega) \\ E_2(j\omega) \\ E_3(j\omega) \\ E_4(j\omega) \\ E_5(j\omega) \end{bmatrix} =$$

$$\begin{bmatrix} I_s(j\omega) - \frac{\Gamma_1}{j\omega}(E_2-E_3) - \frac{\Gamma_2}{j\omega}(E_3-E_4) \\ \frac{\Gamma_1}{j\omega}(E_2-E_3) + \frac{\Gamma_2}{j\omega}(E_3-E_4) - \frac{\Gamma_1}{j\omega}(E_1-E_2) - \frac{\Gamma_1}{j\omega}(E_3-E_4) - \frac{\Gamma_2}{j\omega}(E_4-E_5) \\ \frac{\Gamma_1}{j\omega}(E_1-E_2) + \frac{\Gamma_1}{j\omega}(E_3-E_4) + \frac{\Gamma_2}{j\omega}(E_4-E_5) - \frac{\Gamma_2}{j\omega}(E_1-E_2) - \frac{\Gamma_1}{j\omega}(E_2-E_3) - \frac{\Gamma_1}{j\omega}(E_4-E_5) \\ \frac{\Gamma_2}{j\omega}(E_1-E_2) + \frac{\Gamma_1}{j\omega}(E_2-E_3) + \frac{\Gamma_1}{j\omega}(E_4-E_5) - \frac{\Gamma_2}{j\omega}(E_2-E_3) - \frac{\Gamma_1}{j\omega}(E_3-E_4) \\ \frac{\Gamma_2}{j\omega}(E_2-E_3) + \frac{\Gamma_1}{j\omega}(E_3-E_4) \end{bmatrix}$$

Hence,

$$\begin{bmatrix} G_1+\frac{\Gamma_0}{j\omega}+j\omega(C+C_1) & \frac{-\Gamma_0+\Gamma_1}{j\omega} & \frac{-\Gamma_1+\Gamma_2}{j\omega} - j\omega C_1 & \frac{-\Gamma_2}{j\omega} & 0 \\ \frac{-\Gamma_0+\Gamma_1}{j\omega} & 2\frac{\Gamma_0-2\Gamma_1}{j\omega}+j\omega(C+C_1) & \frac{-\Gamma_0+2\Gamma_1-\Gamma_2}{j\omega} & -j\omega C_1 + \frac{2\Gamma_2-\Gamma_1}{j\omega} & 0 \\ -j\omega C_1 + \frac{\Gamma_2-\Gamma_1}{j\omega} & \frac{2\Gamma_1-\Gamma_0-\Gamma_2}{j\omega} & \frac{2\Gamma_0-2\Gamma_1}{j\omega}+j\omega(C+2C_1) & \frac{-\Gamma_0+2\Gamma_1-\Gamma_2}{j\omega} & -j\omega C_1 + \frac{\Gamma_2-\Gamma_1}{j\omega} \\ \frac{-\Gamma_2}{j\omega} & -j\omega C_1 + \frac{2\Gamma_2-\Gamma_1}{j\omega} & \frac{-\Gamma_0+2\Gamma_1-\Gamma_2}{j\omega} & \frac{2\Gamma_0-2\Gamma_1}{j\omega}+j\omega(C+C_1) & \frac{\Gamma_1-\Gamma_0}{j\omega} \\ 0 & \frac{-\Gamma_2}{j\omega} & -j\omega C_1 + \frac{\Gamma_2-\Gamma_1}{j\omega} & \frac{\Gamma-\Gamma_0}{j\omega} & G_2+\frac{\Gamma_0}{j\omega}+j\omega(C+C_1) \end{bmatrix}$$

$$\begin{bmatrix} E_1(j\omega) \\ E_2(j\omega) \\ E_3(j\omega) \\ E_4(j\omega) \\ E_5(j\omega) \end{bmatrix} = \begin{bmatrix} I_s(j\omega) \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

The spanning tree including the capacitors C can be selected for the cut-set analysis. The cut-sets corresponding to the branches of this tree are the circuit nodes. So, the same equations as the node equations can be used for the cut-set analysis.

Question 2

The linear time-invariant network of Fig. 2, having the shown topological graph, is in the sinusoidal steady state. From the topological graph, the highlighted tree is picked.

