MATHEMATICAL QUESTIONS

Question 1

For the circuit of Fig. 1, find both the phase and line currents, and the phase and line voltages throughout the circuit. Then, calculate the total power dissipated in the load.

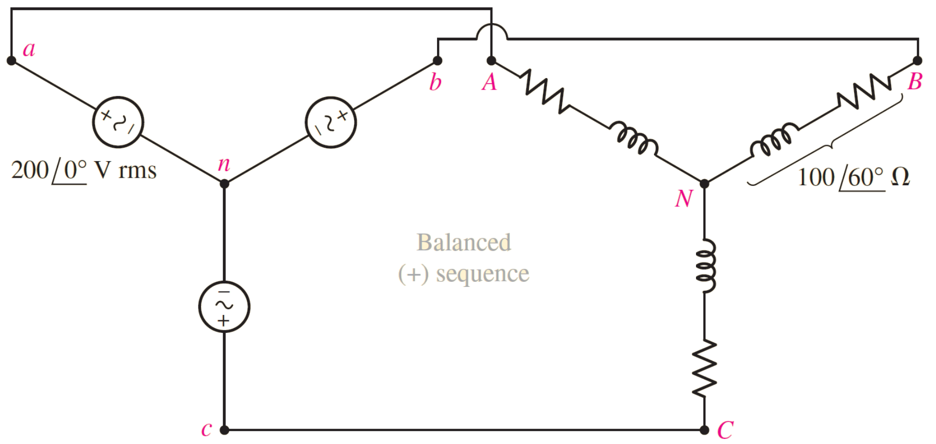

Figure 1: A balanced three-phase three-wire Y-Y connected system.

Since one of the source phase voltages is given and we are told to use the positive phase sequence, the three phase voltages are

$$V_{an} = 200\angle 0^\circ \text{ V} \qquad V_{bn} = 200\angle -120^\circ \text{ V} \qquad V_{cn} = 200\angle -240^\circ \text{ V}$$

The line voltage is $200\sqrt{3} = 346V$; the phase angle of each line voltage can be determined by constructing a phasor diagram, subtracting the phase voltages by invoking the following equations, we find that V_{ab} is $346\angle 30^\circ$, V_{bc} is $346\angle -90^\circ$ and V_{ca} is $346\angle -210^\circ$

$$V_{ab} = \sqrt{3}V_p\angle 30^\circ$$

$$V_{bc} = \sqrt{3}V_p\angle -90^\circ$$

$$V_{ca} = \sqrt{3}V_p\angle -210^\circ$$

The line current for phase A is

$$I_{aA} = \frac{V_{an}}{Z_p} = \frac{200\angle 0^\circ}{100\angle 60^\circ} = 2\angle -60^\circ \text{ A}$$

Since we know this is a balanced three-phase system, we may write the remaining line currents based on I_{aA}

$$I_{bB} = 2\angle(-60^\circ - 120^\circ) = 2\angle -180^\circ \text{ A}$$

Question 1 continued on next page...

$$I_{cC} = 2\angle(-60^\circ - 240^\circ) = 2\angle -300^\circ \text{ A}$$

Finally, the average power absorbed by phase A is $Re\{V_{an} I^*_{aA}\}$, or

$$P_{AN} = 200(2)\cos(0^\circ + 60^\circ) = 200 \text{ W}$$

Thus, the total average power drawn by the three-phase load is 600 W.
The phasor diagram for this circuit is shown in Fig. 2. Once we knew any of the line voltage magnitudes and any of the line current magnitudes, the angles for all three voltages and all three currents could have been obtained by simply reading the diagram.

