MATHEMATICAL QUESTIONS

Question 1

The circuit shown in Fig. 1 is called Sallen active lowpass filter, where the triangle abstracts an op-amp amplification circuit with the gain K.

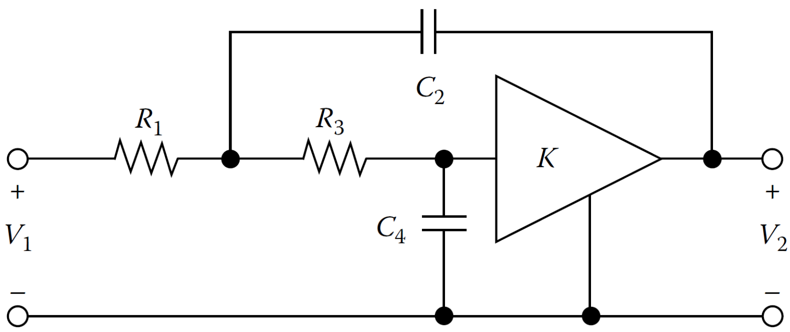

Figure 1: Sallen active lowpass filter.

(a) Find the transfer function of the circuit.

The input voltage of the amplifier is V_2/K. Writing a KCL for the node connecting R_2, R_1, and C_2,

$$\frac{V - V_1}{R_1} + \frac{V_1 - V_2/K}{R_3} + C_2 s(V - V_2) = 0$$

Further,

$$V_2 = K\frac{1/C_4 s}{1/C_4 s + R_3}V$$

Simplifying the equations,

$$H(s) = \frac{V_2(s)}{V_1(s)} = \frac{\frac{K}{R_1 R_3 C_2 C_4}}{s^2 + (\frac{1}{R_3 C_4} + \frac{1}{R_1 C_2} + \frac{1}{R_3 C_2} - \frac{K}{R_3 C_4})s + \frac{1}{R_1 R_3 C_2 C_4}}$$

(b) Find the frequency response of the circuit.

We know that $H(j\omega) = H(s)|_{s=j\omega}$. Thus,

$$H(j\omega) = \frac{V_2(j\omega)}{V_1(j\omega)} = \frac{\frac{K}{R_1 R_3 C_2 C_4}}{-\omega^2 + (\frac{1}{R_3 C_4} + \frac{1}{R_1 C_2} + \frac{1}{R_3 C_2} - \frac{K}{R_3 C_4})j\omega + \frac{1}{R_1 R_3 C_2 C_4}}$$

Question 1 continued on next page…

(c) Draw the approximated frequency response using the corresponding zero-pole diagram.

Various situations for the pole locations and the corresponding frequency responses are plotted in Fig. 2.