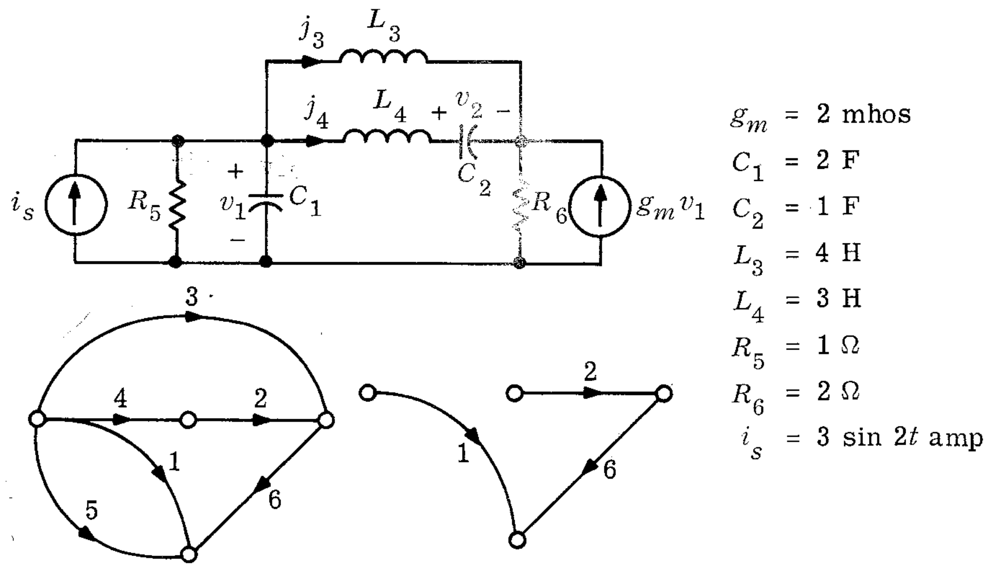

Figure 2: An LTI circuit along with its topological graph and spanning tree.

(a) Write the fundamental loop matrix $\boldsymbol{B}$.

$$\boldsymbol{B} = \begin{bmatrix} -1 & 1 & 0 & 1 & 0 & 1 \\ -1 & 0 & 1 & 0 & 0 & 1 \\ -1 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

(b) Calculate the loop impedance matrix $\boldsymbol{Z}_l$.

$$\boldsymbol{V}(j\omega) = \boldsymbol{Z}_b(j\omega)\boldsymbol{J}(j\omega) + \boldsymbol{V}_s(j\omega) - \boldsymbol{Z}_b(j\omega)\boldsymbol{J}_s(j\omega)$$

$$\begin{bmatrix} V_1(j\omega) \\ V_2(j\omega) \\ V_3(j\omega) \\ V_4(j\omega) \\ V_5(j\omega) \\ V_6(j\omega) \end{bmatrix} = \begin{bmatrix} \frac{1}{C_1 j\omega} & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{C_2 j\omega} & 0 & 0 & 0 & 0 \\ 0 & 0 & L_3 j\omega & 0 & 0 & 0 \\ 0 & 0 & 0 & L_4 j\omega & 0 & 0 \\ 0 & 0 & 0 & 0 & R_5 & 0 \\ \frac{g_m R_6}{j\omega C_1} & 0 & 0 & 0 & 0 & R_6 \end{bmatrix} \begin{bmatrix} J_1(j\omega) \\ J_2(j\omega) \\ J_3(j\omega) \\ J_4(j\omega) \\ J_5(j\omega) \\ J_6(j\omega) \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ R_5 J_s(j\omega) \\ 0 \end{bmatrix}$$

$$\boldsymbol{Z}_l(j\omega) = \boldsymbol{B}\boldsymbol{Z}_b(j\omega)\boldsymbol{B}^T$$

$$\boldsymbol{Z}_l(j\omega) = \begin{bmatrix} \frac{1+g_m R_6}{j\omega C_1} + \frac{1}{j\omega C_2} + L_4 j\omega + R_6 & \frac{1+g_m R_6}{j\omega C_1} + R_6 & \frac{1+g_m R_6}{j\omega C_1} \\ \frac{1+g_m R_6}{j\omega C_1} + R_6 & \frac{1+g_m R_6}{j\omega C_1} + L_3 j\omega + R_6 & \frac{1+g_m R_6}{j\omega C_1} \\ \frac{1}{j\omega C_1} & \frac{1}{j\omega C_1} & \frac{1}{j\omega C_1} + R_5 \end{bmatrix}$$