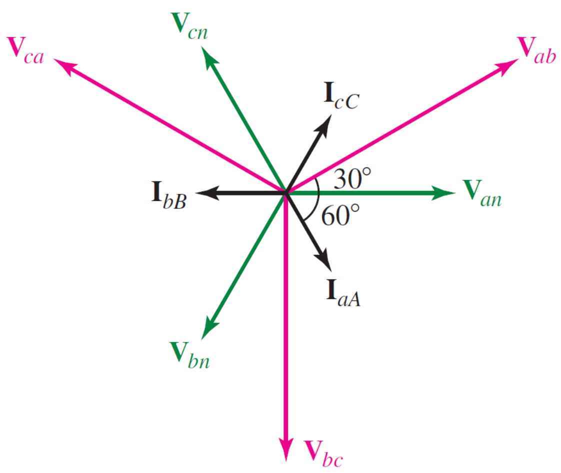

Figure 2: The phasor diagram that applies to the circuit of Fig. 1.

Question 2

The balanced load in Fig. 3 is fed by a balanced three-phase system having $V_{ab} = 230\angle 0^\circ$ V rms and positive phase sequence. Find the reading of each wattmeter and the total power drawn by the load.

Question 2 continued on next page...

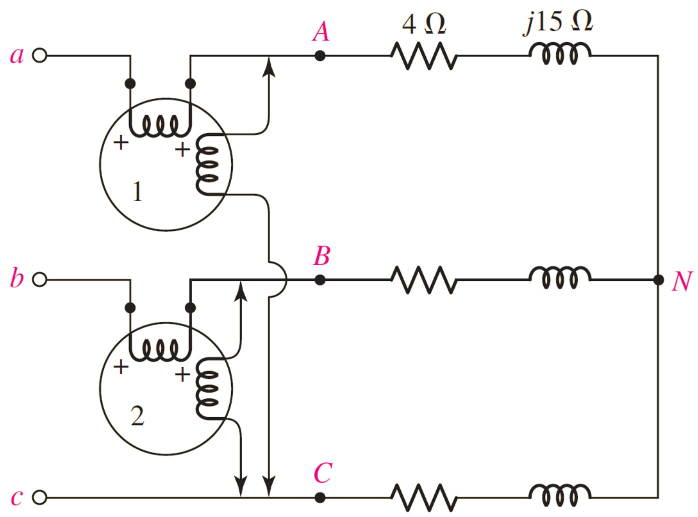

Figure 3: A balanced three-phase system connected to a balanced three-phase load, the power of which is being measured using the two-wattmeter technique.

Since we know to use the positive phase sequence, the line voltages are :

$$V_{ab} = 230\underline{/0^\circ}\text{ V} \qquad V_{bc} = 230\underline{/-120^\circ}\text{ V} \qquad V_{ca} = 230\underline{/120^\circ}\text{ V}$$

Note that $V_{ac} = -V_{ca} = 230\underline{/-60^\circ}$ V. The phase current I_{aA} is given by the phase voltage V_{an} divided by the phase impedance $4 + j15\ \Omega$,

$$I_{aA} = \frac{V_{an}}{Z_p} = \frac{(230/\sqrt{3})\underline{/-30^\circ}}{4 + j15} = 8.554\underline{/-105.1^\circ}\text{ A}$$

We may now compute the power measured by wattmeter 1 as

$$P_1 = |V_{ac}||I_{aA}|\cos(\angle V_{ac} - \angle I_{aA}) = (230)(8.554)\cos(-60^\circ + 105.1^\circ) = 1389\text{ W}$$

In a similar fashion, we determine that,

$$I_{bB} = \frac{V_{bn}}{Z_p} = \frac{(230/\sqrt{3})\underline{/-150^\circ}}{4 + j15} = 8.554\underline{/134.9^\circ}\text{ A}$$

$$P_2 = |V_{bc}||I_{bB}|\cos(\angle V_{bc} - \angle I_{bB}) = (230)(8.554)\cos(-120^\circ - 134.9^\circ) = -512.5\text{ W}$$

Thus, the total average power absorbed by the load is

$$P = P_1 + P_2 = 876.5\text{ W}$$

Question 3

For the balanced three-phase system shown in Fig. 4, it is determined that 100 W is lost in each wire. If the phase voltage of the source is 400 V, and the load draws 12 kW at a lagging PF of 0.83, determine the wire resistance R_W.