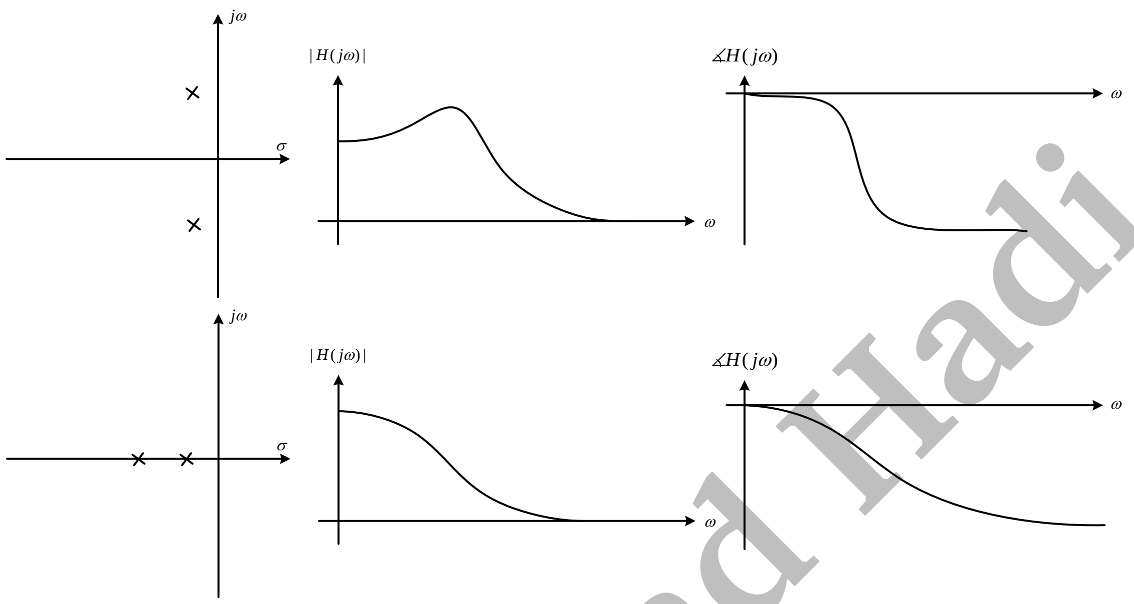

Figure 2: Zero-pole diagram and the corresponding frequency response for the Sallen active lowpass filter.

Question 2

The circuit shown in Fig. 3 is called Sallen active highpass filter, where the triangle abstracts an op-amp amplification circuit with the gain K.

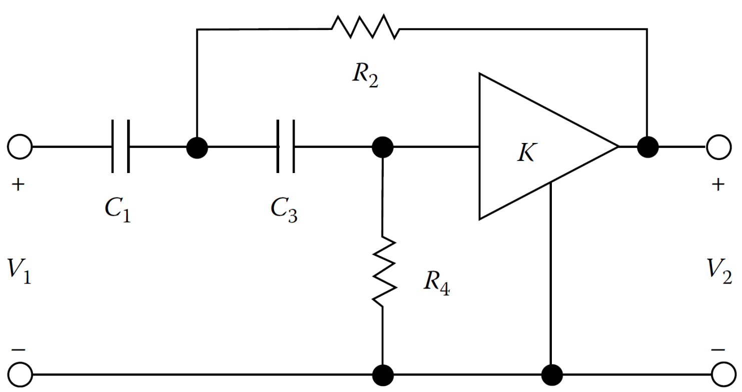

Figure 3: Sallen active highpass filter.

Question 2 continued on next page. . .

(a) *Find the transfer function of the circuit.*

The input voltage of the amplifier is V_2/K. Writing a KCL for the node connecting R_2, C_1, and C_3,

$$(V - V_1)sC_1 + (V - \frac{V_2}{K})sC_3 + \frac{V - V_2}{R_2} = 0$$

Further,

$$V_2 = K\frac{R_4}{1/C_3 s + R_4}V$$

Simplifying the equations,

$$H(s) = \frac{V_2(s)}{V_1(s)} = \frac{Ks^2}{s^2 + s(\frac{1}{C_3R_4} + \frac{1}{C_1R_4} + \frac{1}{C_1R_2} - \frac{K}{C_1R_2}) + \frac{1}{C_1C_3R_2R_4}}$$

(b) *Find the frequency response of the circuit.*

We know that $H(j\omega) = H(s)|_{s=j\omega}$. Thus,

$$H(j\omega) = \frac{-K\omega^2}{\frac{1}{C_1C_3R_2R_4} - \omega^2 + j\omega(\frac{1}{C_3R_4} + \frac{1}{C_1R_4} + \frac{1}{C_1R_2} - \frac{K}{C_1R_2})}$$

(c) *Draw the approximated frequency response using the corresponding zero-pole diagram.*

Various situations for the zero and pole locations and the corresponding frequency responses are plotted in Fig. 4.