(c) *Write the loop equations in terms of voltage and current phasors, that is, $\boldsymbol{Z}_l \boldsymbol{I} = \boldsymbol{E}_s$.*

$$\boldsymbol{Z}_l(j\omega) = \boldsymbol{B}\boldsymbol{Z}_b(j\omega)\boldsymbol{B}^T$$

$$\boldsymbol{Z}_l(j\omega) = \begin{bmatrix} \frac{1+g_m R_6}{j\omega C_1} + \frac{1}{j\omega C_2} + L_4 j\omega + R_6 & \frac{1+g_m R_6}{j\omega C_1} + R_6 & \frac{1+g_m R_6}{j\omega C_1} \\ \frac{1+g_m R_6}{j\omega C_1} + R_6 & \frac{1+g_m R_6}{j\omega C_1} + L_3 j\omega + R_6 & \frac{1+g_m R_6}{j\omega C_1} \\ \frac{1}{j\omega C_1} & \frac{1}{j\omega C_1} & \frac{1}{j\omega C_1} + R_5 \end{bmatrix}$$

$$\boldsymbol{E}_s(j\omega) = \boldsymbol{B}\boldsymbol{Z}_b(j\omega)\boldsymbol{J}_s(j\omega) - \boldsymbol{B}\boldsymbol{V}_s(j\omega) = \begin{bmatrix} 0 \\ 0 \\ -R_5 J_s(j\omega) \end{bmatrix}$$

$$\begin{bmatrix} \frac{1+g_m R_6}{j\omega C_1} + \frac{1}{j\omega C_2} + L_4 j\omega + R_6 & \frac{1+g_m R_6}{j\omega C_1} + R_6 & \frac{1+g_m R_6}{j\omega C_1} \\ \frac{1+g_m R_6}{j\omega C_1} + R_6 & \frac{1+g_m R_6}{j\omega C_1} + L_3 j\omega + R_6 & \frac{1+g_m R_6}{j\omega C_1} \\ \frac{1}{j\omega C_1} & \frac{1}{j\omega C_1} & \frac{1}{j\omega C_1} + R_5 \end{bmatrix} \begin{bmatrix} I_1(j\omega) \\ I_2(j\omega) \\ I_3(j\omega) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ -R_5 J_s(j\omega) \end{bmatrix}$$

(d) *Write the matrix equations required to calculate the branch voltages and branch currents.*

$$\begin{bmatrix} J_1(j\omega) \\ J_2(j\omega) \\ J_3(j\omega) \\ J_4(j\omega) \\ J_5(j\omega) \\ J_6(j\omega) \end{bmatrix} = \boldsymbol{J}(j\omega) = \boldsymbol{B}^T \boldsymbol{I}(j\omega) = \begin{bmatrix} -I_1(j\omega) - I_2(j\omega) - I_3(j\omega) \\ I_1(j\omega) \\ I_2(j\omega) \\ I_1(j\omega) \\ I_3(j\omega) \\ I_1(j\omega) + I_2(j\omega) \end{bmatrix}$$

$$\boldsymbol{V}(j\omega) = \boldsymbol{Z}_b(j\omega)\boldsymbol{J}(j\omega) + \boldsymbol{V}_s(j\omega) - \boldsymbol{Z}_b(j\omega)\boldsymbol{J}_s(j\omega)$$

Question 2 continued on next page. . .