Question 3 continued on next page...

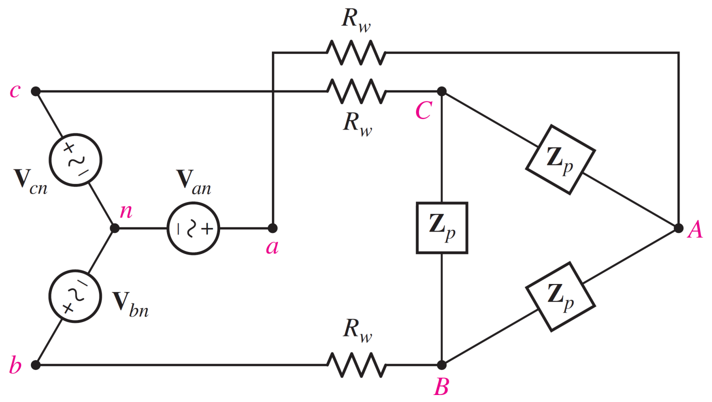

Figure 4: A balanced three-phase three-wire Y-Δ connected system.

Complex power dissipated at wires is

$$P_W = S_W = 3 \times 100 = 300 \text{ VA}$$

The load is lagging with the power factor PF $= \cos(\theta) = 0.83, \theta \geq 0$. So, $\theta = 33.9°$ Reactive power absorbed by the load equals

$$Q_L = P_L tan(\theta) = P_L \tan(\cos^{-1}(\text{PF})) = 8.06 \text{ kVAR}$$

Complex power of the load is

$$S_L = P_L + jQ_L = 12 + j8.06 \text{ kVA}$$

The total complex power dissipated at the load and wires is

$$S_{LW} = S_W + S_L = 0.3 + 12 + j8.06 = 12.3 + j8.06 \text{ kVA}$$

So,

$$|S_{LW}| = 14.71 \text{ kVA}$$

Noting the conservation of the complex power,

$$|S_{LW}| = |S_S| = 3|V_{an}||I_{an}| \Rightarrow |I_{an}| = \frac{|S_S|}{3|V_{an}|} = 12.26 \text{ A}$$

Finally,

$$P_W = 3R_W|I_{an}|^2$$

$$R_W = \frac{P_W}{3|I_{an}|^2} = \frac{300}{3 \times 12.26^2} = 0.67\ \Omega$$

Question 4

For the circuit of Fig. 5, find line currents including the current of the null line. Assume that $(V_{a'n}, V_{b'n}, V_{c'n}) = 1000(1, 1\angle -120°, 1\angle 120°)$ **V rms,** $Z_{ga} = Z_{gb} = Z_{gc} = 2+j8$**,** $Z_{la} = Z_{lb} = Z_{lc} = Z_{ln} = 1 + j2$**,** $Z_{LA} = 19 + j18$**,** $Z_{LB} = 49 - j2$**, and** $Z_{LC} = 29 + j50\ \Omega$**. Repeat the calculations**

Question 4 continued on next page...

for the negative phase sequence $(V_{a'n}, V_{b'n}, V_{c'n}) = 1000(1, 1\angle 120°, 1\angle -120°)$ **V rms.**

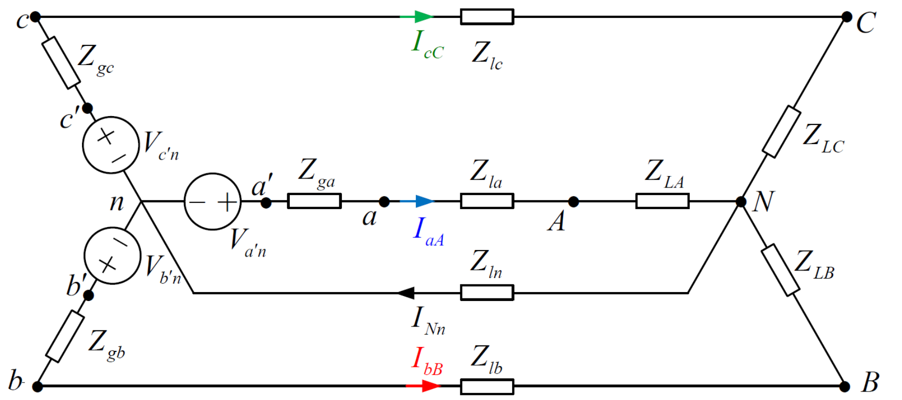

Figure 5: An imbalanced three-phase four-wire Y-Y connected system.