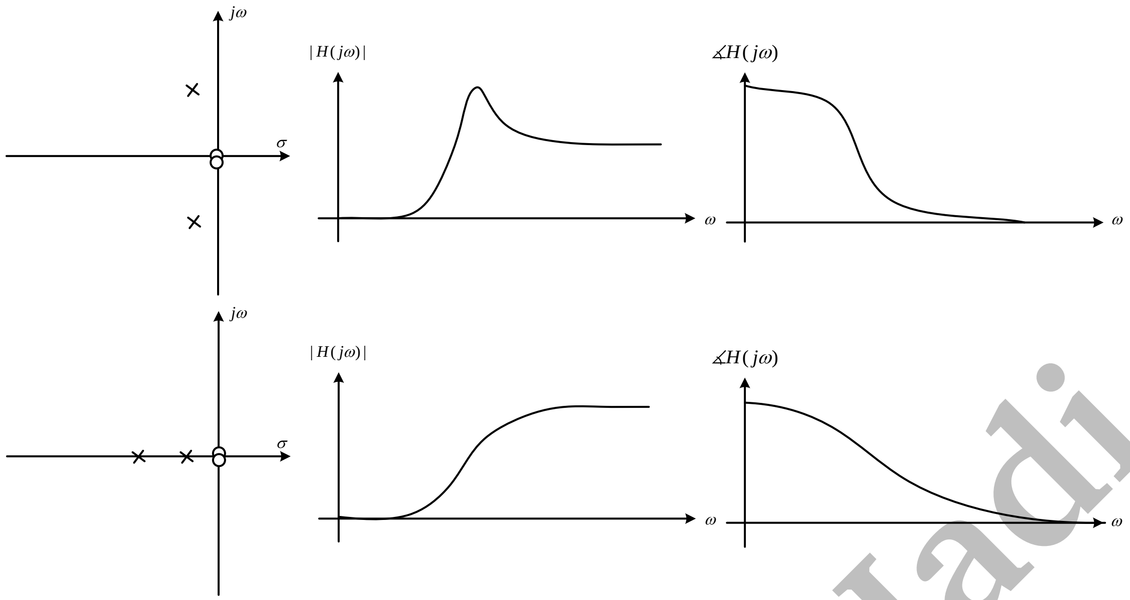

Figure 4: Zero-pole diagram and the corresponding frequency response for the Sallen active highpass filter.

Question 3

Consider the lattice network shown in Fig. 5, where Z and Z' are horizontal and diagonal impedances.

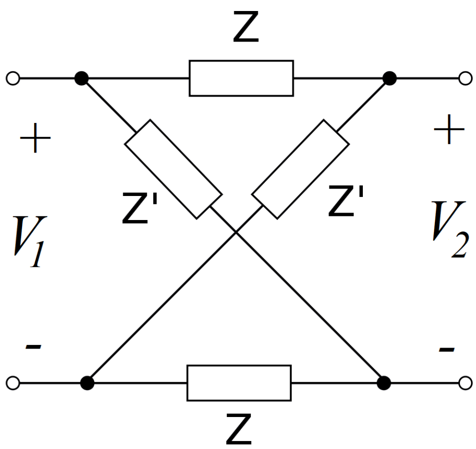

Figure 5: Lattice network.

Question 3 continued on next page...

(a) Find the transfer function $H(s) = \frac{V_2(s)}{V_1(s)}$ of the circuit.

$$V_2 = \frac{Z'}{Z+Z'}V_1 - \frac{Z}{Z+Z'}V_1 \Rightarrow H(s) = \frac{V_2(s)}{V_1(s)} = \frac{Z'(s) - Z(s)}{Z'(s) + Z(s)}$$

(b) Find the frequency response $H(j\omega) = \frac{V_2(j\omega)}{V_1(j\omega)}$ of the circuit.

$$H(j\omega) = H(s)|_{s=j\omega} = \frac{Z'(j\omega) - Z(j\omega)}{Z'(j\omega) + Z(j\omega)}$$

(c) Assume that Z is a resistor of $R\ \Omega$ and Z' is an inductor of L H. Draw the approximated frequency response of the circuit. Can you interpret the filtering response of the circuit?

The corresponding transfer function $H(s) = \frac{sL-R}{sL+R}$ has a zero at $z = \frac{R}{L}$ and a pole at $p = -\frac{R}{L}$. The zero-pole diagram and the corresponding frequency response are plotted in Fig. 6. According to the frequency response, the circuit is an all-pass filter. Note that although the magnitude response is constant, the phase response can still be used for phase compensation applications.

Figure 6: Zero-pole diagram and the corresponding frequency response for the first-order all-pass filter.