$$\begin{bmatrix} V_1(j\omega) \\ V_2(j\omega) \\ V_3(j\omega) \\ V_4(j\omega) \\ V_5(j\omega) \\ V_6(j\omega) \end{bmatrix} = \begin{bmatrix} \frac{1}{C_1 j\omega} & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{C_2 j\omega} & 0 & 0 & 0 & 0 \\ 0 & 0 & L_3 j\omega & 0 & 0 & 0 \\ 0 & 0 & 0 & L_4 j\omega & 0 & 0 \\ 0 & 0 & 0 & 0 & R_5 & 0 \\ \frac{g_m R_6}{j\omega C_1} & 0 & 0 & 0 & 0 & R_6 \end{bmatrix} \begin{bmatrix} J_1(j\omega) \\ J_2(j\omega) \\ J_3(j\omega) \\ J_4(j\omega) \\ J_5(j\omega) \\ J_6(j\omega) \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ R_5 J_s(j\omega) \\ 0 \end{bmatrix}$$

Question 3

Which loop impedance matrix below does belong to a passive LTI RLC circuit? Give your reasons for rejecting any.

$$\begin{bmatrix} 3 & 2 \\ 2 & 5 \end{bmatrix}, \quad \begin{bmatrix} -1 & 2 \\ 2 & 4 \end{bmatrix}, \quad \begin{bmatrix} 1 & 2 \\ 2 & 3 \end{bmatrix}$$

$$\begin{bmatrix} 3+j & -2j \\ -2j & 5+7j \end{bmatrix}, \quad \begin{bmatrix} 3 & -j \\ -j & 2 \end{bmatrix}, \quad \begin{bmatrix} 5 & 7j \\ 6j & 8+3j \end{bmatrix}$$

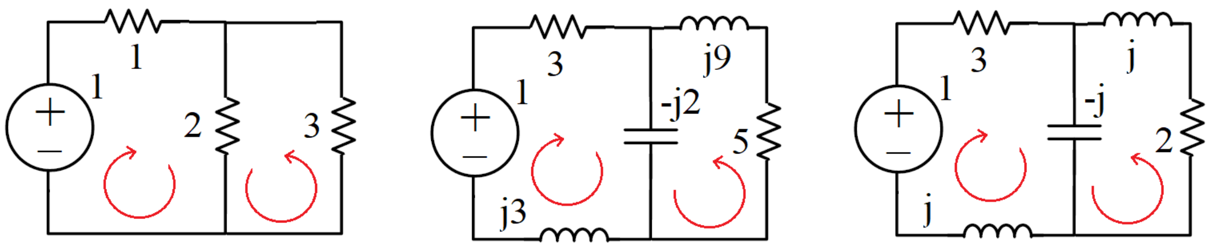

Figure 3: Sample passive LTI RLC circuits for the acceptable loop impedance matrices.

The three first matrices belong to resistove networks. For a passive resistive network, $\det[\boldsymbol{Z}_l] > 0$. $\begin{bmatrix} 3 & 2 \\ 2 & 5 \end{bmatrix}$ can be the loop impedance matrix of a passive RLC circuit such as the left circuit in Fig. 3.

$\begin{bmatrix} -1 & 2 \\ 2 & 4 \end{bmatrix} \equiv \begin{bmatrix} 1 & -2 \\ 2 & 4 \end{bmatrix}$ cannot be the loop impedance matrix of a passive RLC circuit due to asymmetry. Note that the condition $\det[\boldsymbol{Z}_l] = -8 \not> 0$ is also violated.

$\begin{bmatrix} 1 & 2 \\ 2 & 3 \end{bmatrix}$ cannot be the loop impedance matrix of a passive RLC circuit due to negative determinant $\det[\boldsymbol{Z}_l] = -1 \not> 0$. In other word, we have two loops and the total common resistance between theses loops is 2. so, the total impedance in each loop should be equal or greater than 2, which is not the case for the first loop with the total resistance of 1. Note that the resistive network is passive and we cannot have negative resistance.

Question 3 continued on next page. . .

$\begin{bmatrix} 3+j & -2j \\ -2j & 5+7j \end{bmatrix}$ can be the loop impedance matrix of a passive RLC circuit. A sample circuit is the middle circuit in Fig. 3
$\begin{bmatrix} 3 & -j \\ -j & 2 \end{bmatrix}$ can be the loop impedance matrix of a passive RLC circuit. A sample circuit is drawn in right side of Fig. 3.
$\begin{bmatrix} 5 & 7j \\ 6j & 8+3j \end{bmatrix}$ cannot be the loop impedance matrix of a passive RLC circuit due to asymmetry.

