MATHEMATICAL QUESTIONS

Question 1

Show that the circuits shown in Fig. 1 are degenerate, i.e., they may have no or several solutions.

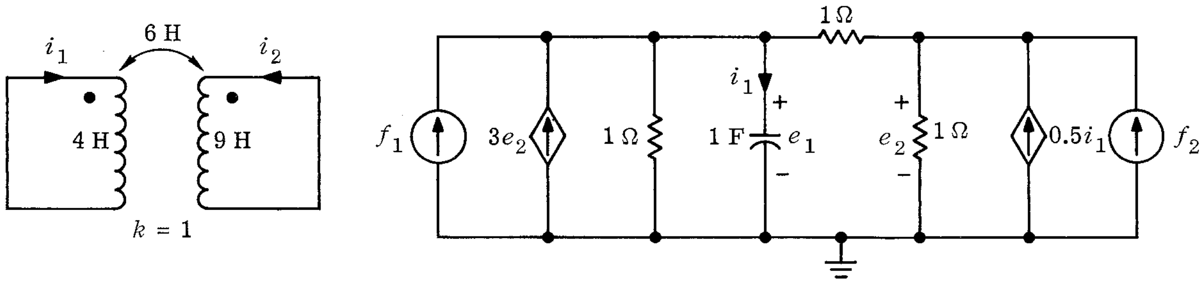

Figure 1: Degenerate LTI circuits.

Assuming $i_1(0) = i_2(0) = 0$ in the left circuit, the mesh equations give

$$4\frac{di_1}{dt} + 6\frac{di_2}{dt} = 0 \xrightarrow{\text{divide by 2}} 2\frac{di_1}{dt} + 3\frac{di_2}{dt} = 0$$

$$6\frac{di_1}{dt} + 9\frac{di_2}{dt} = 0 \xrightarrow{\text{divide by 3}} 2\frac{di_1}{dt} + 3\frac{di_2}{dt} = 0$$

, which are the same equations. You can also check that the inductance matrix L is singular(that is, det$[L] = 0$). This is a direct consequence of the fact that the inductors are ideally coupled. If $f(t)$ is any differentiable function with $f(0) = 0$, then

$$i_1(t) = -3f(t), \quad i_2(t) = 2f(t)$$

constitute a solution which satisfies the initial conditions. So, the circuit has many acceptable solutions.

Now, consider the right Circuit. This example exhibits the kind of difficulty which may occur when a network includes both RLC elements and dependent sources, and when the element values bear special relationships to one another. The node equations for this circuit have the form

$$\frac{d}{dt}e_1 + 2e_1 - 4e_2 = f_1$$

$$\frac{-1}{2}\frac{d}{dt}e_1 - e_1 + 2e_2 = f_2 \xrightarrow{\text{multiply by -2}} \frac{d}{dt}e_1 + 2e_1 - 4e_2 = -2f_2$$

Consequently, this system will have a solution if and only if $f_1(t) = -2f_2(t)$. In other words, unless the independent current sources have their waveforms $f_1(t)$ and $f_2(t)$ related by $f_1(t) = -2f_2(t)$, it is impossible to find branch voltages and branch currents that satisfy

Question 1 continued on next page. . .

the branch equations and Kirchhoff's laws. Now, suppose that $f_1(t) = f_2(t) = 0$ and $e_1(0) = 0$. Note that $f_1(t) = f_2(t) = 0$ satisfies the condition $f_1(t) = -2f_2(t)$. If $g(t)$ is any differentiable function satisfying $g(0) = 0$, then

$$e_1(t) = 4g(t), \quad e_2(t) = g'(t) + 2g(t)$$

constitute a solution. In this case, the circuit has an infinite number of solutions.

Question 2

The results of two measurement scenarios for the reciprocal circuit of Fig. 2 are

$$\begin{cases} v_1(t) = (-6e^{-t} + 14e^{-2t})u(t) \\ v_2(t) = 0 \\ v_3(t) = (-6e^{-t} + 12e^{-2t})u(t) \\ i_1(t) = \delta(t) \\ i_2(t) = -2e^{-2t}u(t) \\ i_3(t) = 0 \end{cases}, \quad \begin{cases} \hat{v}_1(t) = ? \\ \hat{v}_2(t) = 24u(t) \\ \hat{v}_3(t) = (-12e^{-t} + 24e^{-2t})u(t) \\ \hat{i}_1(t) = 0 \\ \hat{i}_2(t) = 24e^{-2t}u(t) \\ \hat{i}_3(t) = 2\delta(t) \end{cases}$$

. Find $\hat{v}_1(t)$ in the second measurement scenario.