Question 4

For the LC ladder network shown in Fig. 7,

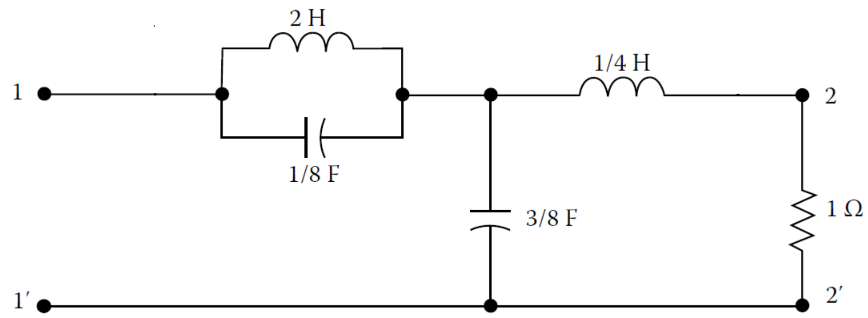

Figure 7: LC ladder network.

(a) *Find the transfer function* $H(s) = \frac{V_{22'}(s)}{V_{11'}(s)}$ *of the circuit.*

Using voltage division rule,

$$V_2(s) = \frac{(\frac{1}{4}s+1)||\frac{8}{3s}}{(\frac{1}{4}s+1)||\frac{8}{3s}+2||\frac{8}{s}}\frac{1}{1+\frac{s}{4}}V_1(s)$$

, which leads to

$$H(s) = \frac{V_2(s)}{V_1(s)} = \frac{s^2+4}{s^3+4s^2+9s+4}$$

(b) *Find the frequency response* $H(j\omega) = \frac{V_{22'}(j\omega)}{V_{11'}(j\omega)}$ *of the circuit.*

$$H(jw) = H(s)\big|_{s=j\omega} = \frac{s^2+4}{s^3+4s^2+9s+4}\bigg|_{s=j\omega} = \frac{-\omega^2+4}{-j\omega^2-4\omega^2+9j\omega+4}$$

(c) *Plot the Bode diagram of the circuit.*

The bode diagram is drawn in Fig. 8.

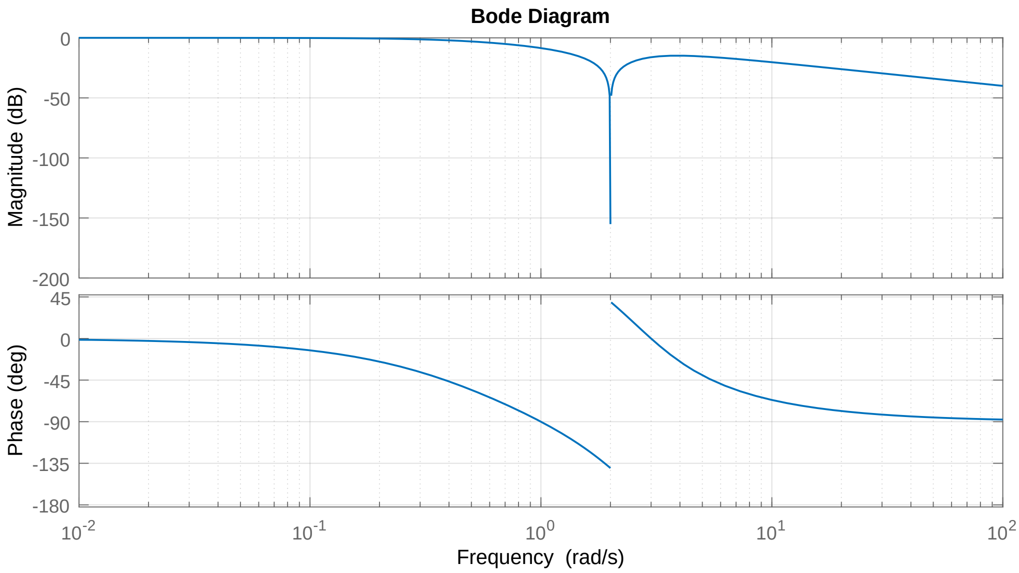

Figure 8: Bode Diagram of the filter of Fig. 7.

SOFTWARE QUESTIONS

Question 5

The circuit shown in Fig. 9 is called biquad active filter. The triangles denote amplifiers with the gains -1 and 2. The amplifiers may be implemented using inverting and non-inverting op-amp circuits. The admittances Y_1, Y_2, Y_3 and Y_4 can be replaced by series or parallel RC circuits. A sample customized configuration is shown in Fig. 10. Depending on the configuration, the circuit provides various filtering responses. Simulate the circuit in PSpice and investigate the filtering response of the circuit for various configurations of Y_1, Y_2, Y_3 and Y_4.