Question 4

For the circuit of Fig. 4,

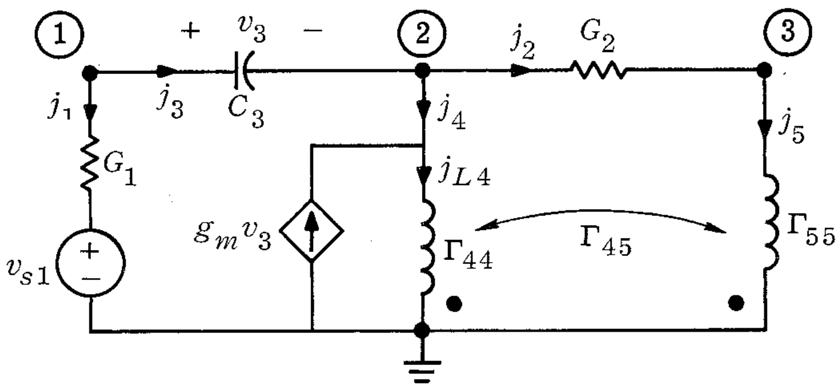

Figure 4: An LTI circuit for which the modified node analysis equations are required.

(a) Write the modified node analysis equations in Laplace domain.

To obtain the differential equations, the currents of the inductors and voltage source should be considered unknown. Let

$$I_4 = J_s, \quad I_5 = J_{L_4}, \quad I_6 = J_{L_5}$$

$$\begin{cases} I_4 + C_3 s(E_1 - E_2) - C_3 v_{c_3}(0) = 0 \\ C_3 s(E_2 - E_1) + C_3 v_{c_3}(0) + I_5 - g_m(E_1 - E_2) + G_2(E_2 - E_3) = 0 \\ G_2(E_3 - E_2) + I_6 = 0 \end{cases}$$

We added three extra unknown variables. So, we nned three more equations.

$$\begin{cases} V_{s_1} + R_1 I_4 = E_1 \\ E_2 = L_{44} s I_5 - L_{44} i_{L_4(0)} + L_{45} s I_6 - L_{45} i_{L_5(0)} \\ E_3 = L_{55} s I_6 - L_{55} i_{L_5(0)} + L_{45} s I_5 - L_{45} i_{L_4(0)} \end{cases}$$

Question 4 continued on next page...

In matrix form,

$$\begin{bmatrix} C_3s & -C_3s & 0 & 1 & 0 & 0 \\ -C_3s-g_m & C_3s+g_m+G_2 & -G_2 & 0 & 1 & 0 \\ 0 & -G_2 & G_2 & 0 & 0 & 1 \\ -1 & 0 & 0 & R_1 & 0 & 0 \\ 0 & 1 & 0 & 0 & -L_{44}s & -L_{45}s \\ 0 & 0 & 1 & 0 & -L_{45}s & -L_{55}s \end{bmatrix} \begin{bmatrix} E_1(s) \\ E_2(s) \\ E_3(s) \\ I_4(s) \\ I_5(s) \\ I_6(s) \end{bmatrix}$$

$$= \begin{bmatrix} C_3v_{c_3}(0) \\ -C_3v_{c_3}(0) \\ 0 \\ -V_{s_1}(s) \\ -L_{44}i_{L_4(0)} - L_{45}i_{L_5(0)} \\ -L_{55}i_{L_5(0)} - L_{45}i_{L_4(0)} \end{bmatrix}$$

(b) Write the modified node analysis equations in time domain.

$$\begin{bmatrix} C_3D & -C_3D & 0 & 1 & 0 & 0 \\ -C_3D-g_m & C_3D+g_m+G_2 & -G_2 & 0 & 1 & 0 \\ 0 & -G_2 & G_2 & 0 & 0 & 1 \\ -1 & 0 & 0 & R_1 & 0 & 0 \\ 0 & 1 & 0 & 0 & -L_{44}D & -L_{45}D \\ 0 & 0 & 1 & 0 & -L_{45}D & -L_{55}D \end{bmatrix} \begin{bmatrix} e_1(t) \\ e_2(t) \\ e_3(t) \\ i_4(t) \\ i_5(t) \\ i_6(t) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ -v_{s_1}(t) \\ 0 \\ 0 \end{bmatrix}$$