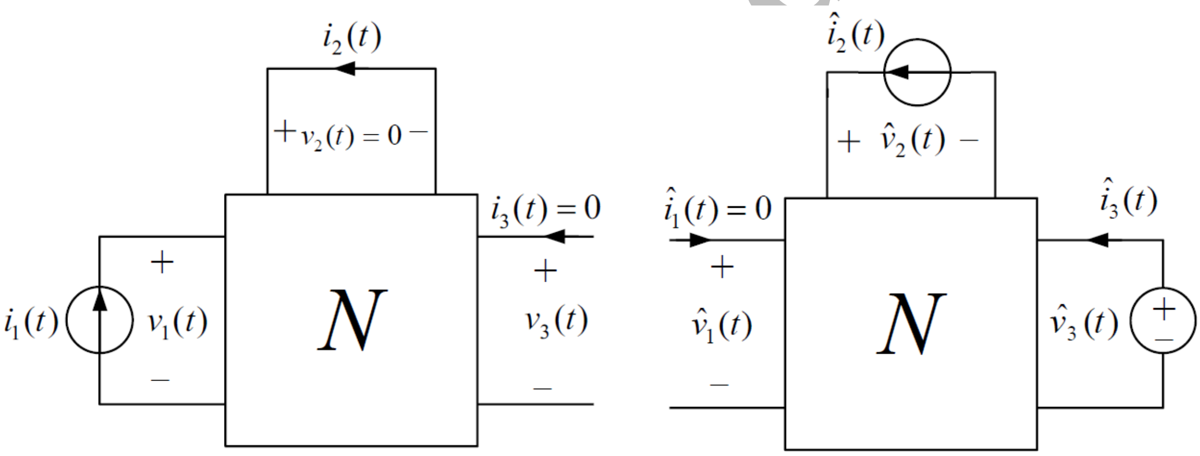

Figure 2: Two-measurement experiment for a reciprocal circuit.

We can calculate the Laplace transform of the measured currents and voltages as

$$V_1(s) = \frac{-6}{s+1} + \frac{14}{s+2}, \quad V_2(s) = 0, \quad V_3(s) = \frac{-6}{s+1} + \frac{12}{s+2}$$

$$I_1(s) = 1, \quad I_2(s) = \frac{-2}{s+2}, \quad I_3(s) = 0$$

$$\hat{V}_2(s) = \frac{24}{s}, \quad \hat{V}_3(s) = \frac{-12}{s+1} + \frac{24}{s+2}$$

Question 2 continued on next page...

$$\hat{I}_1(s) = 0, \quad \hat{I}_2(s) = \frac{24}{s+2}, \quad \hat{I}_3(s) = 2$$

We have from Tellegen's theorem that

$$V_1(s)\hat{I}_1(s) + V_2(s)\hat{I}_2(s) + V_3(s)\hat{I}_3(s) = \hat{V}_1(s)I_1(s) + \hat{V}_2(s)I_2(s) + \hat{V}_3(s)I_3(s)$$

So,

$$0 + 0 + 2(\frac{-6}{s+1} + \frac{12}{s+2}) = \hat{V}_1(s) + \frac{24}{s}\frac{-2}{s+2} + 0 \Rightarrow \hat{V}_1(s) = \frac{24}{s} - \frac{12}{s+1}$$

Or in the time domain,

$$\hat{v}_1(t) = 24u(t) - 12e^{-t}u(t)$$

Question 3

Verify that if the superposition theorem holds for the voltage response v at the nonlinear circuit of Fig. 3 or not.