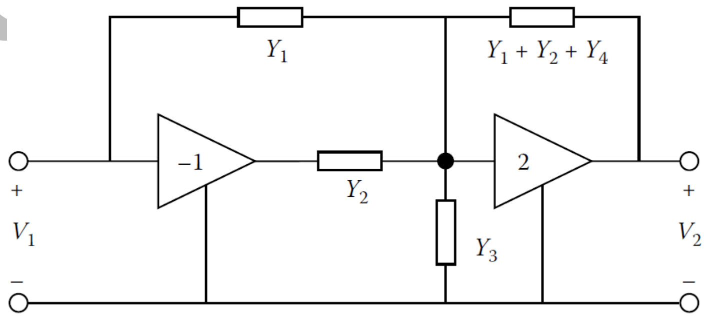

Figure 9: Biquad active filter.

Question 5 continued on next page...

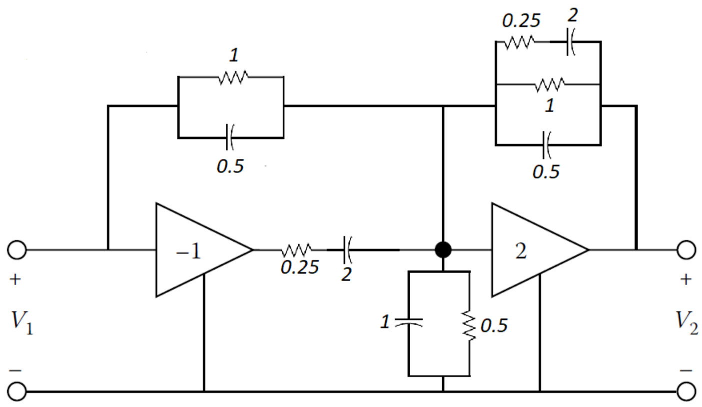

Figure 10: A sample customized realization of the biquad active filter.

It can be shown that the transfer function of the biquad active filter is $H(s) = 2\frac{Y_1-Y_2}{Y_3-Y_4}$. It is the task of filter designer to choose proper configurations for Y_1, Y_2, Y_3 and Y_4 to provide a desired filtering response. Figs. 11-14 show different configurations for sample implementation of LPF, HPF, BPF, and BSF.