(c) Write the modified node analysis equations in Phasor domain.

$$\begin{bmatrix} C_3j\omega & -C_3j\omega & 0 & 1 & 0 & 0 \\ -C_3j\omega-g_m & C_3j\omega+g_m+G_2 & -G_2 & 0 & 1 & 0 \\ 0 & -G_2 & G_2 & 0 & 0 & 1 \\ -1 & 0 & 0 & R_1 & 0 & 0 \\ 0 & 1 & 0 & 0 & -L_{44}j\omega & -L_{45}j\omega \\ 0 & 0 & 1 & 0 & -L_{45}j\omega & -L_{55}j\omega \end{bmatrix} \begin{bmatrix} E_1(j\omega) \\ E_2(j\omega) \\ E_3(j\omega) \\ I_4(j\omega) \\ I_5(j\omega) \\ I_6(j\omega) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ -V_{s_1}(j\omega) \\ 0 \\ 0 \end{bmatrix}$$

Question 5

For the circuit if Fig. 5,

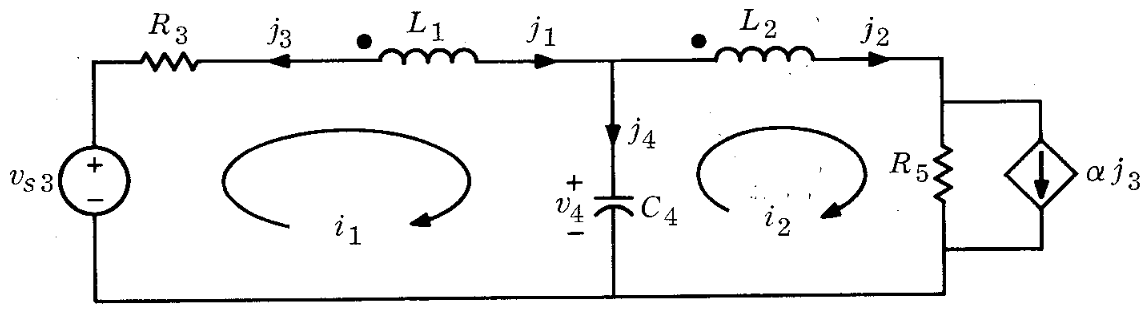

Figure 5: An LTI circuit for which the loop and mesh equations are required.

(a) Write the systematic loop equations in Laplace domain.

Selecting the tree including R_3, v_{s3}, C_4, R_5. We have,

$$\boldsymbol{B} = \begin{bmatrix} 1 & 0 & -1 & 1 & 0 \\ 0 & 1 & 0 & -1 & 1 \end{bmatrix}$$

$$\boldsymbol{B}\boldsymbol{V}(s) = \boldsymbol{0} \Rightarrow \begin{bmatrix} 1 & 0 & -1 & 1 & 0 \\ 0 & 1 & 0 & -1 & 1 \end{bmatrix} \begin{bmatrix} V_1(s) \\ V_2(s) \\ V_3(s) \\ V_4(s) \\ V_5(s) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\boldsymbol{J}(s) = \boldsymbol{B}^T\boldsymbol{I}(s) \Rightarrow \begin{bmatrix} J_1(s) \\ J_2(s) \\ J_3(s) \\ J_4(s) \\ J_5(s) \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ -1 & 0 \\ 1 & -1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} I_1(s) \\ I_2(s) \end{bmatrix}$$

$$\begin{bmatrix} V_1(s) \\ V_2(s) \\ V_3(s) \\ V_4(s) \\ V_5(s) \end{bmatrix} = \begin{bmatrix} L_1 s & Ms & 0 & 0 & 0 \\ Ms & L_2 s & 0 & 0 & 0 \\ 0 & 0 & R_3 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{C_4 s} & 0 \\ 0 & 0 & -R_5\alpha & 0 & R_5 \end{bmatrix} \begin{bmatrix} J_1(s) \\ J_2(s) \\ J_3(s) \\ J_4(s) \\ J_5(s) \end{bmatrix} + \begin{bmatrix} -L_1 i_1(0) - Mi_2(0) \\ -Mi_1(0) - L_2 j_2(0) \\ V_{s_3}(s) \\ \frac{v_4(0)}{s} \\ 0 \end{bmatrix}$$