Writing three KVLs for the three meshes of the circuit,

$$\begin{aligned}-V_{a'n} + Z_{ga}I_{aA} + Z_{la}I_{aA} + Z_{LA}I_{aA} + Z_{ln}I_{Nn} = 0\\ -V_{b'n} + Z_{gb}I_{bB} + Z_{lb}I_{bB} + Z_{LB}I_{bB} + Z_{ln}I_{Nn} = 0\\ -V_{c'n} + Z_{gc}I_{cC} + Z_{lc}I_{cC} + Z_{LC}I_{cC} + Z_{ln}I_{Nn} = 0\\ I_{aA} + I_{bB} + I_{cC} = I_{Nn}\end{aligned}$$

Now,

$$\begin{aligned}Z_{ln}I_{Nn} + (Z_{ga} + Z_{la} + Z_{LA})I_{aA} = V_{a'n}\\ Z_{ln}I_{Nn} + (Z_{gb} + Z_{lb} + Z_{LB})I_{bB} = V_{b'n}\\ Z_{ln}I_{Nn} + (Z_{gc} + Z_{lc} + Z_{LC})I_{cC} = V_{c'n}\\ I_{aA} + I_{bB} + I_{cC} - I_{Nn} = 0\end{aligned}$$

Now, we compute the line currents for the positive phase sequence as

$$\begin{aligned}(1+j2)I_{Nn} + (2+j8+1+j2+19+j18)I_{aA} = (1+j2)I_{Nn} + (22+j28)I_{aA} = 1000\underline{/0°}V\\ (1+j2)I_{Nn} + (2+j8+1+j2+49-j2)I_{bB} = (1+j2)I_{Nn} + (52+j8)I_{bB} = 1000\underline{/-120°}V\\ (1+j2)I_{Nn} + (2+j8+1+j2+29+j50)I_{cC} = (1+j2)I_{Nn} + (32+j60)I_{cC} = 1000\underline{/120°}V\\ I_{aA} + I_{bB} + I_{cC} - I_{Nn} = 0\end{aligned}$$

Solving the system of equations,

$$I_{aA} = 26.53\underline{/-51.8°}A \quad I_{bB} = 19.55\underline{/-131.4°}A \quad I_{cC} = 15.14\underline{/60.7°}A \quad I_{Nn} = 24.8\underline{/-64°}A$$

Similarly, for the negative phase sequence, we have the system of equations below

$$\begin{aligned}(1+j2)I_{Nn} + (2+j8+1+j2+19+j18)I_{aA} = (1+j2)I_{Nn} + (22+j28)I_{aA} = 1000\underline{/0°}V\\ (1+j2)I_{Nn} + (2+j8+1+j2+49-j2)I_{bB} = (1+j2)I_{Nn} + (52+j8)I_{bB} = 1000\underline{/120°}V\\ (1+j2)I_{Nn} + (2+j8+1+j2+29+j50)I_{cC} = (1+j2)I_{Nn} + (32+j60)I_{cC} = 1000\underline{/-120°}V\\ I_{aA} + I_{bB} + I_{cC} - I_{Nn} = 0\end{aligned}$$

, which leads to

$$I_{aA} = 28\underline{/-51.2°}A \quad I_{bB} = 19.2\underline{/111.1°}A \quad I_{cC} = 14.58\underline{/177.6°}A \quad I_{Nn} = 5.13\underline{/-140°}A$$

Question 5

For the circuit of Fig. 6, find the phase and line currents, the phase and line voltages, the apparent, real, and reactive powers generated by the three-phase source, and the apparent, real, and reactive powers absorbed by the three-phase load.