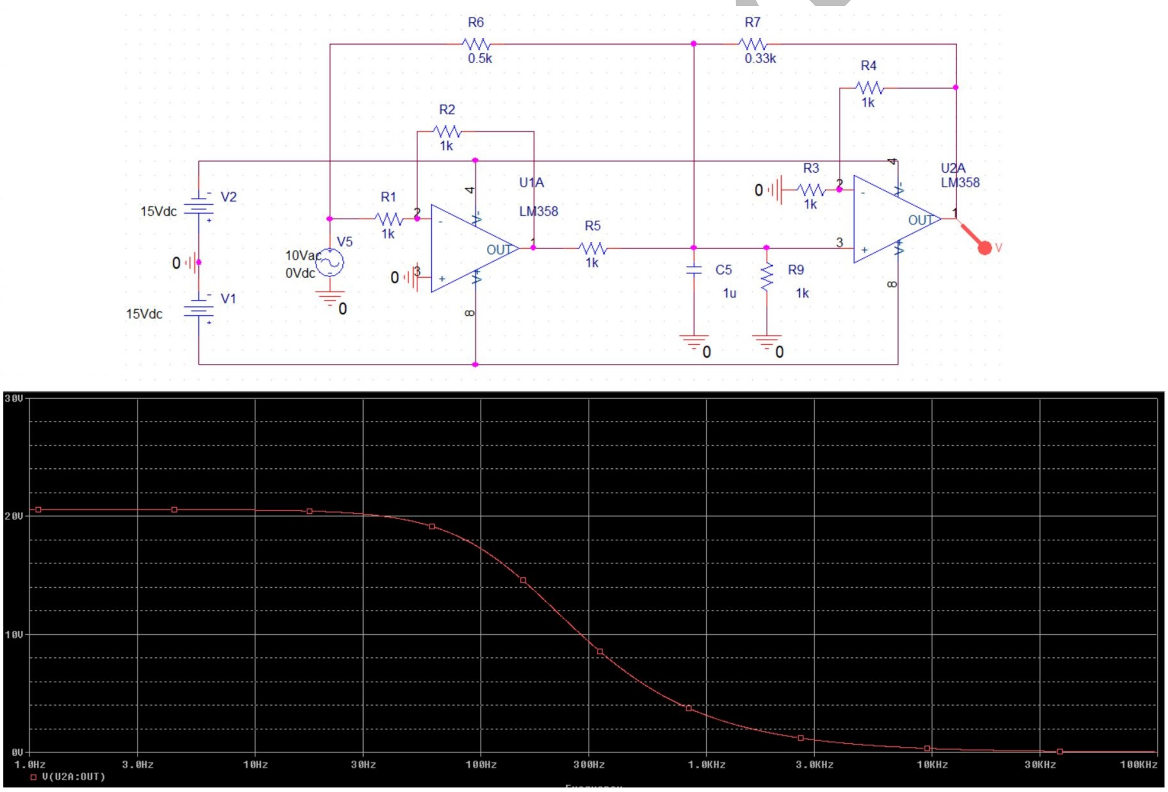

Figure 11: Sample configuration of the biquad filter for implementing a LPF.