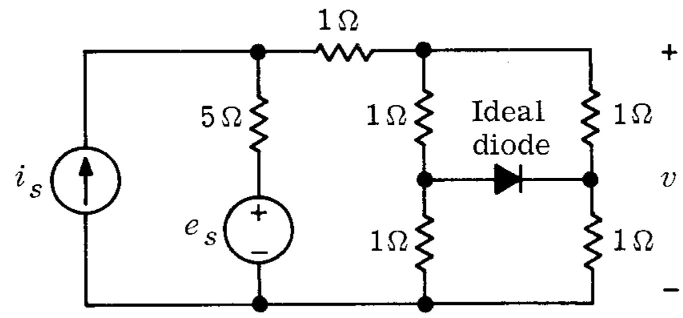

Figure 3: A balanced bridge nonlinear circuit.

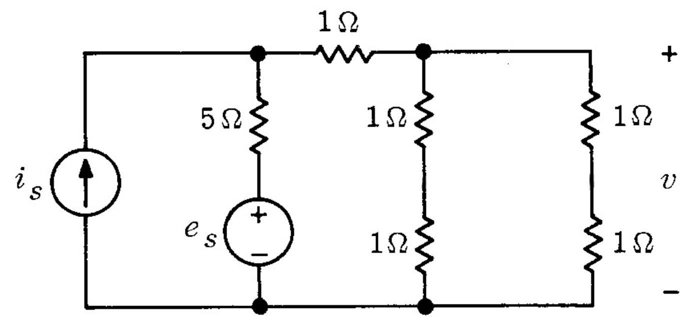

Figure 4: Equivalent circuit for the circuit of Fig. 3.

Question 3 continued on next page…

If we find the Thevenin equivalent circuit seen from the diode,

$$v = \frac{(1+1)||(1+1)}{(1+1)||(1+1)+5+1}(e_s+5i_s) = \frac{e_s+5i_s}{7}, \quad v_{doc} = \frac{1}{1+1}v - \frac{1}{1+1}v = 0$$

Clearly, in the Thevenin equivalent circuit, no current flows through the diode since the driving open circuit voltage is zero. Therefore, the diode is off and we can remove it to get the equivalent circuit shown in Fig. 4. Clearly, the equivalent circuit is an LTI resistive network for which the superposition holds. The circuit shown in Fig. 3 is called balanced Wheatstone bridge.

Question 4

The small-signal model of the transistor amplifier of Fig. 5 is drawn.

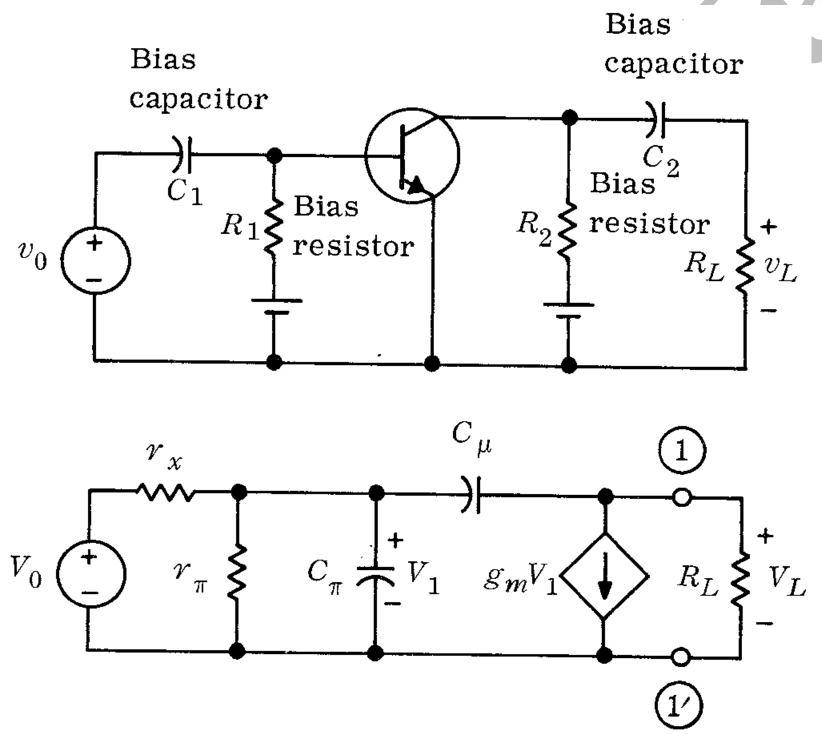

Figure 5: A simple transistor amplifier and its small-signal equivalent circuit.

(a) Find the Thevenin and Norton equivalent circuits seen from port $1-1'$ of the small-signal model.