$$\boldsymbol{Z}_l(s) = \boldsymbol{B}\boldsymbol{Z}_b(s)\boldsymbol{B}^T = \begin{bmatrix} R_3 + L_1 s + \frac{1}{C_4 s} & -\frac{1}{C_4 s} + Ms \\ -\frac{1}{C_4 s} + Ms + \alpha R_5 & R_5 + L_2 s + \frac{1}{C_4 s} \end{bmatrix}$$

$$\boldsymbol{E}_s(s) = \boldsymbol{B}\boldsymbol{Z}_b(s)\boldsymbol{J}_s(s) - \boldsymbol{B}\boldsymbol{V}_s(s) = \begin{bmatrix} V_{s_3}(s) + L_1 i_{L_1}(0) + Mi_{L_2}(0) - \frac{v_4(0))}{s} \\ L_2 i_{L_2}(0) + Mi_{L_1}(0) + \frac{v_4(0)}{s} \end{bmatrix}$$

$$\boldsymbol{Z}_l(s)\boldsymbol{I}(s) = \boldsymbol{E}_s(s)$$

Question 5 continued on next page. . .

(b) Write the systematic mesh equations in Phasor domain.

$$\boldsymbol{M} = \begin{bmatrix} 1 & 0 & -1 & 1 & 0 \\ 0 & 1 & 0 & -1 & 1 \end{bmatrix}$$

$$\boldsymbol{M}\boldsymbol{V}(j\omega) = \boldsymbol{0} \Rightarrow \begin{bmatrix} 1 & 0 & -1 & 1 & 0 \\ 0 & 1 & 0 & -1 & 1 \end{bmatrix} \begin{bmatrix} V_1(j\omega) \\ V_2(j\omega) \\ V_3(j\omega) \\ V_4(j\omega) \\ V_5(j\omega) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\boldsymbol{J}(j\omega) = \boldsymbol{M}^T \boldsymbol{I}(j\omega) \Rightarrow \begin{bmatrix} J_1(j\omega) \\ J_2(j\omega) \\ J_3(j\omega) \\ J_4(j\omega) \\ J_5(j\omega) \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ -1 & 0 \\ 1 & -1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} I_1(j\omega) \\ I_2(j\omega) \end{bmatrix}$$

$$\begin{bmatrix} V_1(j\omega) \\ V_2(j\omega) \\ V_3(j\omega) \\ V_4(j\omega) \\ V_5(j\omega) \end{bmatrix} = \begin{bmatrix} L_1 j\omega & Mj\omega & 0 & 0 & 0 \\ Mj\omega & L_2 j\omega & 0 & 0 & 0 \\ 0 & 0 & R_3 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{C_4 j\omega} & 0 \\ 0 & 0 & -R_5\alpha & 0 & R_5 \end{bmatrix} \begin{bmatrix} J_1(j\omega) \\ J_2(j\omega) \\ J_3(j\omega) \\ J_4(j\omega) \\ J_5(j\omega) \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ V_{s_3}(j\omega) \\ 0 \\ 0 \end{bmatrix}$$

$$\boldsymbol{Z}_m(j\omega) = \boldsymbol{M}\boldsymbol{Z}_b(s)\boldsymbol{M}^T = \begin{bmatrix} R_3 + L_1 j\omega + \frac{1}{C_4 j\omega} & -\frac{1}{C_4 j\omega} + Mj\omega \\ -\frac{1}{C_4 j\omega} + Mj\omega + \alpha R_5 & R_5 + L_2 j\omega + \frac{1}{C_4 j\omega} \end{bmatrix}$$

$$\boldsymbol{E}_s(j\omega) = \boldsymbol{M}\boldsymbol{Z}_b(j\omega)\boldsymbol{J}_s(j\omega) - \boldsymbol{M}\boldsymbol{V}_s(j\omega) = \begin{bmatrix} V_{s_3}(j\omega) \\ 0 \end{bmatrix}$$

$$\boldsymbol{Z}_m(j\omega)\boldsymbol{I}(j\omega) = \boldsymbol{E}_s(j\omega)$$

(c) Write the shortcut mesh equations in time, phasor, and Laplace domains.