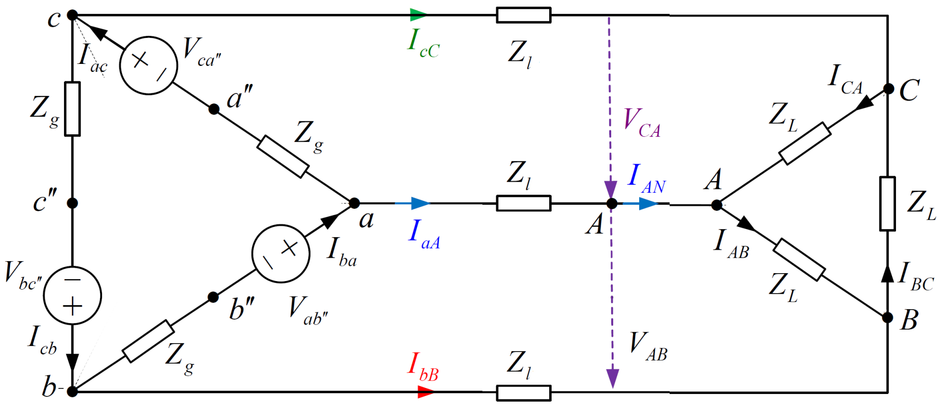

Figure 6: A balanced three-phase three-wire Δ-Δ connected system.

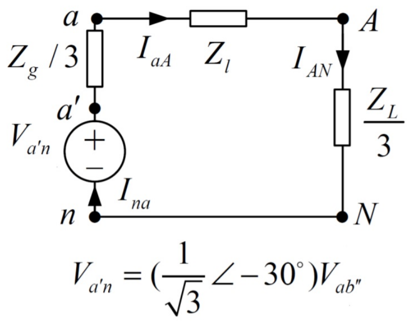

Figure 7: Single-phase equivalent circuit of the three-phase circuit shown in Fig. 6.

First of all we need to draw the single-phase equivalent circuit, as shown in Fig. 7. Note that we have assumed that the phase sequence is positive and used Δ-Y conversion at the load and source to obtain the equivalent circuit. Clearly,

$$\text{KVL} : I_{aA} = \frac{(\frac{1}{\sqrt{3}}\underline{/-30^\circ})V_{ab''}}{\frac{Z_g}{3} + Z_l + \frac{Z_L}{3}}$$

Now, at the load side,

- Line currents

$$(I_{aA}, I_{bB}, I_{cC}) = \frac{(\sqrt{3}\underline{/-30^\circ})V_{ab''}}{Z_g + 3Z_l + Z_L}(1, 1\underline{/-120^\circ}, 1\underline{/+120^\circ})$$

Question 5 continued on next page. . .

- Phase currents

$$(I_{AB}, I_{BC}, I_{CA}) = \frac{1\angle 30^\circ}{\sqrt{3}}(I_{aA}, I_{bB}, I_{cC}) = \frac{V_{ab''}}{Z_g + 3Z_l + Z_L}(1, 1\angle -120^\circ, 1\angle +120^\circ)$$

- Line voltages

$$(V_{AB}, V_{BC}, V_{CA}) = Z_L(I_{AB}, I_{BC}, I_{CA}) = \frac{Z_L V_{ab''}}{Z_g + 3Z_l + Z_L}(1, 1\angle -120^\circ, 1\angle +120^\circ)$$

- Phase voltages

$$(V_{AB}, V_{BC}, V_{CA}) = Z_L(I_{AB}, I_{BC}, I_{CA}) = \frac{Z_L V_{ab''}}{Z_g + 3Z_l + Z_L}(1, 1\angle -120^\circ, 1\angle +120^\circ)$$