For computing the E_{oc}, we use node analysis.

$$(E_{oc} - V_1)C_\mu s + g_m V_1 = 0$$

$$\frac{V_1 - V_0}{r_x} + \frac{V_1}{r_\pi} + V_1 C_\pi s + (V_1 - E_{oc})C_\mu s = 0$$

Question 4 continued on next page...

$$E_{oc} = \frac{(r_\pi C_\mu s - r_\pi g_m)V_0}{C_\mu s(C_\pi r_x r_\pi s - g_m r_x r_\pi + r_x + r_\pi)}$$

Let $\frac{1}{r_\pi} + \frac{1}{r_x} = g_t$. Then,

$$E_{oc} = \frac{\frac{(sC_\mu - g_m)V_0}{r_x}}{s^2 C_\mu C_\pi + sC_\mu(g_t + g_m)}$$

For computing the I_{sc}, we use node analysis.

$$\frac{V_1 - V_0}{r_x} + \frac{V_1}{r_\pi} + V_1 C_\pi s + V_1 C_\mu s = 0$$

$$I_{sc} = (C_\mu s - g_m)V_1$$

which gives

$$I_{sc} = \frac{\frac{(sC_\mu - g_m)V_0}{r_x}}{s(C_\pi + C_\mu) + g_t}$$

For computing Z_{Th}, we know that $Z_{Th} = \frac{E_{oc}}{I_{sc}}$. Thus,

$$Z_{Th} = \frac{E_{oc}}{I_{sc}} = \frac{s(C_\pi + C_\mu) + g_t}{s^2 C_\pi C_\mu + sC_\mu(g_t + g_m)}$$

(b) Find the voltage gain $H(s) = \frac{V_L(s)}{V_0(s)}$.

Using the voltage division rule in the Thevenin equivalent circuit,

$$V_L = E_{oc}\frac{R_L}{Z_{Th} + R_L}$$

$$H(s) = \frac{V_L}{V_0} = \frac{V_L}{E_{oc}}\frac{E_{oc}}{V_0} = \frac{\frac{sC_\mu - g_m}{r_x}}{s^2 C_\mu C_\pi R_L + s(C\pi + C\mu + C_\mu g_t R_L + C_\mu g_m R_L) + g_t} R_L$$

SOFTWARE QUESTIONS

Question 5

An audio file can be considered as the output voltage of a microphone versus time. Develop a MATLAB function that receives an audio file, passes it through the RC lowpass filter $H(j\omega) = \frac{1}{1+j\frac{\omega}{\omega_c}}, \omega_c = \frac{1}{RC}$, and generates a filtered audio file. Listen to the filtered output for different values of ω_c and examine the filtering impact on the quality of the filtered audio.

A sample implementation can be as

```
clear; clc; close all;

% read the input audio file
[x,Fs] = audioread('input_audio.wav');

% cut-off frequency
Fc = 1000;

% take the Fourier transform
L = length(x);
t = 0:1/Fs:L/Fs-1/Fs;
f = Fs .* (-L/2:L/2-1) ./ L;
X = fftshift(fft(x))/L;

% apply the filter
Y = LPF(X,f,Fc);

% take the inverse Fourier transform
y = ifft(Y*L);

% save the filted audioe file
audiowrite('filtered_audio.wav',abs(y),Fs);

% filtet implementation
function [Y,fo] = LPF(X, fi, fc)
F = 1./(1+fi'/fc*1i);
Y = repmat(F,[1 size(X,2)]) .* X;
fo = fi;
end
```

Feel free to run the mfile for various wave files and cut-off frequencies.

BONUS QUESTIONS

Question 6

Return your answers by filling the LaTeX template of the assignment. If you want to add a circuit schematic, you can draw it directly using TikZ package, or draw it in a secondary application such as Microsoft Visio and then, import it as a figure.

EXTRA QUESTIONS

Question 7

Feel free to solve the following questions from the book *"Basic Circuit Theory"* by C. Desoer and E. Kuh.

Question 7 continued on next page. . .

1. **Chapter 16, question 3.**
2. **Chapter 16, question 4.**
3. **Chapter 16, question 5.**
4. **Chapter 16, question 8.**
5. **Chapter 16, question 16.**
6. **Chapter 16, question 17.**
7. **Chapter 16, question 18.**