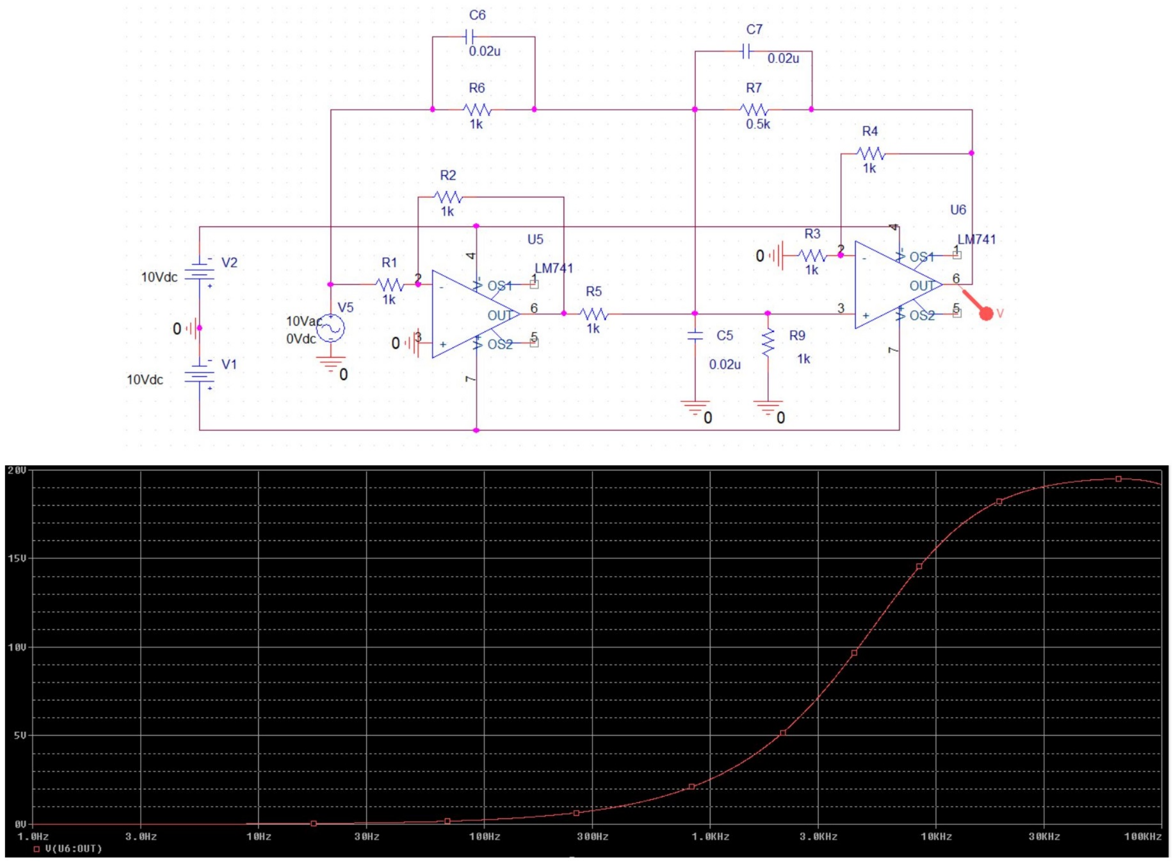

Figure 12: Sample configuration of the biquad filter for implementing a HPF.

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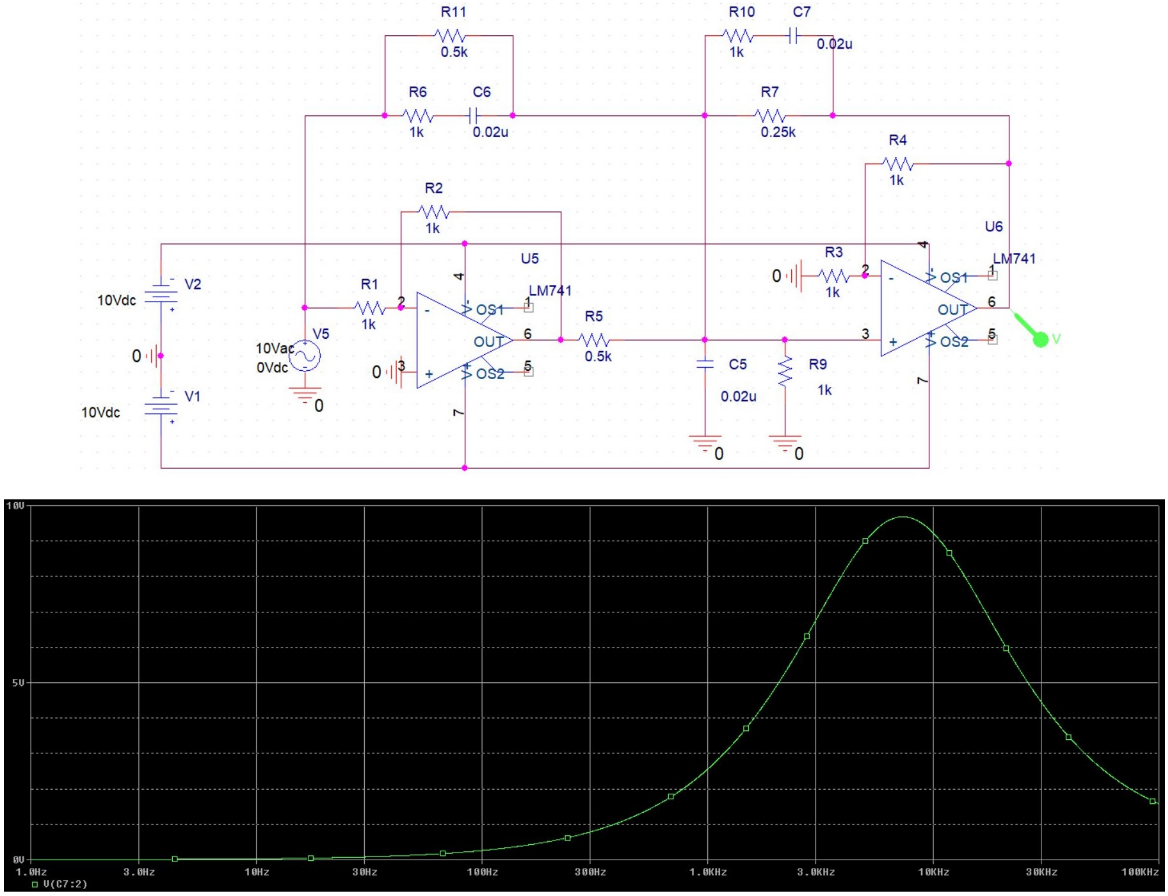

Figure 13: Sample configuration of the biquad filter for implementing a BPF.

Question 5 continued on next page...