- Apparent power

$$S_L = V_{AB}I_{AB}^* + V_{CA}I_{CA}^* + V_{BC}I_{BC}^* = 3V_{AB}I_{AB}^* = 3|\frac{V_{ab''}}{Z_g + 3Z_l + Z_L}|^2 Z_L$$

- Real power

$$P_L = 3|Z_L||\frac{V_{ab''}}{Z_g + 3Z_l + Z_L}|^2 \cos(\angle Z_L)$$

- Reactive power

$$Q_L = 3|Z_L||\frac{V_{ab''}}{Z_g + 3Z_l + Z_L}|^2 \sin(\angle Z_L)$$

Similarly, at the source,

- Line currents

$$(I_{aA}, I_{bB}, I_{cC}) = \frac{(\sqrt{3}\angle -30^\circ)V_{ab''}}{Z_g + 3Z_l + Z_L}(1, 1\angle -120^\circ, 1\angle +120^\circ)$$

- Phase currents

$$(I_{ba}, I_{cb}, I_{ac}) = \frac{1\angle 30^\circ}{\sqrt{3}}(I_{aA}, I_{bB}, I_{cC}) = \frac{V_{ab''}}{Z_g + 3Z_l + Z_L}(1, 1\angle -120^\circ, 1\angle +120^\circ)$$

- Line voltages

$$\begin{aligned}(V_{ab}, V_{bc}, V_{ca}) &= (V_{ab''}, V_{bc''}, V_{ca''}) - Z_g(I_{ba}, I_{cb}, I_{ac}) \\ &= \frac{(3Z_l + Z_L)V_{ab''}}{Z_g + 3Z_l + Z_L}(1, 1\angle -120^\circ, 1\angle +120^\circ)\end{aligned}$$

- Phase voltages

$$\begin{aligned}(V_{ab}, V_{bc}, V_{ca}) &= (V_{ab''}, V_{bc''}, V_{ca''}) - Z_g(I_{ba}, I_{cb}, I_{ac}) \\ &= \frac{(3Z_l + Z_L)V_{ab''}}{Z_g + 3Z_l + Z_L}(1, 1\angle -120^\circ, 1\angle +120^\circ)\end{aligned}$$

Question 5 continued on next page. . .

- Apparent power

$$S_s = V_{ab}I_{ba}^* + V_{ca}I_{ac}^* + V_{bc}I_{cb}^* = 3V_{ab}I_{ba}^* = 3|\frac{V_{ab''}}{Z_g + 3Z_l + Z_L}|^2(3Z_l + Z_L)$$

- Real power

$$P_s = 3|3Z_l + Z_L||\frac{V_{ab''}}{Z_g + 3Z_l + Z_L}|^2 \cos(\angle 3Z_l + Z_L)$$

- Reactive power

$$Q_s = 3|3Z_l + Z_L||\frac{V_{ab''}}{Z_g + 3Z_l + Z_L}|^2 \sin(\angle 3Z_l + Z_L)$$

SOFTWARE QUESTIONS

Question 6

Each phase of a three-phase induction motor can be modeled with the circuit of Fig. 8. In fact, in each phase, the equivalent load impedance of

$$Z_p = R_1 + jX_1 + \frac{1}{\frac{1}{R_c} + \frac{1}{jX_M} + \frac{1}{jX_2 + R_2/s}}$$

is seen, where the slit s determines the relative difference of the rotor and synchronous speeds. Use the PSIM simulation tool to compare the current and power of three-phase induction motors for Δ and Y connections, as shown in Fig. 9. Assume that $R_1 = 0.641\ \Omega$, $X_1 = 1.106\ \Omega$, $R_2 = 0.332\ \Omega$, $X_2 = 0.464\ \Omega$, $X_M = 26.3\ \Omega$, $R_C = 1\ M\Omega$, and $s = 0.022$. Also assume that the source has a Y connection with a phase voltage of 220 V rms. You might plot current and power curves versus time for Δ and Y connections in the motor to facilitate the comparison.