In Laplace domain,

$$\begin{bmatrix} R_3 + L_1 s + \frac{1}{C_4 s} & -\frac{1}{C_4 s} \\ -\frac{1}{C_4 s} & R_5 + L_2 s + \frac{1}{C_4 s} \end{bmatrix} \begin{bmatrix} I_1(s) \\ I_2(s) \end{bmatrix} =$$

$$\begin{bmatrix} V_{s_3}(s) - MsI_2(s) + L_1 i_{L_1}(0) + Mi_{L_2}(0) - \frac{v_4(0)}{s} \\ -MsI_1(s) + L_2 i_{L_2}(0) + Mi_{L_1}(0) + \frac{v_4(0)}{s} + \alpha R_5 I_1(s) \end{bmatrix}$$

$$\begin{bmatrix} R_3 + L_1 s + \frac{1}{C_4 s} & -\frac{1}{C_4 s} + Ms \\ -\frac{1}{C_4 s} + Ms + \alpha R_5 & R_5 + L_2 s + \frac{1}{C_4 s} \end{bmatrix} \begin{bmatrix} I_1(s) \\ I_2(s) \end{bmatrix} = \begin{bmatrix} V_{s_3}(s) + L_1 i_{L_1}(0) + Mi_{L_2}(0) - \frac{v_4(0))}{s} \\ L_2 i_{L_2}(0) + Mi_{L_1}(0) + \frac{v_4(0)}{s} \end{bmatrix}$$

In phasor domain,

$$\begin{bmatrix} R_3 + L_1 j\omega + \frac{1}{c_4 j\omega} & -\frac{1}{c_4 j\omega} + Mj\omega \\ -\frac{1}{c_4 j\omega} + Mj\omega + \alpha R_5 & R_5 + L_2 j\omega + \frac{1}{c_4 j\omega} \end{bmatrix} \begin{bmatrix} I_1(j\omega) \\ I_2(j\omega) \end{bmatrix} = \begin{bmatrix} V_{s_3}(j\omega) \\ 0 \end{bmatrix}$$

In time domain,

$$\begin{bmatrix} R_3 + L_1 D + \frac{D^{-1}}{c_4} & -\frac{D^{-1}}{c_4} + MD \\ -\frac{D^{-1}}{c_4} + MD + \alpha R_5 & R_5 + L_2 D + \frac{D^{-1}}{c_4} \end{bmatrix} \begin{bmatrix} i_1(t) \\ i_2(t) \end{bmatrix} = \begin{bmatrix} v_{s3}(t) - v_4(0) \\ v_4(0) \end{bmatrix}$$

SOFTWARE QUESTIONS

Question 6

Write a MATLAB function that returns the time-domain solution of the Laplace matrix equation $\boldsymbol{A}(s)\boldsymbol{X}(s) = \boldsymbol{I}(s)$, where $\boldsymbol{A}(s)$ is a square matrix whose elements are polynomials of s and $\boldsymbol{I}(s)$ is a vector whose elements are real fractional functions of s. You might use the symbolic math features of MATLAB.

Using the symbolic calculations in MATLAB, the required function can be simply implemented as

```
function x = LDES(A,I)

x = ilaplace(inv(A)*I);

end
```

You may use the following mfile to call the developed function and see its results.

```
A = [s+1 -s;-s s+2];
I=[s;1];
x = LDES(A,I)
```

BONUS QUESTIONS

Question 7

Return your answers by filling the LaTeXtemplate of the assignment. If you want to add a circuit schematic, you can draw it directly using TikZ package, or draw it in a secondary application such as Microsoft Visio and then, import it as a figure.

EXTRA QUESTIONS

Question 8

Feel free to solve the following questions from the book *"Basic Circuit Theory"* by C. Desoer and E. Kuh.

1. **Chapter 10, question 5.**
2. **Chapter 10, question 6.**
3. **Chapter 10, question 7.**
4. **Chapter 10, question 8.**
5. **Chapter 10, question 12.**
6. **Chapter 10, question 13.**
7. **Chapter 10, question 14.**
8. **Chapter 11, question 5.**
9. **Chapter 11, question 8.**