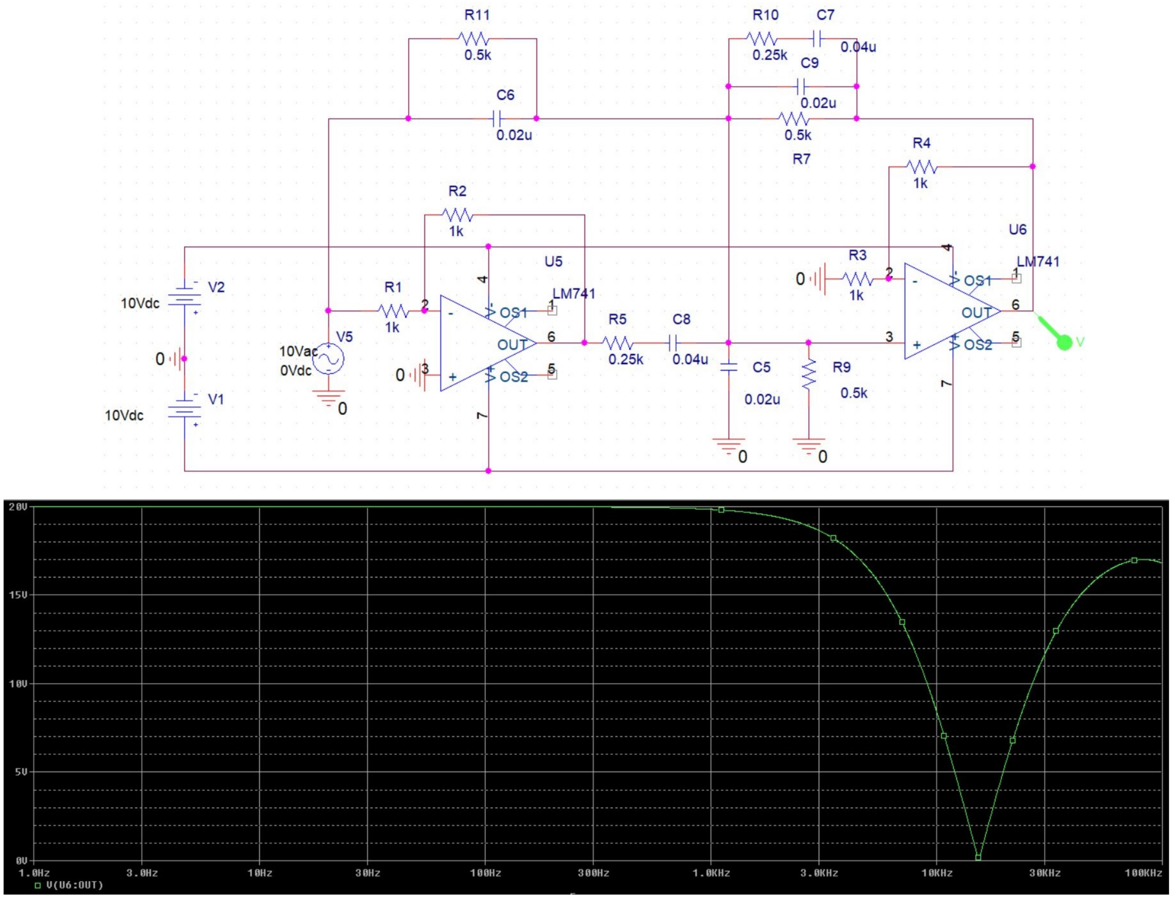

Figure 14: Sample configuration of the biquad filter for implementing a BSF.

BONUS QUESTIONS

Question 6

Return your answers by filling the LaTeX template of the assignment. If you want to add a circuit schematic, you can draw it directly using TikZ package, or draw it in a secondary application such as Microsoft Visio and then, import it as a figure.

EXTRA QUESTIONS

Question 7 continued on next page...

Question 7

Feel free to solve the following questions from the book *"Basic Circuit Theory"* by C. Desoer and E. Kuh.

1. **Chapter 15, question 4.**
2. **Chapter 15, question 5.**
3. **Chapter 15, question 6.**
4. **Chapter 15, question 7.**
5. **Chapter 15, question 10.**