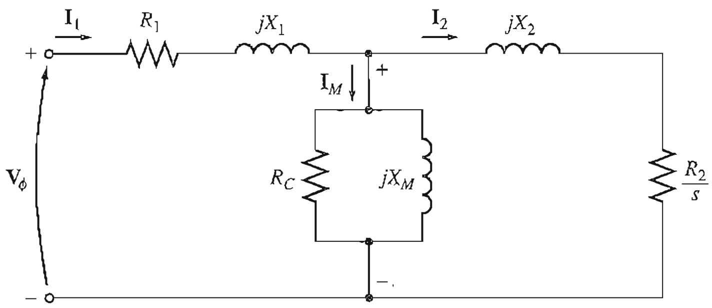

Figure 8: Per-phase equivalent circuit of an induction motor.

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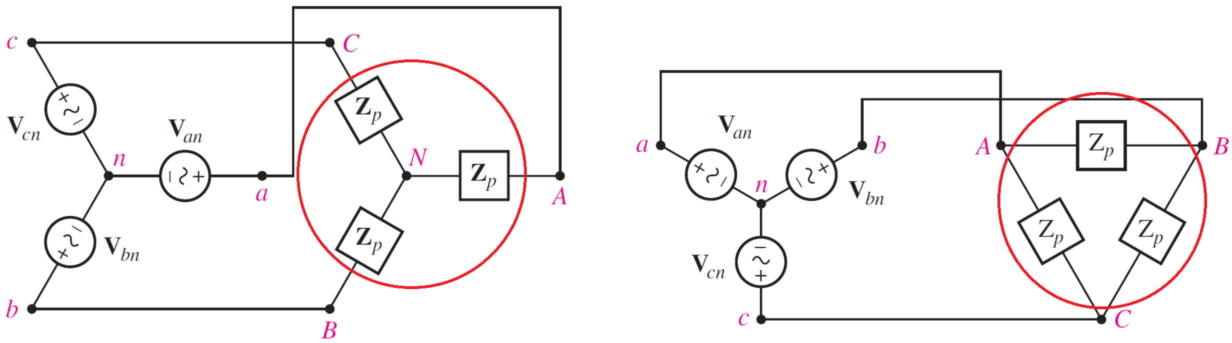

Figure 9: A three-phase induction motor in Y and Δ connections.

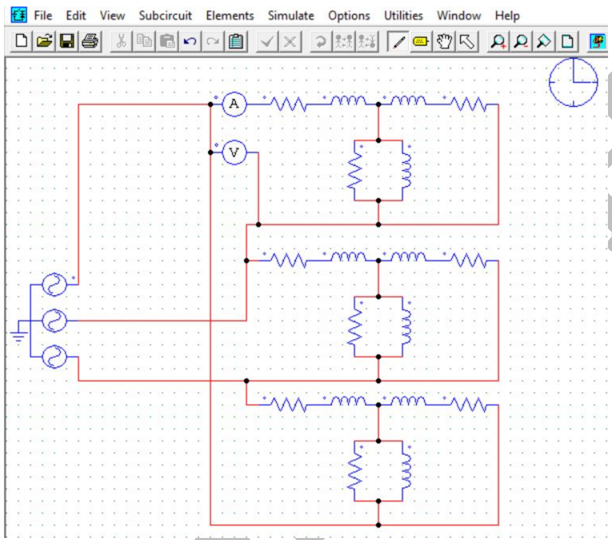

Figure 10: Simulation of Δ connection in PSIM.

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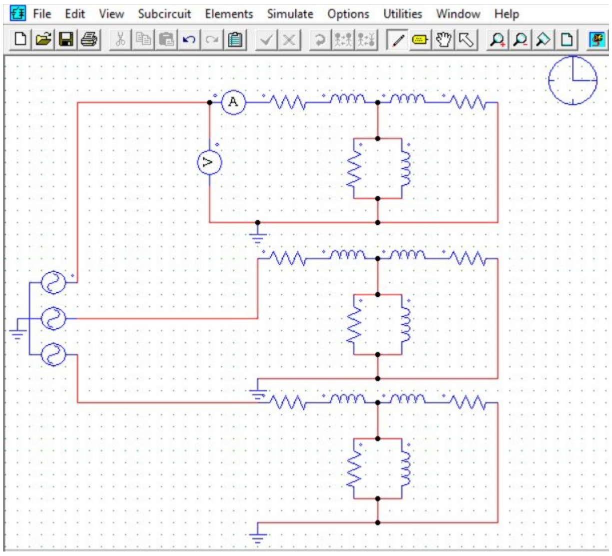

Figure 11: Simulation of Y connection in PSIM.

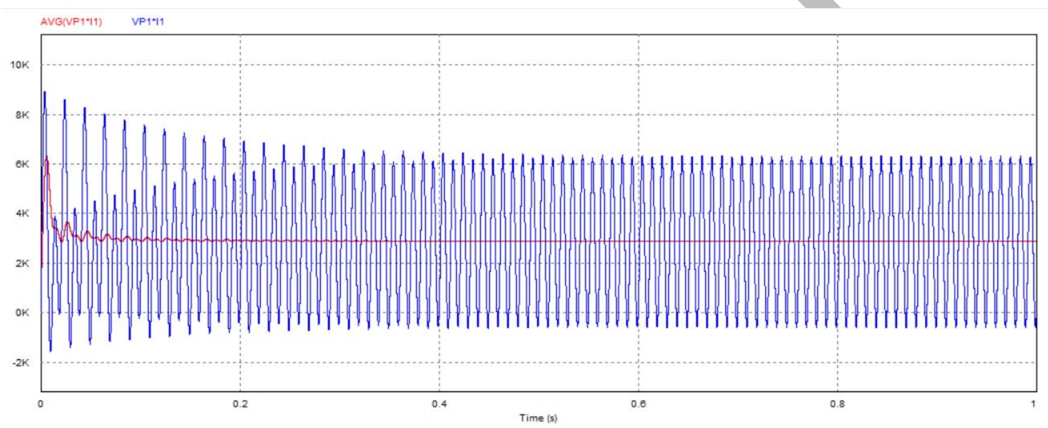

Figure 12: Per phase instantaneous power for Δ connection.

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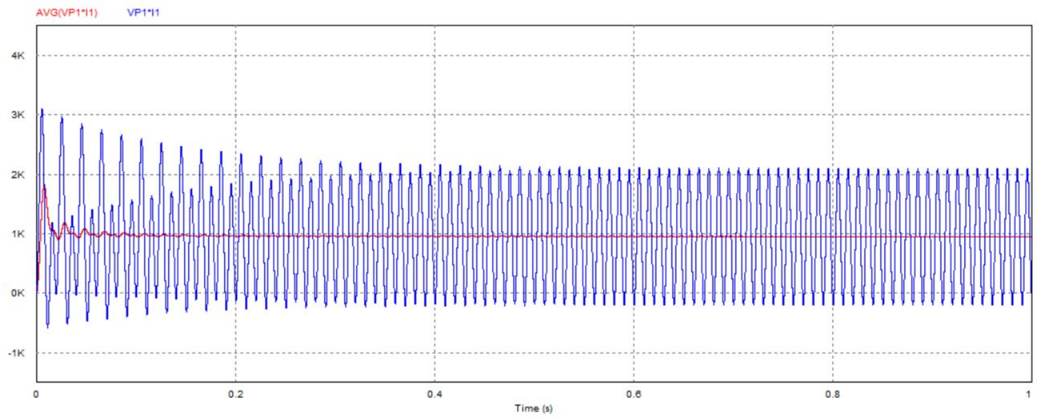

Figure 13: Per phase instantaneous power for Y connection.

Figs. 10 and 11 shows the schematic of simulation in PSIM for Δ and Y connections while Figs. 12 and 13 shows the corresponding instantaneous and average per phase power. Clearly, the average power for Δ connection is three times the average power for Y connection.

BONUS QUESTIONS

Question 7

Return your answers by filling the LaTeX template of the assignment.